Advances in Difference Equations
a SpringerOpen Journal

RESEARCH

Open Access

Permanence, partial survival, extinction, and global attractivity of a nonautonomous harvesting Lotka–Volterra commensalism model incorporating partial closure for the populations

Yu Liu[1], Xiangdong Xie[2*] and Qifa Lin[2]

*Correspondence:
latexfzu@126.com
[2]Department of Mathematics, Ningde Normal University, Ningde, P.R. China
Full list of author information is available at the end of the article

Abstract

We propose and study a nonautonomous harvesting Lotka–Volterra commensalism model incorporating partial closure for the populations. By using the differential inequality theory we obtain sufficient conditions that ensure the extinction, partial survival, and permanence of the system. By applying the fluctuation lemma we establish sufficient conditions that ensure the extinction of one of the components and the stability of the the other one. For the permanent case, by constructing a suitable Lyapunov function we obtain some sufficient conditions for the globally attractivity of the positive solution of the system. Examples, together with their numeric simulations, show the feasibility of the main results. To ensure the stable coexistence of the two species, the harvesting area should be carefully restricted.

MSC: 34C25; 92D25; 34D20; 34D40

Keywords: Commensalism model; Harvesting; Partial closure; Different inequality; Extinction; Permanence; Partial survival; Lyapunov function; Fluctuation lemma

1 Introduction

During the last decade, many scholars [1–11] investigated the dynamic behavior of the mutualism model, and many excellent results were obtained. For example, Chen, Xie, and Chen [1] showed that the stage structure of the species can lead to the extinction of the mutualism model, despite the cooperation between the species; Chen, Chen, and Li [3] showed that the feedback control variables have no influence on the persistent property of a kind of mutualism model, and in this direction, some similar results was established in [6, 8]; several scholars [2, 4, 7, 10, 11] investigated the stability property of the positive equilibrium of the cooperative system, Xie, Chen, and Xue [10] showed that if the harvesting effort is limited, then the cooperative system admits a unique positive equilibrium, which is globally attractive.

Commensalism, which describes a symbiotic interaction between two populations where one population gets benefit from the other while the other is neither harmed nor benefited due to the interaction with the previous species [12], has not arisen the attention of the scholars, since the model seems simple and can be seen as a particular case of the mutualism model. Only recently scholars paid attention to such a kind of relationship; see [12–20] and the references therein. Topics such as the existence of the positive periodic solution [17], the existence of a positive almost periodic solution [14], the existence and stability of the positive equilibrium [16], the influence of the impulsive [15] were investigated, and many excellent results were obtained. However, as was pointed out by Georgescu and Maxin [20], "One would think that the stability of the coexisting equilibria for two-species models of commensalism would follow immediately from the corresponding results for models of mutualism, when these results are available, . . . , However, this is not actually the case". Hence, it is necessary to do some further works on commensalism model.

Sun and Sun [18] proposed the following commensalism system:

$$\begin{aligned} \frac{dx}{dt} &= r_1 x\left(1 - \frac{x}{K_1} + \alpha \frac{y}{K_1}\right), \\ \frac{dy}{dt} &= r_2 y\left(1 - \frac{y}{K_2}\right), \end{aligned} \tag{1.1}$$

where r_1, r_2, K_1, K_2, α are all positive constants. By linearizing the system at equilibrium the authors investigated the local stability property of the equilibria of the system. They showed that the unique positive equilibrium of the system is locally asymptotically stable, whereas the other three boundary equilibria are all unstable.

Recently, Xue, Han, Yang et al. [21] argued that the nonautonomous model is more suitable, since the coefficients of the system vary with time. They proposed the following two species nonautonomous commensalism model:

$$\begin{aligned} \frac{dN_1}{dt} &= N_1\bigl(a(t) - b(t)N_1 + c(t)N_2\bigr), \\ \frac{dN_2}{dt} &= N_2\bigl(d(t) - e(t)N_2\bigr). \end{aligned} \tag{1.2}$$

The authors gave a set of sufficient conditions that ensure the existence of a unique globally attractive positive periodic solution.

On the other hand, to obtain the resource for the development of the human being, harvest of the species is necessary. During the last decades, many scholars investigated the influence of the harvesting to predator–prey or competition system; see [22–27] and the references therein. Chakraborty, Das, and Kar [24] argued that it is necessary to harvest the population but harvesting should be regulated, so that both the ecological sustainability and conservation of the species can be implemented in a long run. They proposed the following harvesting predator–prey model:

$$\begin{aligned} \frac{dx}{dt} &= rx\left(1 - \frac{x}{K}\right) - \frac{\alpha xy}{a + bx + cy} - q_1 mEx, \\ \frac{dy}{dt} &= sy\left(1 - \frac{y}{L}\right) + \frac{\beta xy}{a + bx + cy} - q_2 mEy. \end{aligned} \tag{1.3}$$

Some interesting results concerned with the boundedness of the system, existence of equilibria, and local and global stability of the positive equilibrium were obtained.

Stimulated by the works of Xue, Han, Yang et al. [21] and Chakraborty, Das, and Kar [24], in this paper, we propose the following nonautonomous nonselective harvesting Lotka–Volterra commensalism model incorporating partial closure for the populations:

$$\begin{aligned} \frac{dN_1(t)}{dt} &= N_1(t)\bigl(a(t) - b(t)N_1(t) + c(t)N_2(t)\bigr) - q_1(t)F(t)m(t)N_1(t), \\ \frac{dN_2(t)}{dt} &= N_2(t)\bigl(d(t) - e(t)N_2(t)\bigr) - q_2(t)F(t)m(t)N_2(t), \\ N_1(0) &> 0, \qquad N_2(0) > 0, \end{aligned} \tag{1.4}$$

where $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$, $q_1(t)$, $q_2(t)$, $F(t)$, and $m(t)$ are positive constants, and $a(t)$, $b(t)$, $c(t)$, $d(t)$, $e(t)$ have the same meaning as in system (1.2); $F(t)$ is the combined fishing effort used to harvest, and $m(t)$ $(0 < m(t) < 1)$ is the fraction of the stock available for harvesting.

