Lu et al. *Advances in Difference Equations* (2018) 2018:266
https://doi.org/10.1186/s13662-018-1734-4

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH

Open Access

Structure of solitary wave solutions of the nonlinear complex fractional generalized Zakharov dynamical system

Dianchen Lu[1], Aly R. Seadawy[2,3*] and Mostafa M.A. Khater[1]

*Correspondence:
Aly742001@yahoo.com
[2]Mathematics Department, Faculty of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia
[3]Mathematics Department, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt
Full list of author information is available at the end of the article

Abstract

The analytical and solitary traveling solutions of the nonlinear complex fractional generalized Zakharov equations are investigated. The nonlinear complex fractional generalized Zakharov equations describe the interaction between dispersive and non-dispersive waves in one dimension. Analytical and solitary traveling wave solutions were obtained through applying a generalized Kudryashov and a novel $(\frac{G'}{G})$-expansion methods. Novel solutions were the results of our investigated model, which illustrated the effectiveness and the power of the obtained methods in regards to accuracy, power, and effectiveness compared to the previously used methods.

Keywords: Nonlinear complex fractional generalized-Zakharov system; Generalized Kudryashov methods; Novel $(\frac{G'}{G})$-expansion method; Solitary traveling wave solutions

1 Introduction

The nonlinear complex fractional generalized-Zakharov system characterizes the proliferation of Langmuir waves in the ionized plasma. This model was discovered by Irving Langmuir and Lewi Tonks in the 1920s. Langmuir waves are known as plasma oscillations, they represent the fast oscillation of the electron density in conducting media. This oscillation can describe the instability in the dielectric function of free electron gas. While the ionization is defined as an atom which could be positively or negatively charged through acquiring or losing an electron or more, and that is always synchronized with other chemical shifts.

The nonlinear model of nonlinear complex fractional generalized-Zakharov system is very important and has many applications related to this phenomenon. Long time ago, researchers were trying to discover new physical features of the nonlinear model. Vladimir Zakharov in 1972 was the first who introduced this system. Then, further research was done to continue this trend, and the results obtained were analytical and solitary traveling wave solutions. In 1996, Malfliet et al. used the tanh method [1]; in 2009, Borhanifar et al. used the Exp-function method [2]; in 2003, Li et al. used the improved tanh method and symbolic computation [3]; in 2004, Yong et al. used the Riccati equation method [4]; in 2012, Naher Hasibun et al. used the Exp-function method [5]; in 2007, Abdou used the extended tanh method [6]; in 2015, Tuluce Demiray et al. used the extended trial equation method [7]; in 2007, Zhang et al. used a direct algebraic method [8, 9]; in 2008 Wazwaz and Abdul-Majid used the extended tanh method [10]; in 2009, Zhang et al. used the new generalized algebraic method [11]; in 2005, Wazwaz and Abdul-Majid used the sine-cosine algorithm [12]; in 2011, Kabir et al. used the modified Kudryashov method [13]; in 2009, Layeni used the rational auxiliary equation method [14]; in 2010, Betchewe et al. used [15], and so on [16–20].

From the above discussed studies, lots of methods were derived to solve the nonlinear partial differential equation models as follows: the $(\frac{G'}{G})$-expansion method extended the $(\frac{G'}{G})$-expansion method, the tanh-function method extended a simple equation method, the modified simple equation method extended the Jacobian method, a novel $(\frac{G'}{G})$-expansion method, and so on [21–55].

This paper is regulated as follows: In Sect. 2, we apply a generalized Kudryashov method and a novel $(\frac{G'}{G})$-expansion method to get the exact and solitary traveling wave solutions of the nonlinear complex fractional generalized-Zakharov equations. In Sect. 3, conclusion is presented and discussed.

