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Permanence and global stability of a May cooperative system with strong and weak cooperative partners

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Abstract

In this paper, a May cooperative system with strong and weak cooperative partners is proposed. First, by using differential inequality theory, we obtain the permanence and non-permanence of the system. Second, we discuss the existence of the positive equilibrium point and boundary equilibrium point, after that, by constructing suitable Lyapunov functions, it is shown that the equilibrium points are globally asymptotically stable in the positive octant. Finally, examples together with their numerical simulations show the feasibility of the main results.

MSC: 34C05; 34C25

Keywords: Cooperative system; Strong; Weak; Global stability

1 Introduction

Cooperative system is an important system in the field of biology, and the importance of the system is the same as for prey–predator and competitive systems. Many scholars have done research on the cooperative ecosystem (see [1–12]). May [1] described a cooperative system with the following equations:

$$\begin{aligned} \frac{dx_1}{dt} &= r_1x_1\left(1-\frac{x_1}{a_1+b_1x_2}-c_1x_1\right),\\ \frac{dx_2}{dt} &= r_2x_2\left(1-\frac{x_2}{a_2+b_2x_1}-c_2x_2\right), \end{aligned} \tag{1.1}$$

where x_1, x_2 are the densities of the species x_1, x_2 at time t, respectively, r_i refers to the intrinsic rate of population x_i, $i = 1,2$, and b_i, $i = 1,2$, refers to the coefficients of cooperation, r_i, a_i, b_i, c_i, $i = 1,2$ are positive constants. His research shows that the cooperative system has a unique positive equilibrium point and it is globally asymptotically stable.

Cui and Chen [2] think that a non-autonomous form is more reasonable. They put forward the following cooperation system:

$$\begin{aligned} \frac{dx_1}{dt} &= r_1(t)x_1\left(1-\frac{x_1}{a_1(t)+b_1(t)x_2}-c_1(t)x_1\right),\\ \frac{dx_2}{dt} &= r_2(t)x_2\left(1-\frac{x_2}{a_2(t)+b_2(t)x_1}-c_2(t)x_2\right), \end{aligned} \tag{1.2}$$

where the function $r_i(t)$, $a_i(t)$, $b_i(t)$, $c_i(t)$, $i = 1, 2$ are continuous functions and bounded above and below by positive constants. Under the premise of $r_i(t)$, $a_i(t)$, $b_i(t)$, $c_i(t)$, $i = 1, 2$ are periodic function, they get the sufficient conditions which guarantee the global asymptotic stability of positive periodic solutions of this system.

In view of the influence of time delay, species interactions and feedback, Chen, Liao and Huang [3] proposed the following n-species cooperation system:

$$\begin{aligned} \frac{dx_i(t)}{dt} &= r_i(t)x_i(t)\left[1 - \frac{x_i(t)}{a_i(t) + \sum_{j=1,j\neq i}^{n} b_{ij}(t)\int_{-T_{ij}}^{0} K_{ij}(s)x_j(t+s)\,ds} - c_i(t)x_i(t)\right] \\ &\quad - d_i(t)u_i(t)x_i(t) - e_i(t)x_i(t)\int_{-\tau_i}^{0} H_i(s)u_i(t+s)\,ds, \\ \frac{du_i(t)}{dt} &= -\alpha_i(t)u_i(t) + \beta_i(t)x_i(t) + r_i(t)\int_{-\eta_i}^{0} G_i(s)x_i(t+s)\,ds, \end{aligned} \tag{1.3}$$

where $x_i(t)$, $i = 1, \dots, n$ is the density of cooperation species X_i, $u_i(t)$, $i = 1, \dots, n$, is the feedback control variable. The authors obtained the sufficient conditions which guarantee the permanence by using differential inequality theory. For more work as regards the system, we can refer to [4–6].

In the real world, individual organisms are associated with a strong and weak differential. Mohammadi [13] proposed a Leslie–Gower predator–prey model:

$$\begin{aligned} \frac{dH_1}{dt} &= (r_1 - bH_1 - \alpha H_2)H_1, \\ \frac{dH_2}{dt} &= (\alpha H_1 - c_1 - c_2 P)H_2, \\ \frac{dP}{dt} &= \left(r_2 - \frac{a_2 P}{H_2}\right)P, \end{aligned} \tag{1.4}$$

where r_1, b_1, α, c_1, c_2, r_2, a_2 are positive constants, the predators can distinguish between strong and weak prey and predator eats only weak prey, when a prey becomes weak, it does not become strong again; by constructing a suitable Lyapunov function, it is shown that the unique equilibrium point is stable in the positive octant.

Conversely, in many cooperative ecosystems, partners like strong partners, because the strong partners are more conducive to their survival. This shows that the cooperative object should only be part instead of the whole.

There are two populations:

The partner H, whose total density is H, is divided into two categories H_1, H_2. H_1 denotes the strong partner density and H_2 denotes the weak. Of the other partner, the total density is P.

The May cooperative model (1.1) is our basic model and we consider the following assumptions to improve the model:

(A_1) The partner P can distinguish between strong partner H_1 and weak partner H_2 and the partner P cooperates only with strong partner H_1.