As for as an ecosystem is concerned, there are the most important three topics: permanence, extinction, and global attractivity, which reflect the existence of the species in the long run, the extinction of the species, and the species maintained in a stable state. During the last decades, there are many excellent results on these three topics; see [28–37] and the references therein. For example, Shi, Li, and Chen [30] studied the extinction property of a competition system with infinite delay and feedback controls; Chen, Xie, and Li [31] investigated the partial extinction of the predator–prey model with stage structure; Chen, Chen, and Huang [32] investigated the extinction property of the nonlinear competition system with Beddington–DeAngelis functional response; Xie, Xue, Wu et al. [33] studied the extinction property of a nonlinear toxic substance competition system; Chen, Ma, and Zhang [34] showed that if the refuge is restricted to suitable area, then the Lotka–Volterra predato–prey system can admit a unique positive equilibrium, which is globally attractive. In this paper, we also focus our attention on the persistency, extinction, and stability of system (1.4).

The paper is arranged as follows. We will investigate the extinction, partial survival and persistency of system (1.4) in the next section. In Sect. 3, we investigate the global stability property of the solutions of the system. Two examples, together with their numeric simulations, are presented in Sect. 4 to show the feasibility of the main results. We end this paper by a brief discussion.

2 Extinction and persistency of the system

For the rest of the paper, for a bounded continuous function g defined on R, let

$$g^L = \inf_{t\in[0,+\infty)} g(t) \quad \text{and} \quad g^M = \sup_{t\in[0,+\infty)} g(t).$$

Lemma 2.1 ([28]) *If $a > 0$, $b > 0$, and $\dot{x} \geq x(b - ax)$ for $t \geq 0$ and $x(0) > 0$, then*

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If $a > 0$, $b > 0$, and $\dot{x} \leq x(b - ax)$ for $t \geq 0$ and $x(0) > 0$, then

$$\limsup_{t\to+\infty} x(t) \leq \frac{b}{a}.$$

Lemma 2.2 *The domain* $R_+^2 = \{(x,y) | x > 0, y > 0\}$ *is invariant with respect to* (1.4).

Proof Since

$$N_1(t) = N_1(0)\exp\left\{\int_0^t \Gamma_1(s)\,ds\right\} > 0, \qquad N_2(t) = N_2(0)\exp\left\{\int_0^t \Gamma_2(s)\,ds\right\} > 0,$$

where

$$\Gamma_1(s) = a(s) - q_1(s)F(s)m(s) - b(s)N_1(s) + c(s)N_2(s),$$
$$\Gamma_2(s) = d(s) - e(s)N_2(s) - q_2(s)F(s)m(s),$$

the assertion of the lemma immediately follows for all $t \in [0, +\infty)$. □

Theorem 2.1 *Let* $(N_1(t), N_2(t))^T$ *be any solution of system* (1.4). *Assume that*

$$d^M < q_2^L F^L m^L, \qquad a^M < q_1^L F^L m^L. \tag{2.1}$$

Then

$$\lim_{t\to+\infty} N_1(t) = 0, \qquad \lim_{t\to+\infty} N_2(t) = 0,$$

that is, both species will be driven to extinction.

Proof It follows from (2.1) that there exists a small enough $\varepsilon > 0$ such that

$$a^M + c^M\varepsilon < q_1^L F^L m^L. \tag{2.2}$$

Let $(N_1(t), N_2(t))^T$ be any solution of system (1.4). From the second equation of system (1.4) it follows that

$$\dot{N}_2(t) \le \left(d^M - q_2^L F^L m^L\right)N_2(t). \tag{2.3}$$

So

$$N_2(t) \le N_2(0)\exp\left\{\left(d^M - q_2^L F^L m^L\right)t\right\} \to 0 \quad \text{as } t \to 0. \tag{2.4}$$

For $\varepsilon > 0$ as in (2.2), it follows from (2.4) that there exists a large enough T_1 such that

$$N_2(t) < \varepsilon \quad \text{for all } t \ge T_1. \tag{2.5}$$

From the first equation and (2.5) it follows that

$$\dot{N}_1(t) \le N_1(t)\left(a^M + c^M\varepsilon - q_1^L F^L m^L - b^L N_1(t)\right). \tag{2.6}$$

So

$$N_1(t) \le N_1(T_1)\exp\left\{\left(a^M + c^M\varepsilon - q_1^L F^L m^L\right)(t - T_1)\right\} \to 0 \quad \text{as } t \to 0. \tag{2.7}$$

This ends the proof of Theorem 2.1. □

Theorem 2.2 *Let $(N_1(t), N_2(t))^T$ be any solution of system (1.4). Assume that*

$$d^M < q_2^L F^L m^L, \qquad a^L > q_1^M F^M m^M. \tag{2.8}$$

Then

$$\frac{a^L - q_1^M F^M m^M}{b^M} \le \liminf_{t\to+\infty} N_1(t) \le \limsup_{t\to+\infty} N_1(t) \le \frac{a^M - q_1^L F^L m^L}{b^L},$$
$$\lim_{t\to+\infty} N_2(t) = 0,$$

that is, the first species is permanent, and the second species will be driven to extinction.