2 Formulation for the nonlinear complex fractional generalized-Zakharov equations

Consider the nonlinear complex fractional generalized-Zakharov equations in the form [21–23]:

$$\begin{cases} i\frac{D^{\alpha}P}{Dt^{\alpha}} + P_{xx} - 2\delta|P|^2P + 2PQ = 0, \\ \frac{D^{2\alpha}Q}{Dt^{2\alpha}} - Q_{xx} + (|P|^2)_{xx} = 0. \end{cases} \tag{1}$$

Using the following traveling wave transformation $[P(x,t) = Y(x,t)e^{i\phi}, \phi = kx + \frac{ct^{\alpha}}{\alpha}, Y(x,t) = Y(\xi), Q(x,t) = Q(\xi), \xi = x - \frac{2kt^{\alpha}}{\alpha}]$ on system (1), we obtain

$$\begin{cases} Y'' - (c+k^2)Y - 2\delta Y^3 + 2YQ = 0, \\ (4k^2-1)Q'' + (Y^2)'' = 0. \end{cases} \tag{2}$$

Integrating twice the second equation in system (2), where the constant of integration equals zero, we get $[Q(\xi) = \frac{Y^2}{1-4k^2}]$. Substituting $Q(\xi)$ into the first equation of system (2), we obtain

$$Y'' - (c+k^2)Y + \left(\frac{2}{1-4k^2} - 2\delta\right)Y^3 = 0. \tag{3}$$

Balancing Eq. (3) by using the relation between the highest order derivative term and the nonlinear term, we get $Y''\&Y^3 \Rightarrow N+2 = 3N \Rightarrow N = 1$.

2.1 Exact and solitary wave solution of the nonlinear complex fractional generalized-Zakharov equations by using a generalized Kudryashov method

We use a generalized Kudryashov method on the nonlinear complex fractional generalized-Zakharov equations that suppose the general solution of it in the following form:

$$Y(\xi) = \frac{a_0 + a_1E(\xi) + a_2E^2(\xi)}{b_0 + b_1E(\xi)}. \tag{4}$$

Substituting Eq. (4) and its derivatives into Eq. (3) and equating the coefficient of different power of $E^i(\xi)$ to zero, we obtain a system of algebraic equations by solving it with any computer program like (Maple, Mathematica and so on). We get the following:

$$k=\sqrt{1-c},\qquad a_0=0,\qquad a_1=-b_1\sqrt{\frac{4c-3}{4c\delta-3\delta-1}},\qquad a_2=b_1\sqrt{\frac{4c-3}{4c\delta-3\delta-1}},$$
$$b_0=-\frac{b_1}{2},$$

where $[\frac{4c-3}{4c\delta-3\delta-1}>0, c\neq 1]$. Consequently, the solitary wave solution for Eqs. (1) is as follows:

$$P(x,t)=\frac{2\sqrt{\frac{4c-3}{-1+(4c-3)\delta}}\alpha \mathrm{e}^{\frac{x\alpha-2kt^{\alpha}}{\alpha}}}{\alpha^2\mathrm{e}^{\frac{-4kt^{\alpha}+2x\alpha}{\alpha}}-1}e^{i(kx+\frac{ct^{\alpha}}{\alpha})}, \tag{5}$$

$$Q(x,t)=\frac{(-4c+3)\alpha^2\mathrm{e}^{\frac{-4kt^{\alpha}+2x\alpha}{\alpha}}}{(4c\delta-3\delta-1)(\mathrm{e}^{\frac{-4kt^{\alpha}+2x\alpha}{\alpha}}\alpha^2-1)^2(-1+c)}. \tag{6}$$

2.2 Exact and solitary wave solution of the nonlinear complex fractional generalized-Zakharov equations by using novel $(\frac{G'}{G})$-expansion method

We use a novel $(\frac{G'}{G})$-expansion method on the nonlinear complex fractional generalized-Zakharov equations that suppose the general solution of it in the following form:

$$Y(\xi)=\frac{a_{-1}}{(d+\frac{G'}{G})}+a_0+a_1\left(d+\frac{G'}{G}\right). \tag{7}$$

Substituting Eq. (7) and its derivatives into Eq. (3) and equating the coefficient of different power of $(d+\frac{G'}{G})^i$ to zero, we obtain a system of algebraic equations by solving it with any computer program like (Maple, Mathematica, Matlap and so on). We get the following:

Family 1

$$\mu=\frac{1}{4}\frac{2k^2+\lambda^2+2c}{\nu-1},\qquad a_{-1}=0,$$
$$a_0=\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}},\qquad a_1=-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}.$$