(A_2) When provided with food resources, the weak partner H_2 has no negative influence on the stronger partner, that is to say, the weak partners can only eat the food after the strong ones have used enough.

(A_3) Due to the lack of sufficient food resources, once it becomes weak, the weak partner H_2 and their descendants will no longer be strong.

(A_4) The rate of becoming weak is described by the simple mass action $\alpha H_1 H_2$.

By the above assumptions, we propose a model as follows:

$$\begin{aligned} \frac{dH_1}{dt} &= r_1 H_1\left(1 - \frac{H_1}{a_1 + b_1 P} - c_1 H_1 - \frac{\alpha H_2}{r_1}\right), \\ \frac{dH_2}{dt} &= H_2(\alpha H_1 + d - eH_2), \\ \frac{dP}{dt} &= r_2 P\left(1 - \frac{P}{a_2 + b_2 H_1} - c_2 P\right), \end{aligned} \tag{1.5}$$

where r_i, a_i, b_i, c_i, d, $i = 1,2$ are positive constants.

The structure of this article as follows. In Sect. 2 we will introduce several useful lemmas and prove permanence and non-permanence. In Sect. 3 we will discuss the existence of the equilibrium point. In Sect. 4 global stability of equilibrium points is studied. In Sect. 5 two examples are given to show the feasibility of our results. We end this paper by a brief discussion.

2 Permanence and non-permanence

In view of the actual ecological implications of system (1.5), we assume that the initial value $H_i(0) > 0$, $i = 1,2$, $P(0) > 0$ in system (1.5). Obviously, any solution of system (1.5) remains positive for all $t \geq 0$.

Lemma 2.1 (see [14]) *Let $a > 0$, $b > 0$.*

(I) *If $\frac{dx}{dt} \geq x(b - ax)$, then $\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}$ for $t \geq 0$ and $x(0) > 0$.*

(II) *If $\frac{dx}{dt} \leq x(b - ax)$, then $\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}$ for $t \geq 0$ and $x(0) > 0$.*

Lemma 2.2 (see [15]) *Let $a > 0$, $b > 0$.*

If $\frac{dx}{dt} \leq x(-b - ax)$, then $\lim_{t\to+\infty} x(t) = 0$ for $t \geq 0$ and $x(0) > 0$.

Theorem 2.1 *If the assumptions* (B_1) *and* (B_2) *hold,*

(B_1) $M = 1 - \frac{\alpha d}{r_1 e} > 0$,

(B_2) $1 > \frac{\alpha^2(a_1c_2+b_1)}{r_1e(a_1c_1c_2+b_1c_1+c_2)}$,

then system (1.5) *is permanent.*

Proof Let $(H_1(t), H_2(t), P(t))^T$ be any positive solution of system (1.5), from the second equation of system (1.5), it follows that

$$\frac{dH_2}{dt} \geq H_2(d - eH_2).$$

According to Lemma 2.1, we have

$$\liminf_{t\to+\infty} H_2(t) \geq \frac{d}{e} \overset{\text{def}}{=} H_2^i > 0. \tag{2.1}$$

For any positive constant ε small enough, it follows from (2.1) that there exists a large enough $T_1 > 0$ such that

$$H_2(t) > H_2^i - \varepsilon, \quad t \geq T_1. \tag{2.2}$$

From the third equation, we have

$$\frac{dP}{dt} \leq r_2 P(1 - c_2 P).$$

According to Lemma 2.1, we have

$$\limsup_{t\to+\infty} P(t) \leq \frac{1}{c_2} \stackrel{\text{def}}{=} P^s > 0. \tag{2.3}$$

For any positive constant ε small enough, it follows from (2.3) that there exists a large enough $T_2 > T_1$ such that

$$P(t) \leq P^s + \varepsilon, \quad t \geq T_2. \tag{2.4}$$

By applying (2.2) and (2.4), from the first equation of system (1.5), we have

$$\frac{dH_1}{dt} \leq r_1 H_1 \left(1 - \frac{H_1}{a_1 + b_1(P^s + \varepsilon)} - c_1 H_1 - \frac{\alpha(H_2^i - \varepsilon)}{r_1}\right), \quad t \geq T_2.$$

According to Lemma 2.1, we have

$$\limsup_{t\to+\infty} H_1(t) \leq \left(1 - \frac{\alpha(H_2^i - \varepsilon)}{r_1}\right) \frac{a_1 + b_1(P^s + \varepsilon)}{1 + a_1 c_1 + b_1 c_1 (P^s + \varepsilon)}.$$

Letting $\varepsilon \to 0$ and by applying (2.1) and (2.3)

$$\limsup_{t\to+\infty} H_1(t) \leq \frac{a_1 c_2 + b_1}{a_1 c_1 c_2 + b_1 c_1 + c_2} M \stackrel{\text{def}}{=} H_1^s > 0. \tag{2.5}$$