Proof From the first inequality of (2.8), similarly to the analysis of (2.3)–(2.4), for any solution $(N_1(t), N_2(t))^T$ of system (1.4), we obtain

$$N_2(t) \le N_2(0) \exp\bigl\{\bigl(d^M - q_2^L F^L m^L\bigr)t\bigr\} \to 0 \quad \text{as } t \to 0. \tag{2.9}$$

For $\varepsilon > 0$ small enough, it follows from (2.9) that there exists a large enough T_2 such that

$$N_2(t) < \varepsilon \quad \text{for all } t \ge T_2. \tag{2.10}$$

From the first equation and (2.10) it follows that

$$\dot{N}_1(t) \le N_1(t)\bigl(a^M + c^M\varepsilon - q_1^L F^L m^L - b^L N_1(t)\bigr). \tag{2.11}$$

It follows from Lemma 2.1 and (2.11) that

$$\limsup_{t\to+\infty} N_1(t) \le \frac{a^M + c^M\varepsilon - q_1^L F^L m^L}{b^L}. \tag{2.12}$$

Since $\varepsilon > 0$ is an arbitrary small positive constant, letting $\varepsilon \to 0$ in (2.12) leads to

$$\limsup_{t\to+\infty} N_1(t) \le \frac{a^M - q_1^L F^L m^L}{b^L}. \tag{2.13}$$

From the first equation we also have

$$\dot{N}_1(t) \ge N_1(t)\bigl(a^L - q_1^M F^M m^M - b^M N_1(t)\bigr). \tag{2.14}$$

It follows from Lemma 2.1 and (2.14) that

$$\liminf_{t\to+\infty} N_1(t) \ge \frac{a^L - q_1^M F^M m^M}{b^M}. \tag{2.15}$$

Relations (2.9), (2.13), and (2.15) show that the statement of Theorem 2.2 holds. This ends the proof of the theorem. □

Theorem 2.3 *Let* $(N_1(t), N_2(t))^T$ *be any solution of system* (1.4). *Assume that*

$$d^L > q_2^M F^M m^M, \qquad a^M + c^M \frac{d^M - q_2^L F^L m^L}{e^L} < q_1^L F^L m^L. \tag{2.16}$$

Then

$$\lim_{t\to+\infty} N_1(t) = 0,$$
$$\frac{d^L - q_2^M F^M m^M}{e^M} \le \liminf_{t\to+\infty} N_2(t) \le \limsup_{t\to+\infty} N_2(t) \le \frac{d^M - q_2^L F^L m^L}{e^L},$$

that is, the second species is permanent, whereas the first species will be driven to extinction.

Proof It follows from (2.16) that there exists a small enough $\varepsilon > 0$ such that

$$a^M + c^M \left(\frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon \right) < q_1^L F^L m^L. \tag{2.17}$$

Let $(N_1(t), N_2(t))^T$ be any positive solution of system (1.4). From the second equation of system (1.4) it follows that

$$\dot{N}_2(t) \le N_2(t) \left(d^M - q_2^L F^L m^L - e^L N_2(t) \right). \tag{2.18}$$

It follows from Lemma 2.1 and (2.18) that

$$\limsup_{t\to+\infty} N_2(t) \le \frac{d^M - q_2^L F^L m^L}{e^L}. \tag{2.19}$$

For $\varepsilon > 0$ as in (2.17), it follows from (2.19) that there exists $T_3 > 0$ such that

$$N_2(t) < \frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon \quad \text{for all } t \ge T_3. \tag{2.20}$$

Again, from the second equation of system (1.4) we also have

$$\dot{N}_2(t) \ge N_2(t) \left(d^L - q_2^M F^M m^M - e^M N_2(t) \right). \tag{2.21}$$

It follows from Lemma 2.1 and (2.21) that

$$\liminf_{t\to+\infty} N_2(t) \ge \frac{d^L - q_2^M F^M m^M}{e^M}. \tag{2.22}$$

From the first equation and (2.20), for $t \ge T_3$, it follows that

$$\dot{N}_1(t) \le \left(a^M + c^M \left(\frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon \right) - q_1^L F^L m^L \right) N_1(t). \tag{2.23}$$

So

$$N_1(t) \le N_1(T_3) \exp\left\{ \Gamma_1^\varepsilon (t - T_3) \right\} \to 0 \quad \text{as } t \to +\infty, \tag{2.24}$$

where

$$\Gamma_1^\varepsilon = a^M + c^M\left(\frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon\right) - q_1^L F^L m^L.$$

It immediately follows from (2.19), (2.22), and (2.24) that the statement of Theorem 2.3 holds. This ends the proof of the theorem. □

Theorem 2.4 *Let $(N_1(t), N_2(t))^T$ be any solution of system (1.4). Assume that*

$$d^L > q_2^M F^M m^M, \qquad a^L + c^L \frac{d^L - q_2^M F^M m^M}{e^M} > q_1^M F^M m^M. \tag{2.25}$$

Then the system is permanent, that is, there exist positive constants m_i, M_i, $i = 1, 2$, independent of the solutions of (1.4), such that

$$m_1 \le \liminf_{t\to+\infty} N_1(t) \le \limsup_{t\to+\infty} N_1(t) \le M_1,$$
$$m_2 \le \liminf_{t\to+\infty} N_2(t) \le \limsup_{t\to+\infty} N_2(t) \le M_2,$$

where

$$\begin{aligned} m_1 &= \frac{a^L + c^L \frac{d^L - q_2^M F^M m^M}{e^M} - q_1^M F^M m^M}{b^M}; \\ M_1 &= \frac{a^M + c^M \frac{d^M - q_2^L F^L m^L}{e^L} - q_1^L F^L m^L}{b^L}; \\ m_2 &= \frac{d^L - q_2^M F^M m^M}{e^M}; \qquad M_2 = \frac{d^M - q_2^L F^L m^L}{e^L}. \end{aligned} \tag{2.26}$$

Proof It follows from (2.25) that, indeed, for enough small $\varepsilon > 0$, namely, for

$$\varepsilon < \frac{a^L + c^L(\frac{d^L - q_2^M F^M m^M}{e^M}) - q_1^M F^M m^M}{c^L}, \tag{2.27}$$

we have the inequality

$$a^L + c^L\left(\frac{d^L - q_2^M F^M m^M}{e^M} - \varepsilon\right) > q_1^M F^M m^M. \tag{2.28}$$

From (2.28) we can easily see that

$$a^M + c^M\left(\frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon\right) > q_1^L F^L m^L. \tag{2.29}$$

Let $(N_1(t), N_2(t))^T$ be any solution of system (1.4). From the second equation of system (1.4), applying the first inequality of (2.25), similarly to the analysis of (2.18)–(2.22), we can show that

$$m_2 \overset{\text{def}}{=} \frac{d^L - q_2^M F^M m^M}{e^M} \le \liminf_{t\to+\infty} N_2(t) \le \limsup_{t\to+\infty} N_2(t) \le \frac{d^M - q_2^L F^L m^L}{e^L} \overset{\text{def}}{=} M_2. \tag{2.30}$$

For any positive constant $\varepsilon > 0$ small enough, which satisfies (2.27) and $\varepsilon < \frac{d^L - q_2^M F^M m^M}{e^M}$, there exists a large enough $T_4 > 0$ such that

$$\frac{d^L - q_2^M F^M m^M}{e^M} - \varepsilon < N_2(t) < \frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon \quad \text{for all } t \ge T_4. \tag{2.31}$$

From the first equation of (1.4) and (2.31), for $t \ge T_4$, it follows that

$$\dot{N}_1(t) \le \left(a^M + c^M\left(\frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon\right) - q_1^L F^L m^L - b^L N_1(t)\right) N_1(t). \tag{2.32}$$

Applying Lemma 2.1 to (2.32) leads to

$$\limsup_{t\to+\infty} N_1(t) \le \frac{a^M + c^M(\frac{d^M - q_2^L F^L m^L}{e^L} + \varepsilon) - q_1^L F^L m^L}{b^L}.$$

Setting $\varepsilon \to 0$ in this inequality leads to

$$\limsup_{t\to+\infty} N_1(t) \le \frac{a^M + c^M \frac{d^M - q_2^L F^L m^L}{e^L} - q_1^L F^L m^L}{b^L} \overset{\text{def}}{=} M_1. \tag{2.33}$$

From the first equation of (1.4) and (2.31), for $t \ge T_4$, we also have

$$\dot{N}_1(t) \ge \left(a^L + c^L\left(\frac{d^L - q_2^M F^M m^M}{e^M} - \varepsilon\right) - q_1^M F^M m^M - b^M N_1(t)\right) N_1(t). \tag{2.34}$$

Applying Lemma 2.1 to (2.34) leads to

$$\liminf_{t\to+\infty} N_1(t) \ge \frac{a^L + c^L(\frac{d^L - q_2^M F^M m^M}{e^M} - \varepsilon) - q_1^M F^M m^M}{b^M}.$$

Setting $\varepsilon \to 0$ in this inequality leads to

$$\liminf_{t\to+\infty} N_1(t) \ge \frac{a^L + c^L \frac{d^L - q_2^M F^M m^M}{e^M} - q_1^M F^M m^M}{b^M} \overset{\text{def}}{=} m_1. \tag{2.35}$$

Relations (2.30), (2.33), and (2.35) show that the statement of Theorem 2.4 holds. This ends the proof of the theorem. □

3 Global attractivity

In Sect. 2, we discussed the persistent or extinction property of the system, which means that the solutions of the system are bounded above and below by some positive constants or the species will be driven to extinction. One of the interesting problems is to give sufficient conditions to ensure the global attractivity of the positive solution of the system. Before we state the main results of this section, we need to introduce two useful lemmas.

Lemma 3.1 (Fluctuation lemma, [35, Lemma 4]) *Let $x(t)$ be a bounded differentiable function on (α,∞). Then there exist sequences $\tau_n \to \infty$ and $\sigma_n \to \infty$ such that*

$$\text{(a)}\quad \dot{x}(\tau_n) \to 0 \quad \textit{and} \quad x(\tau_n) \to \limsup_{t\to\infty} x(t) = \overline{x} \quad \textit{as } n \to \infty,$$

$$\text{(b)}\quad \dot{x}(\sigma_n) \to 0 \quad \textit{and} \quad x(\sigma_n) \to \liminf_{t\to\infty} x(t) = \underline{x} \quad \textit{as } n \to \infty.$$

For the logistic equation

$$\dot{x}(t) = x(t)\bigl(r(t) - a(t)x(t)\bigr), \tag{3.1}$$

from Lemma 2.1 of Zhao and Chen [36] we have the following:

Lemma 3.2 *Suppose that $r(t)$ and $a(t)$ are bounded above and below by positive constants. Then any positive solutions of Eq. (3.1) are defined on $[0,+\infty)$, bounded above and below by positive constants, and globally attractive.*

Theorem 3.1 *Under the assumptions of Theorem 2.2, let $N(t) = (N_1(t), N_2(t))^T$ be any positive solution of system (1.4). Then the species N_2 will be driven to extinction, that is, $N_2(t) \to 0$ as $t \to +\infty$, and $N_1(t) \to N_1^*(t)$ as $t \to +\infty$, where $N_1^*(t)$ is any positive solution of*

$$\frac{dN_1(t)}{dt} = N_1(t)\bigl(a(t) - q_1(t)F(t)m(t) - b(t)N_1(t)\bigr). \tag{3.2}$$

Proof Let $N(t) = (N_1(t), N_2(t))^T$ be any positive solution of system (1.4). By Theorem 2.2 the species N_2 will be driven to extinction, that is, $N_2(t) \to 0$ as $t \to +\infty$. To finish the proof of Theorem 3.1, it suffices to show that $N_1(t) \to N_1^*(t)$ as $t \to +\infty$, where $N_1^*(t)$ is any positive solution of system (3.2). It follows from Theorem 2.2 and Lemma 3.2 that there exists $T_5 > 0$ such that

$$\frac{1}{2}\frac{a^L - q_1^M F^M m^M}{b^M} < N_1(t) < \frac{3}{2}\frac{a^M - q_1^L F^L m^L}{b^L}$$

and

$$\eta_1 < N_1^*(t) < \eta_2,$$

where η_i, $i = 1, 2$, are two positive constants independent of the solution of system (3.2). Let $w(t) = (N_1(t))^{-1}$, $w^*(t) = (N_1^*(t))^{-1}$, and $z(t) = w(t) - w^*(t)$. Then

$$\begin{aligned}
\dot{w}(t) &= -\bigl(a(t) - q_1(t)F(t)m(t)\bigr)w(t) + b(t) + c(t)w(t)N_2(t),\\
\dot{w^*}(t) &= -\bigl(a(t) - q_1(t)F(t)m(t)\bigr)w^*(t) + b(t).
\end{aligned}$$