For any positive constant ε small enough, it follows from (2.5) that there exists a large enough $T_3 > T_2$ such that

$$H_1(t) \leq H_1^s + \varepsilon, \quad t \geq T_3. \tag{2.6}$$

Then the second equation of (1.5) leads to

$$\frac{dH_2}{dt} \leq H_2\big(\alpha\big(H_1^s + \varepsilon\big) + d - eH_2\big), \quad t \geq T_3.$$

According to Lemma 2.1, we have

$$\limsup_{t\to+\infty} H_2(t) \leq \frac{\alpha(H_1^s + \varepsilon) + d}{e}.$$

Letting $\varepsilon \to 0$ in the above inequality leads to

$$\limsup_{t\to+\infty} H_2(t) \le \frac{\alpha H_1^s + d}{e} \stackrel{\text{def}}{=} H_2^s. \tag{2.7}$$

For any positive constant ε small enough, it follows from (2.7) that there exists a large enough $T_4 > T_3$ such that

$$H_2(t) \le H_2^s + \varepsilon, \quad t \ge T_4. \tag{2.8}$$

Then substituting (2.8) into the first equation of (1.5), we have

$$\frac{dH_1}{dt} \ge r_1 H_1 \left(1 - \frac{H_1}{a_1} - c_1 H_1 - \frac{\alpha(H_2^s + \varepsilon)}{r_1}\right), \quad t \ge T_4.$$

According to Lemma 2.1, we have

$$\liminf_{t\to+\infty} H_1(t) \ge \left(1 - \frac{\alpha(H_2^s + \varepsilon)}{r_1}\right) \frac{a_1}{a_1 c_1 + 1}.$$

Letting $\varepsilon \to 0$ and by applying (2.5) and (2.7)

$$\liminf_{t\to+\infty} H_1(t) \ge M - \frac{\alpha^2 H_1^s}{r_1 e} \stackrel{\text{def}}{=} H_1^i > 0. \tag{2.9}$$

From the third equation of system(1.5), it follows that

$$\frac{dP}{dt} \ge r_2 P\left(1 - \frac{P}{a_2} - c_2 P\right).$$

According to Lemma 2.1, we have

$$\liminf_{t\to+\infty} P(t) \ge \frac{a_2}{a_2 c_2 + 1} \stackrel{\text{def}}{=} P^i. \tag{2.10}$$

(2.1), (2.3), (2.5), (2.7), (2.9) and (2.10) show that if the assumptions (B_1), (B_2) hold, then system (1.5) is permanent. □

Theorem 2.2 *If the assumption* (B_3) *holds,*
(B_3) $M = 1 - \frac{\alpha d}{r_1 e} < 0$,
then the weak partners H_2 *and partners* P *are permanent, the strong partners* H_1 *are non-permanent.*

Proof Let $(H_1(t), H_2(t), P)^T$ be any positive solutions of system (1.5) for $t \ge 0$.
From the proof Theorem 2.1, we know

$$\frac{dH_1}{dt} \le r_1 H_1 \left(1 - \frac{H_1}{a_1 + b_1(P^s + \varepsilon)} - c_1 H_1 - \frac{\alpha(H_2^i - \varepsilon)}{r_1}\right), \quad t \ge T_2.$$

Noting that condition (B_3) implies that $1 - \frac{\alpha(H_2^i - \varepsilon)}{r_1} < 0$.

According to Lemma 2.2, we have

$$\lim_{t\to+\infty} H_1(t) = 0. \tag{2.11}$$

By applying (2.11), from the second equation of system (1.5), it is easy to prove that

$$\lim_{t\to+\infty} H_2(t) = \frac{d}{e}. \tag{2.12}$$

(2.3), (2.10), (2.11) and (2.12) show that if the assumptions (B_3) hold, then the weak partners H_2 and partners P are permanent, the strong partners H_1 are non-permanent. □

3 Existence of equilibrium point

Theorem 3.1 *If the assumption* (B_1) *holds, then system* (1.5) *have a unique positive equilibrium point.*

Proof We determine the positive equilibrium of the system (1.5) through solving the following equations:

$$\begin{cases} 1 - \frac{H_1}{a_1+b_1P} - c_1H_1 - \frac{\alpha H_2}{r_1} = 0, \\ \alpha H_1 + d - eH_2 = 0, \\ 1 - \frac{P}{a_2+b_2H_1} - c_2P = 0. \end{cases} \tag{3.1}$$

Here we transform Eqs. (3.1) into the following form:

$$\begin{cases} 1 - \frac{H_1}{a_{11}+b_{11}P} - c_{11}H_1 = 0, \\ 1 - \frac{P}{a_2+b_2H_1} - c_2P = 0, \\ \alpha H_1 + d - eH_2 = 0, \end{cases} \tag{3.2}$$

where $a_{11} = Ma_1, b_{11} = Mb_1, c_{11} = (c_1 + \frac{\alpha d}{r_1 e})/M$, from the first and second equations of (3.2), we have

$$DH_1^2 + EH_1 + F = 0, \tag{3.3}$$

where

$$D = b_2(a_{11}c_{11}c_2 + b_{11}c_{11} + c_2), \qquad F = -a_{11}(a_2c_2 + b_2 + 1),$$
$$E = \big[(a_2c_2 + 1) + c_{11}(a_{11} + a_{11}a_2c_2 + a_2b_{11}) - b_2(a_{11}c_2 + b_{11})\big].$$

From the terms D and F of (3.3), we know that there is a unique positive solutions H_1^*. Substitute H_1^* into the second and third equations of (3.1). Then system (1.5) has a unique positive equilibrium point $E_1(H_1^*, H_2^*, P^*)$. □

Theorem 3.2 *Clearly, the system* (1.5) *has an equilibrium point* $E_2(0, H_{2*}, P_*)$.