It follows that z satisfies

$$z'(t) = -\big(a(t) - q_1(t)F(t)m(t)\big)z(t) + c(t)w(t)N_2(t), \quad t \geq T_5. \tag{3.3}$$

From this analysis, for $t \geq T_5$, we have

$$0 < \left(\frac{3}{2}\frac{a^M - q_1^L F^L m^L}{b^L}\right)^{-1} < w(t) < \left(\frac{1}{2}\frac{a^L - q_1^M F^M m^M}{b^M}\right)^{-1}$$

and

$$0 < \eta_2^{-1} \leq w^*(t) \leq \eta_1^{-1}.$$

Thus $z(t)$ is a bounded differentiable function. By the fluctuation lemma (Lemma 3.1) there exist sequences $\tau_n \to \infty$ and $\sigma_n \to \infty$ such that $z(\tau_n) \to \overline{z}$, $z'(\tau_n) \to 0$; $z(\sigma_n) \to \underline{z}$, $z'(\sigma_n) \to 0$ as $n \to \infty$. We will show that $\overline{z} = \underline{z} = 0$. From (3.3) we have

$$z(t) = \frac{c(t)w(t)N_2(t)}{a(t) - q_1(t)F(t)m(t)} - \frac{z'(t)}{a(t) - q_1(t)F(t)m(t)}.$$

Noting that

$$0 < \frac{c(t)w(t)}{a(t) - q_1(t)F(t)m(t)} \leq \frac{c^M(\frac{1}{2}\frac{a^L - q_1^M F^M m^M}{b^M})^{-1}}{a^L - q_1^M F^M m^M},$$
$$0 < \frac{1}{a(t) - q_1(t)F(t)m(t)} \leq \frac{1}{a^L - q_1^M F^M m^M},$$

and $\lim_{t\to\infty} N_2(t) = 0$, we can see that

$$\begin{aligned}
&\lim_{n\to\infty} \frac{c(\tau_n)w(\tau_n)N_2(\tau_n)}{a(\tau_n) - q_1(\tau_n)F(\tau_n)m(\tau_n)} \\
&\quad = \lim_{n\to\infty} \frac{c(\sigma_n)w(\sigma_n)N_2(\sigma_n)}{a(\sigma_n) - q_1(\sigma_n)F(\sigma_n)m(\sigma_n)} \\
&\quad = \lim_{n\to\infty} \frac{z'(\tau_n)}{a(\tau_n) - q_1(\tau_n)F(\tau_n)m(\tau_n)} \\
&\quad = \lim_{n\to\infty} \frac{z'(\sigma_n)}{a(\sigma_n) - q_1(\sigma_n)F(\sigma_n)m(\sigma_n)} = 0.
\end{aligned}$$

Hence $\overline{z} = \underline{z} = 0$. Since

$$\big|N_1(t) - N_1^*(t)\big| = \big|w^*(t) - w(t)\big|\big(N_1^*(t)N_1(t)\big)$$

and both $N_1(t)$ and $N_1^*(t)$ are bounded functions, we have

$$\lim_{t\to\infty} N_1(t) = N_1^*(t),$$

as required. This completes the proof. □

Theorem 3.2 *Under the assumptions of Theorem 2.3, let $N(t) = (N_1(t), N_2(t))^T$ be any positive solution of system (1.4). Then the species N_1 will be driven to extinction, that is, $N_1(t) \to 0$ as $t \to +\infty$, and $N_2(t) \to N_2^*(t)$ as $t \to +\infty$, where $N_2^*(t)$ is any positive solution of*

$$\frac{dN_2(t)}{dt} = N_2(t)\big(d(t) - q_2(t)F(t)m(t) - e(t)N_2(t)\big). \tag{3.4}$$

Proof Let $N(t) = (N_1(t), N_2(t))^T$ be any positive solution of system (1.4). By Theorem 2.3 the species N_1 will be driven to extinction, that is, $N_1(t) \to 0$ as $t \to +\infty$. On the other hand, noting that the second equation of system (1.4) is independent of $N_1(t)$, from Lemma 3.2 it immediately follows that $N_2(t) \to N_2^*(t)$ as $t \to +\infty$, where $N_2^*(t)$ is any positive solution of system (3.4). This ends the proof of Theorem 3.2. □

Theorem 3.3 *In addition to (2.25), assume further that*

$$e^L > c^M. \tag{3.5}$$

Let $N(t) = (N_1(t), N_2(t))^T$ and $N^*(t) = (N_1^*(t), N_2^*(t))^T$ be any two positive solutions of system (1.4). Then

$$\lim_{t\to+\infty}\big(\left|N_1(t) - N_1^*(t)\right| + \left|N_2(t) - N_2^*(t)\right|\big) = 0.$$

Proof Let $N(t) = (N_1(t), N_2(t))^T$ and $N^*(t) = (N_1^*(t), N_2^*(t))^T$ be any two positive solutions of system (1.4). For any small enough positive constant $\varepsilon > 0$, it then follows from Theorem 2.4 that there exists a large enough T_6 such that, for all $t \geq T_6$,

$$\begin{aligned} &N_1(t), N_1^*(t) < M_1 + \varepsilon, \qquad N_2(t), N_2^*(t) < M_2 + \varepsilon, \\ &N_1(t), N_1^*(t) > m_1 - \varepsilon, \qquad N_2(t), N_2^*(t) > m_2 - \varepsilon. \end{aligned} \tag{3.6}$$

Now let

$$V(t) = \left|\ln N_1(t) - \ln N_1^*(t)\right| + \left|\ln N_2(t) - \ln N_2^*(t)\right|.$$

Then, by (3.5), for $t > T_6$, we have

$$\begin{aligned} D^+V(t) &\leq -b(t)\left|N_1(t) - N_1^*(t))\right| + c(t)\left|N_2(t) - N_2^*(t)\right| \\ &\quad - e(t)\left|N_2(t) - N_2^*(t)\right| \\ &\leq -b^L\left|N_1(t) - N_1^*(t))\right| - \big(e^L - c^M\big)\left|N_2(t) - N_2^*(t)\right|. \end{aligned} \tag{3.7}$$

Integrating both sides of (3.7) on the interval $[T_6, t)$, we have

$$\begin{aligned} &V(t) - V(T_6) \\ &\quad \leq \int_{T_6}^t \big[-b^L\left|N_1(s) - N_1^*(s)\right| - \big(e^L - c^M\big)\left|N_2(s) - N_2^*(s)\right|\big]\,ds \quad \text{for } t \geq T_6. \end{aligned} \tag{3.8}$$

It follows from (3.8) that

$$V(t)+\min\{b^L,e^L-c^M\}\int_{T_6}^{t}\bigl[\bigl|N_1(s)-N_1^*(s)\bigr|+\bigl|N_2(s)-N_2(s)\bigr|\bigr]\,ds \\ \le V(T_6)\quad \text{for } t\ge T_6. \tag{3.9}$$

Therefore $V(t)$ is bounded on $[T_6,+\infty)$, and also

$$\int_{T_6}^{t}\bigl[\bigl|N_1(s)-N_1^*(s)\bigr|+\bigl|N_2(s)-N_2(s)\bigr|\bigr]\,ds<+\infty. \tag{3.10}$$

By (3.6), $|N_1(t)-N_1^*(t)|$ and $|N_2(t)-N_2^*(t)|$ are bounded on $[T_6,+\infty)$. On the other hand, it is easy to see that $\dot N_1(t)$, $\dot N_2(t)$, $\dot N_1^*(t)$, and $\dot N_2^*(t)$ are bounded for $t\ge T_6$. Therefore $|N_1(t)-N_1^*(t)|$, $|N_2(t)-N_2^*(t)|$ are uniformly continuous on $[T_6,+\infty)$. By the Barbălat lemma we conclude that

$$\lim_{t\to+\infty}\bigl[\bigl|N_1(t)-N_1^*(t)\bigr|+\bigl|N_2(t)-N_2^*(t)\bigr|\bigr]=0.$$

This ends the proof of Theorem 3.3. □

4 Numerical simulations

Example 4.1 Consider the following system:

$$\begin{aligned}
\frac{dN_1(t)}{dt}&=N_1(t)\left(\frac12+\frac14\cos t-N_1(t)+N_2(t)\right)-\frac12N_1(t),\\
\frac{dN_2(t)}{dt}&=N_2(t)\left(\frac12+\frac14\sin t-N_2(t)\right)-\frac12N_2(t),\\
N_1(0)&>0,\qquad N_2(0)>0.
\end{aligned} \tag{4.1}$$

Corresponding to system (1.4), here we take

$$a(t)=\frac12+\frac14\cos t,\qquad d(t)=\frac12+\frac14\sin t, \tag{4.2}$$

$$q_1(t)=q_2(t)=F(t)=c(t)=e(t)=1,\qquad m(t)=\frac12. \tag{4.3}$$

We can easily verify that

$$a^M<q_1^LF^Lm^L,\qquad d^M<q_2^LF^Lm^L.$$

Hence, all the conditions of Theorem 2.1 hold, and it follows from Theorem 2.1 that both species will be driven to extinction. Numeric simulations (Figs. 1 and 2) also support this assertion.

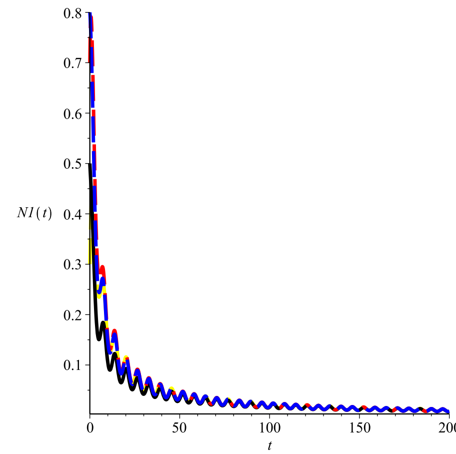

Figure 1 Numeric simulations of the first component system (4.1) with initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 0.5), (0.3, 0.7)$, and $(0.7, 0.9)$

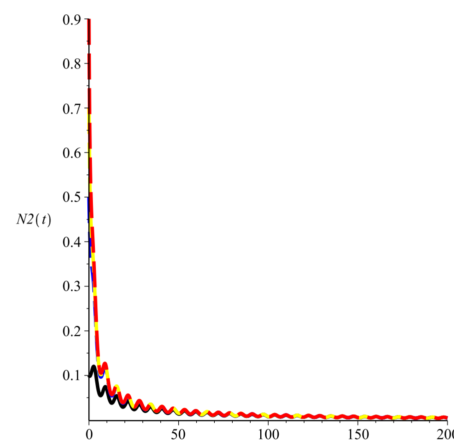

Figure 2 Numeric simulations of the second component of system (4.1) with initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 0.5), (0.3, 0.7)$, and $(0.7, 0.9)$

Example 4.2 Consider the following system:

$$\begin{aligned} \frac{dN_1(t)}{dt} &= N_1(t)\left(\frac{1}{2} + \frac{1}{4}\cos t - N_1(t) + \frac{1}{4}N_2(t)\right) - \frac{1}{50}N_1(t), \\ \frac{dN_2(t)}{dt} &= N_2(t)\left(\frac{1}{2} + \frac{1}{4}\sin t - N_2(t)\right) - \frac{1}{50}N_2(t), \\ N_1(0) &> 0, \qquad N_2(0) > 0. \end{aligned} \tag{4.4}$$