4 Global stability

Theorem 4.1 *If the assumptions* (B_1) *and* (B_4) *hold,*

(B_4) $\alpha^2 < \frac{a_2 r_1 c_1 e}{a_2+b_2H_1^s}$,

then the positive equilibrium of system (1.5) *is globally asymptotically stable.*

Proof Inspired by the idea of Li, Han and Chen [7] and Leon [8], the following Lyapunov function is presented:

We define $L : \{(H_1, H_2, P) \in R_+^3 : H_1 > 0, H_2 > 0, P > 0\} \to R$ by

$$L(H_1,H_2,P) = \eta_1 \int_{H_1^*}^{H_1} \frac{(\theta - H_1^*)}{(a_2+b_2\theta)\theta}\,d\theta + \eta_2 \int_{P^*}^{P} \frac{(\theta - P^*)}{(a_1+b_1\theta)\theta}\,d\theta + \eta_3\left(H_2 - H_2^* - H_2^* \ln\frac{H_2}{H_2^*}\right),$$

where $\eta_1 = 1$, $\eta_2 = r_1 b_1 H_1^*(a_2 + b_2H_1^*)/r_2 b_2 P^*(a_1 + b_1P^*)$, $\eta_3 = 1/a_2$. The function $L(H_1, H_2, P)$ is defined, continuous and positive definite for all $H_1, H_2, P > 0$. and the minimum $L(H_1, H_2, P) = 0$ occurs at the equilibrium point $(H_1^*, H_2^*, P^*)^T$. Calculating the derivative of L along the solution $(H_1(t), H_2(t), P(t))^T$ of the system (1.5), we have

$$\begin{aligned}
\frac{dL}{dt} &= \frac{\eta_1 r_1 (H_1 - H_1^*)}{a_2+b_2H_1}\left(1 - \frac{H_1}{a_1+b_1P} - c_1H_1 - \frac{\alpha H_2}{r_1}\right) \\
&\quad + \eta_3\big(H_2 - H_2^*\big)(\alpha H_1 + d - eH_2) + \frac{\eta_2 r_2 (P-P^*)}{a_1+b_1P}\left(1 - \frac{P}{a_2+b_2H_1} - c_2P\right) \\
&= \frac{\eta_1 r_1 (H_1 - H_1^*)}{a_2+b_2H_1}\left(-\frac{\alpha(H_2-H_2^*)}{r_1} - c_1\big(H_1 - H_1^*\big) + \frac{H_1^*}{a_1+b_1P^*} - \frac{H_1}{a_1+b_1P}\right) \\
&\quad + \eta_3\big(H_2 - H_2^*\big)\big(\alpha\big(H_1 - H_1^*\big) - e\big(H_2 - H_2^*\big)\big) \\
&\quad + \frac{\eta_2 r_2 (P-P^*)}{a_1+b_1P}\left(-c_2\big(P-P^*\big) + \frac{P^*}{a_2+b_2H_1^*} - \frac{P}{a_2+b_2H_1}\right) \\
&= \frac{\eta_1 r_1 (H_1 - H_1^*)}{a_2+b_2H_1}\left(-\frac{\alpha(H_2-H_2^*)}{r_1} - c_1\big(H_1 - H_1^*\big) - \frac{(H_1-H_1^*)}{a_1+b_1P}\right. \\
&\quad \left. + \frac{b_1H_1^*(P-P^*)}{(a_1+b_1P)(a_1+b_1P^*)}\right) + \eta_3\big(H_2 - H_2^*\big)\big(\alpha\big(H_1 - H_1^*\big) - e\big(H_2 - H_2^*\big)\big) \\
&\quad + \frac{\eta_2 r_2 (P-P^*)}{a_1+b_1P}\left(-c_2\big(P-P^*\big) - \frac{(P-P^*)}{a_2+b_2H_1} + \frac{b_2P^*(H_1-H_1^*)}{(a_2+b_2H_1^*)(a_2+b_2H_1)}\right) \\
&\le A(t) + B(t),
\end{aligned}$$

where

$$\begin{aligned}
A(t) &= \frac{1}{(a_1+b_1P)(a_2+b_2H_1)}\left(-\eta_1 r_1\big(H_1 - H_1^*\big)^2 - \eta_2 r_2\big(P-P^*\big)^2\right. \\
&\quad \left. + \left(\frac{\eta_1 r_1 b_1 H_1^*}{a_1+b_1P^*} + \frac{\eta_2 r_2 b_2 P^*}{a_2+b_2H_1^*}\right)\big(H_1 - H_1^*\big)\big(P-P^*\big)\right), \\
B(t) &= -\frac{\alpha}{a_2+b_2H_1}\big(H_1 - H_1^*\big)\big(H_2 - H_2^*\big) - \frac{c_1 r_1}{a_2+b_2H_1}\big(H_1 - H_1^*\big)^2 \\
&\quad + \eta_3\alpha\big(H_1 - H_1^*\big)\big(H_2 - H_2^*\big) - \eta_3 e\big(H_2 - H_2^*\big)^2.
\end{aligned}$$

Now, we prove $A(t)$, $B(t)$ are negative definite.

Let $A(t) \stackrel{\text{def}}{=} \frac{1}{(a_1+b_1P)(a_2+b_2H_1)} Y^T A Y$ where $Y = ((H_1 - H_1^*), (P - P^*))^T$ and

$$A = \begin{pmatrix} -\eta_1 r_1 & \frac{\eta_1 r_1 b_1 H_1^*}{2(a_1+b_1P^*)} + \frac{\eta_2 r_2 b_2 P^*}{2(a_2+b_2H_1^*)} \\ \frac{\eta_1 r_1 b_1 H_1^*}{2(a_1+b_1P^*)} + \frac{\eta_2 r_2 b_2 P^*}{2(a_2+b_2H_1^*)} & -\eta_2 r_2 \end{pmatrix}. \tag{4.1}$$

Note first that both of the off-diagonal elements of matrix A are negative and

$$\begin{aligned} &\eta_1\eta_2 r_1 r_2 - \left(\frac{\eta_1 r_1 b_1 H_1^*}{2(a_1+b_1P^*)} + \frac{\eta_2 r_2 b_2 P^*}{2(a_2+b_2H_1^*)}\right)^2 \\ &\quad = \eta_1\eta_2 r_1 r_2\left(1 - \frac{b_1H_1^*}{a_1+b_1P^*}\frac{b_2P^*}{a_2+b_2H_1^*}\right) + \left(\frac{\eta_1 r_1 b_1 H_1^*}{2(a_1+b_1P^*)} - \frac{\eta_2 r_2 b_2 P^*}{2(a_2+b_2H_1^*)}\right)^2 \\ &\quad = \eta_1\eta_2 r_1 r_2\left(1 - \frac{b_1H_1^*}{a_1+b_1P^*}\frac{b_2P^*}{a_2+b_2H_1^*}\right) > 0, \end{aligned}$$

thus $A(t) \le 0$.

Noting that $ab \le \frac{\theta a^2}{2} + \frac{b^2}{2\theta}, \theta > 0$, it follows

$$\begin{aligned} B(t) &\le \frac{\alpha}{a_2+b_2H_1}\left(\frac{1}{2\theta_1}(H_1-H_1^*)^2 + \frac{\theta_1}{2\theta_1}(H_2-H_2^*)^2 - r_1c_1(H_1-H_1^*)^2\right) \\ &\quad + \frac{\eta_3\alpha}{2\theta_2}(H_1-H_1^*)^2 + \frac{\eta_3\alpha\theta_2}{2}(H_2-H_2^*)^2 - \eta_3 e(H_2-H_2^*)^2 \\ &\le -\left(\frac{r_1c_1}{a_2+b_2H_1^s} - \frac{\alpha}{2a_2\theta_1} - \frac{\eta_3\alpha}{2\theta_2}\right)(H_1-H_1^*)^2 \\ &\quad - \left(\eta_3 e - \frac{\eta_3\alpha\theta_2}{2} - \frac{\alpha\theta_1}{2a_2}\right)(H_2-H_2^*)^2. \end{aligned}$$

Denote $\delta_1 = \frac{r_1c_1}{a_2+b_2H_1^s} - \frac{\alpha}{2a_2\theta_1} - \frac{\eta_3\alpha}{2\theta_2}$ and $\delta_2 = \eta_3 c_2 - \frac{\eta_3\alpha\theta_2}{2} - \frac{\alpha\theta_1}{2a_2}$. Then taking

$$\eta_3 = \frac{1}{a_2}, \qquad \theta_1 = \theta_2 = \frac{2\alpha e a_2 + 2\alpha e b_2 H_1^s}{a_2 r_1 c_1 c_2 + a_2\alpha^2 + b_2 H_1^s\alpha^2}$$

gives

$$\delta_1 = \frac{e(a_2r_1c_1e - a_2\alpha^2 - b_2H_1^s\alpha^2)}{a_2(a_2r_1c_1e + a_2\alpha^2 + b_2H_1^s\alpha^2)}, \qquad \delta_2 = \frac{a_2r_1c_1e - a_2\alpha^2 - b_2H_1^s\alpha^2}{2a_2e(a_2+b_2H_1^s)}.$$

From (B_4), we know that $\delta_i > 0$, $i = 1, 2$. It is easy to see that $B(t) \le 0$.