Corresponding to system (1.4), here we take

$$a(t) = \frac{1}{2} + \frac{1}{4}\cos t, \qquad d(t) = \frac{1}{2} + \frac{1}{4}\sin t, \tag{4.5}$$

$$q_1(t) = q_2(t) = F(t) = e(t) = 1, \qquad c(t) = \frac{1}{4}, m(t) = \frac{1}{50}. \tag{4.6}$$

We can easily verify that all the conditions of Theorems 2.4 and 3.3 hold, and it follows from Theorems 2.4 and 3.3 that the system is permanent and that any positive solution of system (4.4) is globally attractive. Numeric simulations (Figs. 3 and 4) also support this assertion.

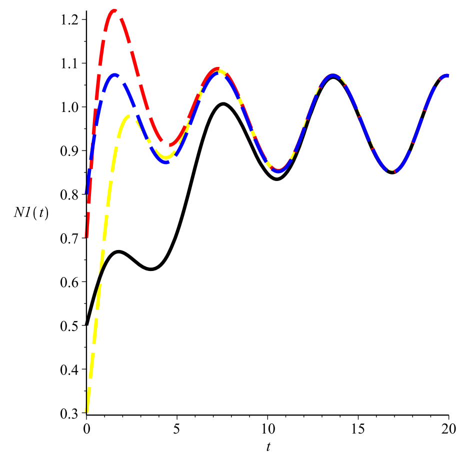

Figure 3 Numeric simulations of the first component of system (4.4) with initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 0.5), (0.3, 0.7)$, and $(0.7, 0.9)$

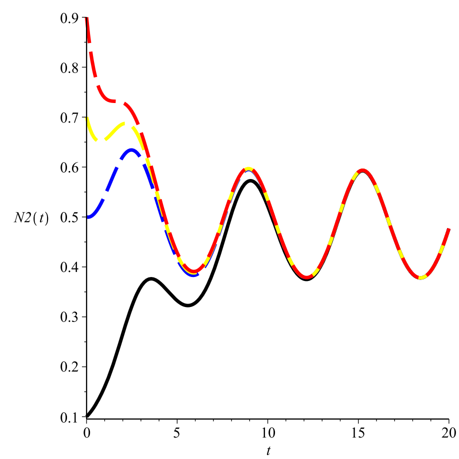

Figure 4 Numeric simulations of the second component of system (4.4) with initial conditions $(x(0), y(0)) = (0.5, 0.1), (0.8, 0.5), (0.3, 0.7)$, and $(0.7, 0.9)$

5 Discussion

Recently, many scholars [12–21] studied the dynamic behavior of the commensalism model; however, none of them consider the influence of harvesting. Stimulated by the recent works of Chakraborty, Das, and Kar [24], we propose a nonautonomous nonselective commensalism model incorporating partial closure to the population.

We focus our attention on the persistent and extinction property of the system, Theorems 2.1–2.4 show that, depending on the area that can be harvested, the the system may exhibit permanent, extinction, or partial survival phenomenon, that is, the introducing of harvesting makes the dynamic behavior of the system complicated. Theorem 2.4 shows that if the harvesting area is small enough (i.e., m is small enough), then two species can coexist in the long run. If we further assume that the intrinsic competition rate ($e(t)$) is larger than the cooperative between the two species ($c(t)$), then two species can coexist in a stable state. Such a result may help us in designing the reserve area of the species.

It seems interesting to incorporate the time delay to system (1.4) and study the influence of the time delay. We leave this for future study.

Acknowledgements
The authors are grateful to anonymous referees for their excellent suggestions, which greatly improved the presentation of the paper.

Funding
This work is supported by the National Natural Science Foundation of China under Grant (11601085) and the Natural Science Foundation of Fujian Province (2017J01400).

Competing interests
The authors declare that there is no conflict of interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Author details
[1]College of Mathematics and Computer Science, Fuzhou University, Fuzhou, P.R. China. [2]Department of Mathematics, Ningde Normal University, Ningde, P.R. China.

Publisher's Note
Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 9 February 2018 Accepted: 18 May 2018 Published online: 19 June 2018