Obvious, $\frac{dL}{dt} < 0$ for all $H_1 > 0$, $H_2 > 0$, $P > 0$ except the equilibrium point (H_1^*, H_2^*, P^*) where $\frac{dL}{dt} = 0$. According to the Lyapunov asymptotic stability theorem [9], the equilibrium point (H_1^*, H_2^*, P^*) is globally asymptotically stable in the interior of R_+^3. This completes the proof. □

Theorem 4.2 *If the assumptions* (B_3) *and* (B_5) *hold,*

(B_5) $\alpha^2 < r_1c_1e$,

then the equilibrium point $E_2(0, H_{2*}, P_*)$ *system is globally asymptotically stable.*

Proof We define $L : \{(H_1, H_2, P) \in R_+^3 : H_1 > 0, H_2 > 0, P > 0\} \to R$ by

$$L(H_1, H_2, P) = \eta_1 \int_0^{H_1} \frac{1}{(a_2 + b_2\theta)}\, d\theta + \eta_2 \int_{P_*}^{P} \frac{(\theta - P_*)}{(a_1 + b_1\theta)\theta}\, d\theta + \eta_3\left(H_2 - H_{2*} - H_{2*}\ln\frac{H_2}{H_{2*}}\right),$$

where $\eta_1 = 1$, $\eta_2 = r_1 a_2^2/r_2 b_2^2 P_*^2$, $\eta_3 = 1/a_2$. The function $L(H_1, H_2, P)$ is defined, continuous and positive definite for all $H_1, H_2, P > 0$. and the minimum $L(H_1, H_2, P) = 0$ occurs at the equilibrium point $(H_1^*, H_2^*, P^*)^T$. Calculating the derivative of L along the solution $(H_1(t), H_2(t), P(t))^T$ of the system (1.5), we have

$$\begin{aligned}
\frac{dL}{dt} &= \frac{\eta_1 r_1 H_1}{a_2 + b_2 H_1}\left(1 - \frac{H_1}{a_1 + b_1 P} - c_1 H_1 - \frac{\alpha H_2}{r_1}\right) + \eta_3(H_2 - H_{2*})(\alpha H_1 + d - eH_2) \\
&\quad + \frac{\eta_2 r_2 (P - P_*)}{a_1 + b_1 P}\left(1 - \frac{P}{a_2 + b_2 H_1} - c_2 P\right) \\
&= \frac{\eta_1 r_1 H_1}{a_2 + b_2 H_1}\left(-\frac{\alpha(H_2 - H_{2*})}{r_1} - c_1 H_1 - \frac{H_1}{a_1 + b_1 P}\right) \\
&\quad + \eta_3(H_2 - H_{2*})\bigl(\alpha H_1 - e(H_2 - H_{2*})\bigr) \\
&\quad + \frac{\eta_2 r_2 (P - P_*)}{a_1 + b_1 P}\left(-c_2(P - P_*) + \frac{P_*}{a_2} - \frac{P}{a_2 + b_2 H_1}\right) \\
&= \frac{\eta_1 r_1 H_1}{a_2 + b_2 H_1}\left(-\frac{\alpha(H_2 - H_{2*})}{r_1} - c_1 H_1 - \frac{H_1}{a_1 + b_1 P}\right) \\
&\quad + \eta_3(H_2 - H_{2*})\bigl(\alpha H_1 - e(H_2 - H_{2*})\bigr) \\
&\quad + \frac{\eta_2 r_2 (P - P_*)}{a_1 + b_1 P}\left(-c_2(P - P_*) - \frac{(P - P_*)}{a_2 + b_2 H_1} + \frac{b_2 P_* H_1}{a_2(a_2 + b_2 H_1)}\right) \\
&\le C(t) + D(t),
\end{aligned}$$

where

$$\begin{aligned}
C(t) &= \frac{1}{(a_1 + b_1 P)(a_2 + b_2 H_1)}\left(-\eta_1 r_1 H_1^2 - \eta_2 r_2 (P - P_*)^2 + \frac{\eta_2 r_2 b_2 P_*}{a_2} H_1 (P - P_*)\right), \\
D(t) &= \left(\eta_3 \alpha - \frac{\alpha}{a_2 + b_2 H_1}\right) H_1 (H_2 - H_{2*}) - \frac{c_1 r_1}{a_2 + b_2 H_1} H_1^2 - \eta_3 e (H_2 - H_{2*})^2.
\end{aligned}$$

Now, we prove $C(t)$, $D(t)$ is negative definite.

Let $C(t) \overset{\text{def}}{=} \frac{1}{(a_1+b_1P)(a_2+b_2H_1)} Y^T C Y$ where $Y = ((H_1), (P - P_*))^T$

$$C = \begin{pmatrix} -\eta_1 r_1 & \frac{\eta_2 r_2 b_2 P^*}{2a_2} \\ \frac{\eta_2 r_2 b_2 P^*}{2a_2} & -\eta_2 r_2 \end{pmatrix}. \tag{4.2}$$

Note first that both of the off-diagonal elements of matrix A are negative and

$$\eta_1 \eta_2 r_1 r_2 - \left(\frac{\eta_2 r_2 b_2 P^*}{2a_2}\right)^2 = \eta_2 r_2 \left(\eta_1 r_1 - \frac{\eta_2 r_2 b_2^2 P_*^2}{4a_2^2}\right) = \frac{3\eta_2 r_1 r_2}{4} > 0,$$

thus $C(t) \le 0$.

Noting that $ab \le \frac{\theta a^2}{2} + \frac{b^2}{2\theta}$, $\theta > 0$, from (2.11), for sufficiently small constant $\varepsilon_0 > 0$, there is an integer $T > 0$ such that if $t > T$, $b_2 H_1(t) < \varepsilon_0$, it follows that

$$\begin{aligned} B(t) &\le \frac{\alpha}{a_2 + b_2 H_1}\left(\frac{1}{2\theta_1}H_1^2 + \frac{\theta_1}{2}(H_2 - H_{2*})^2 - r_1 c_1 H_1^2\right) \\ &\quad + \frac{\eta_3\alpha}{2\theta_2}H_1^2 + \frac{\eta_3\alpha\theta_2}{2}(H_2 - H_{2*})^2 - \eta_3 e (H_2 - H_{2*})^2 \\ &\le -\left(\frac{r_1 c_1}{a_2 + \varepsilon_0} - \frac{\alpha}{2a_2\theta_1} - \frac{\eta_3\alpha}{2\theta_2}\right)H_1^2 \\ &\quad - \left(\eta_3 e - \frac{\eta_3\alpha\theta_2}{2} - \frac{\alpha\theta_1}{2a_2}\right)\left(H_2 - H_2^*\right)^2. \end{aligned}$$

Denote $\delta_1 = \frac{r_1 c_1}{a_2+\varepsilon_0} - \frac{\alpha}{2a_2\theta_1} - \frac{\eta_3\alpha}{2\theta_2}$ and $\delta_2 = \eta_3 c_2 - \frac{\eta_3\alpha\theta_2}{2} - \frac{\alpha\theta_1}{2a_2}$. Then taking

$$\eta_3 = \frac{1}{a_2}, \qquad \theta_1 = \theta_2 = \frac{2\alpha e a_2 + 2\alpha e \varepsilon_0}{a_2 r_1 c_1 c_2 + a_2 + \alpha^2\varepsilon_0},$$

gives

$$\delta_1 = \frac{e(a_2 r_1 c_1 e - a_2\alpha^2 - \alpha^2\varepsilon_0)}{a_2(a_2 r_1 c_1 e + a_2\alpha^2 + \alpha^2\varepsilon_0)}, \qquad \delta_2 = \frac{a_2 r_1 c_1 e - a_2\alpha^2 - \alpha^2\varepsilon_0}{2a_2 e(a_2 + \varepsilon_0)}.$$

From (B_5), we know that, for sufficiently small constant $\varepsilon_0 > 0$, $\delta_i > 0$, $i = 1, 2$. It is easy to see that $D(t) \le 0$.

Obviously, $\frac{dL}{dt} < 0$ for all $H_1 > 0$, $H_2 > 0$, $P > 0$ except the equilibrium point $(0, H_{2*}, P_*)$ where $\frac{dL}{dt} = 0$. According to the Lyapunov asymptotic stability theorem [9], the equilibrium point $(0, H_{2*}, P_*)$ is globally asymptotically stable in the interior of R_+^3. This completes the proof. □

5 Example and numeric simulation

Consider the following system:

$$\begin{aligned} \frac{dH_1}{dt} &= 3H_1\left(1 - \frac{H_1}{2+2P} - 2H_1 - \frac{H_2}{10}\right), \\ \frac{dH_2}{dt} &= H_2(0.3H_1 + 2 - 2H_2), \\ \frac{dP}{dt} &= 2P\left(1 - \frac{P}{2+0.8H_1} - 1.5P\right). \end{aligned} \tag{5.1}$$

By calculation, we have $M = 1 - \frac{\alpha d}{r_1 e} = 0.9 > 0$, $\alpha^2 = 0.09 < 8 < \frac{a_2 r_1 c_1 e}{a_2 + b_2 H_1^s}$, and it is easy to see that the conditions (B_1) and (B_4) are verified. It follows from Theorem 4.1 that there is a unique positive equilibrium point $(H_1^*, H_2^*, P^*) = (0.383868, 1.057580, 0.517211)$ of system (5.1) and it is globally asymptotically stable. Our numerical simulation supports our result (see Fig. 1).