References
1. Chen, F.D., Xie, X.D., Chen, X.F.: Dynamic behaviors of a stage-structured cooperation model. Commun. Math. Biol. Neurosci. **2015**, Article ID 4 (2015)
2. Chen, F.D., Xie, X.D.: Study on the Dynamic Behaviors of Cooperation Population Modeling. Science Press, Beijing (2014)
3. Chen, L.J., Chen, L.J., Li, Z.: Permanence of a delayed discrete mutualism model with feedback controls. Math. Comput. Model. **50**, 1083–1089 (2009)
4. Yang, K., Miao, Z.S., Chen, F.D., Xie, X.D.: Influence of single feedback control variable on an autonomous Holling-II type cooperative system. J. Math. Anal. Appl. **435**(1), 874–888 (2016)
5. Chen, L.J., Xie, X.D.: Permanence of an *N*-species cooperation system with continuous time delays and feedback controls. Nonlinear Anal., Real World Appl. **12**, 34–38 (2001)
6. Chen, L.J., Xie, X.D., Chen, L.J.: Feedback control variables have no influence on the permanence of a discrete *N*-species cooperation system. Discrete Dyn. Nat. Soc. **2009**, Article ID 306425 (2009)
7. Yang, K., Xie, X., Chen, F.: Global stability of a discrete mutualism model. Abstr. Appl. Anal. **2014**, Article ID 709124 (2014)
8. Li, Y.K., Zhang, T.W.: Permanence of a discrete *n*-species cooperation system with time-varying delays and feedback controls. Math. Comput. Model. **53**, 1320–1330 (2011)
9. Liu, Z.J., Wu, J.H., Tan, R.H., Chen, Y.P.: Modeling and analysis of a periodic delayed two-species model of facultative mutualism. Appl. Math. Comput. **217**, 893–903 (2010)
10. Xie, X.D., Chen, F.D., Xue, Y.L.: Note on the stability property of a cooperative system incorporating harvesting. Discrete Dyn. Nat. Soc. **2014**, Article ID 327823 (2014)
11. Xie, X.D., Chen, F.D., Yang, K., Xue, Y.L.: Global attractivity of an integrodifferential model of mutualism. Abstr. Appl. Anal. **2014**, Article ID 928726 (2014)
12. Hari Prasad, B., Pattabhi Ramacharyulu, N.Ch.: Discrete model of commensalism between two species, I. J. Mod. Educ. Comput. Sci. **8**, 40–46 (2012)
13. Wu, R.X., Li, L., Lin, Q.F.: A Holling type commensal symbiosis model involving Allee effect. Commun. Math. Biol. Neurosci. **2018**, Article ID 6 (2018)
14. Xue, Y.L., Xie, X.D., Chen, F.D., et al.: Almost periodic solution of a discrete commensalism system. Discrete Dyn. Nat. Soc. **2015**, Article ID 295483 (2015)
15. Miao, Z.S., Xie, X.D., Pu, L.Q.: Dynamic behaviors of a periodic Lotka–Volterra commensal symbiosis model with impulsive. Commun. Math. Biol. Neurosci. **2015**, Article ID 3 (2015)
16. Wu, R.X., Lin, L., Zhou, X.Y.: A commensal symbiosis model with Holling type functional response. J. Math. Comput. Sci. **16**, 364–371 (2016)
17. Lin, Q.F.: Dynamic behaviors of a commensal symbiosis model with non-monotonic functional response and non-selective harvesting in a partial closure. Commun. Math. Biol. Neurosci. **2018**, Article ID 4 (2018)
18. Sun, G.C., Sun, H.: Analysis on symbiosis model of two populations. J. Weinan Normal Univ. **28**(9), 6–8 (2013)
19. Zhu, Z.F., Chen, Q.L.: Mathematic analysis on commensalism Lotka–Volterra model of populations. J. Jixi Univ. **8**(5), 100–101 (2008)
20. Georgescu, P., Maxin, D.: Global stability results for models of commensalism. Int. J. Biomath. **10**(3), 1750037 (2017)
21. Xue, Y.L., Han, R.Y., Yang, L.Y., Chen, F.D.: On the existence and stability of positive periodic solution of a nonautonomous commensal symbiosis model of two populations. J. Shangming Univ. **32**(2), 32–37 (2015)
22. Chen, B.G.: Dynamic behaviors of a non-selective harvesting Lotka–Volterra amensalism model incorporating partial closure for the populations. Adv. Differ. Equ. **2018**, 111 (2018)
23. Chen, L., Chen, F.: Global analysis of a harvested predator–prey model incorporating a constant prey refuge. Int. J. Biomath. **3**(02), 177–189 (2010)
24. Chakraborty, K., Das, S., Kar, T.K.: On non-selective harvesting of a multispecies fishery incorporating partial closure for the populations. Appl. Math. Comput. **221**, 581–597 (2013)

25. Kar, T.K., Chaudhuri, K.S.: On non-selective harvesting of two competing fish species in the presence of toxicity. Ecol. Model. **161**, 125–137 (2003)
26. Leard, B., Rebaza, J.: Analysis of predator–prey models with continuous threshold harvesting. Appl. Math. Comput. **217**(12), 5265–5278 (2011)
27. Chakraborty, K., Jana, S., Kar, T.K.: Global dynamics and bifurcation in a stage structured prey–predator fishery model with harvesting. Appl. Math. Comput. **218**(18), 9271–9290 (2012)
28. Chen, F., Li, Z., Huang, Y.J.: Note on the permanence of a competitive system with infinite delay and feedback controls. Nonlinear Anal., Real World Appl. **8**, 680–687 (2007)
29. Chen, F.D., Xie, X.D., Miao, Z.S., et al.: Extinction in two species nonautonomous nonlinear competitive system. Appl. Math. Comput. **274**, 119–124 (2016)
30. Shi, C., Li, Z., Chen, F.: Extinction in a nonautonomous Lotka–Volterra competitive system with infinite delay and feedback controls. Nonlinear Anal., Real World Appl. **13**(5), 2214–2226 (2012)
31. Chen, F., Xie, X., Li, Z.: Partial survival and extinction of a delayed predator–prey model with stage structure. Appl. Math. Comput. **219**(8), 4157–4162 (2012)
32. Chen, F., Chen, X., Huang, S.: Extinction of a two species non-autonomous competitive system with Beddington–DeAngelis functional response and the effect of toxic substances. Open Math. **14**(1), 1157–1173 (2016)
33. Xie, X., Xue, Y., Wu, R., et al.: Extinction of a two species competitive system with nonlinear inter-inhibition terms and one toxin producing phytoplankton. Adv. Differ. Equ. **2016**, Article ID 258 (2016)
34. Chen, F., Ma, Z., Zhang, H.: Global asymptotical stability of the positive equilibrium of the Lotka–Volterra prey–predator model incorporating a constant number of prey refuges. Nonlinear Anal., Real World Appl. **13**(6), 2790–2793 (2012)
35. Montes De Oca, F., Vivas, M.: Extinction in two dimensional Lotka–Volterra system with infinite delay. Nonlinear Anal., Real World Appl. **7**(5), 1042–1047 (2006)
36. Zhao, J.D., Chen, W.C.: The qualitative analysis of N-species nonlinear prey-competition systems. Appl. Math. Comput. **149**, 567–576 (2004)
37. Lin, Q.X., Xie, X.D., et al.: Dynamical analysis of a logistic model with impulsive Holling type-II harvesting. Adv. Differ. Equ. **2018**, 112 (2018)