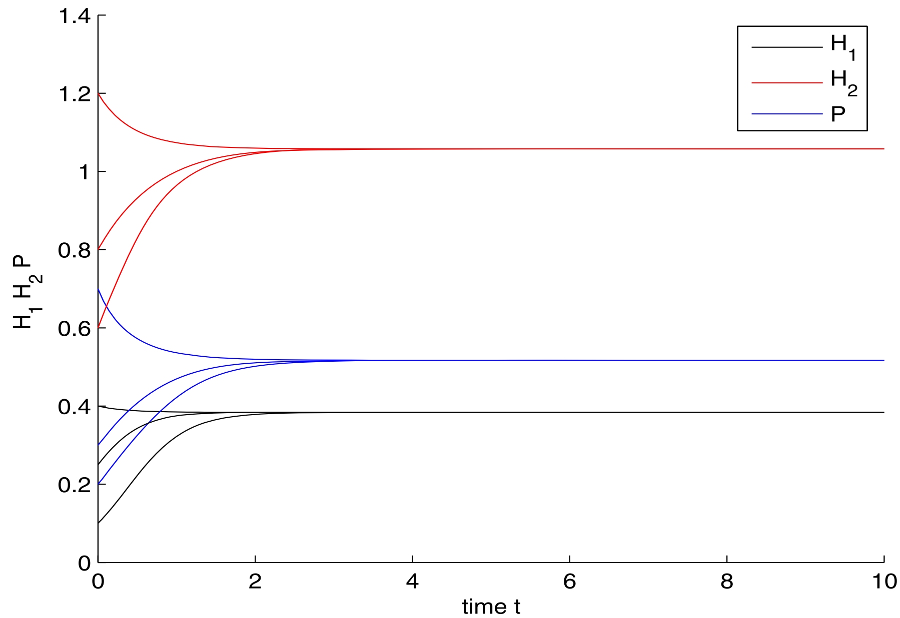

Figure 1 Dynamical behavior of system (5.1) with initial values $(0.4, 1.2, 0.7)^T$, $(0.1, 0.8, 0.2)^T$ and $(0.25, 0.6, 0.3)^T$

Consider the following system:

$$\begin{aligned} \frac{dH_1}{dt} &= 3H_1\left(1 - \frac{H_1}{2+2P} - 2H_1 - \frac{3.5H_2}{3}\right), \\ \frac{dH_2}{dt} &= H_2(3.5H_1 + 2 - 2H_2), \\ \frac{dP}{dt} &= 2P\left(1 - \frac{P}{2+0.8H_1} - 1.5P\right). \end{aligned} \tag{5.2}$$

By calculation, we have $M = 1 - \frac{\alpha d}{r_1 e} = -0.1 < 0$, $\alpha^2 = 10.89 < \frac{a_2 r_1 c_1 e}{a_2 + b_2 H_1^S} = 12$, it is easy to see that the conditions (B_3) and (B_5) are verified. It follows from Theorem 3.1 that there is a unique positive equilibrium point $(H_{1*}, H_{2*}, P_*) = (0, 1, 0.5)$ of system (5.2) and it is globally asymptotically stable. Our numerical simulation supports our result (see Fig. 2).

6 Discussion

In this paper, a May cooperative system with strong and weak cooperative partners is studied. We obtained the sufficient conditions that guarantee the permanence, non-permanence and the global stability of the equilibrium points. By comparing the conditions of (B_1) and (B_3), we found that as α becomes larger and larger, the strong partner changes from persistent to extinct. The ecological explanation is that more and more the strong partner become a weak partner, thus we have extinction of the strong. The above numerical simulations also supports this conclusion.

At the end of this paper, we point out that conditions (B_1) and (B_3) can be weakened or even are unnecessary. This problem is very interesting and worthy of further study in the future.

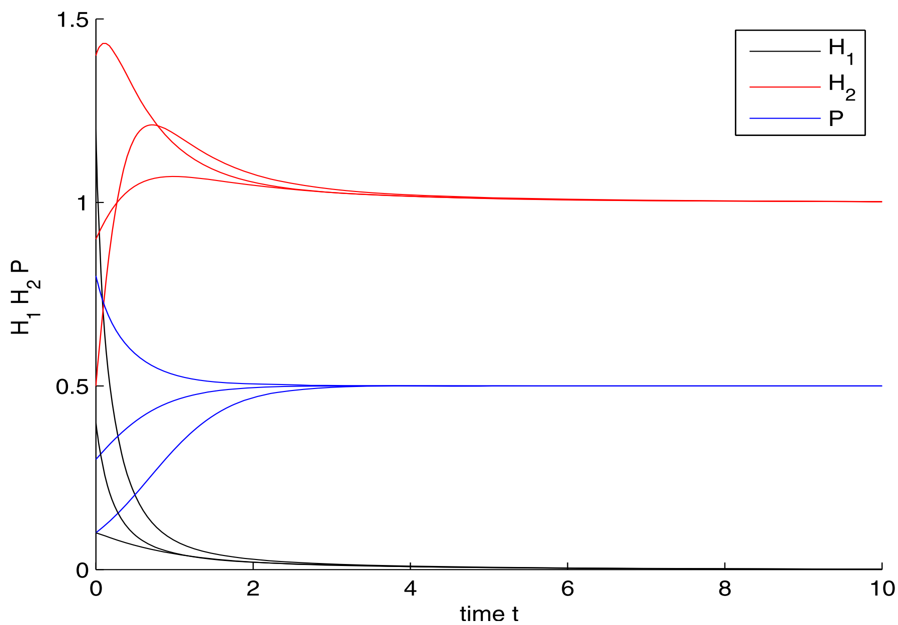

Figure 2 Dynamical behavior of system (5.2) with initial values $(0.4, 1.4, 0.8)^T$, $(0.1, 0.9, 0.3)^T$ and $(1.2, 0.5, 0.1)^T$

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Competing interests
The authors declare that there is no conflict of interests.

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All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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