Consequently, the solitary wave solution for Eqs. (1) is as follows:

- *When* $(\Omega=\lambda^2-4\lambda\mu+4\mu>0)$ *and* $(\lambda(\nu-1)\neq 0)$ *or* $(\mu(\nu-1)\neq 0)$*:*

$$P_{11}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}\left(c-\frac{2kt^{\alpha}}{\alpha}\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^{\alpha}}{\alpha})}, \tag{8}$$

$$Q_{11}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right]^2. \tag{9}$$

$$P_{12}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{10}$$

$$Q_{12}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right]^2. \tag{11}$$

$$P_{13}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\left(\tanh\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\pm i\,\mathrm{sech}\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{12}$$

$$Q_{13}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\left(\tanh\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\pm i\,\mathrm{sech}\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{13}$$

$$P_{14}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\left(\coth\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\pm \mathrm{csch}\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{14}$$

$$Q_{14}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{\Omega}\left(\coth\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\pm \mathrm{csch}\left(\sqrt{\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{15}$$

$$P_{15}(x,t)=\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\Bigg(d-\frac{1}{4(\nu-1)}\Bigg(2\lambda+\sqrt{\Omega}\Bigg(\tanh\Bigg(\frac{\sqrt{\Omega}}{4}\Bigg(c-\frac{2kt^\alpha}{\alpha}\Bigg)\Bigg)\pm\coth\Bigg(\frac{\sqrt{\Omega}}{4}\Bigg(c-\frac{2kt^\alpha}{\alpha}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})},\tag{16}$$

$$Q_{15}(x,t)=\frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\Bigg(d-\frac{1}{4(\nu-1)}\Bigg(2\lambda+\sqrt{\Omega}\Bigg(\tanh\Bigg(\frac{\sqrt{\Omega}}{4}\Bigg(c-\frac{2kt^\alpha}{\alpha}\Bigg)\Bigg)\pm\coth\Bigg(\frac{\sqrt{\Omega}}{4}\Bigg(c-\frac{2kt^\alpha}{\alpha}\Bigg)\Bigg)\Bigg)\Bigg)\Bigg)\Bigg]^2.\tag{17}$$

$$P_{16}(x,t)=\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\Bigg(d+\frac{1}{2(\nu-1)}\Bigg(-\lambda+\frac{\pm\sqrt{\Omega(A^2+B^2)}-A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\Bigg)\Bigg)\Bigg]\times e^{i(kx+\frac{ct^\alpha}{\alpha})},\tag{18}$$

$$Q_{16}(x,t)=\frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\Bigg(d+\frac{1}{2(\nu-1)}\Bigg(-\lambda+\frac{\pm\sqrt{\Omega(A^2+B^2)}-A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\Bigg)\Bigg)\Bigg]^2.\tag{19}$$

$$P_{17}(x,t)=\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\Bigg(d+\frac{1}{2(\nu-1)}\Bigg(-\lambda+\frac{\pm\sqrt{\Omega(A^2+B^2)}+A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\Bigg)\Bigg)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})},\tag{20}$$

$$Q_{17}(x,t)=\frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\times\Bigg(d+\frac{1}{2(\nu-1)}\Bigg(-\lambda+\frac{\pm\sqrt{\Omega(A^2+B^2)}+A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\Bigg)\Bigg)\Bigg]^2.\tag{21}$$

$$
\begin{aligned}
P_{18}(x,t) &= \left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (22)\\
Q_{18}(x,t) &= \frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]^2. \qquad (23)\\
P_{19}(x,t) &= \left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (24)\\
Q_{19}(x,t) &= \frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]^2. \qquad (25)\\
P_{110}(x,t) &= \left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\right)\right]\\
&\quad\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (26)\\
Q_{110}(x,t) &= \frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\right)\right]^2. \qquad (27)\\
P_{111}(x,t) &= \left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\right)\right]\\
&\quad\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (28)\\
Q_{111}(x,t) &= \frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.\\
&\quad\left.\times\left(d+\frac{2\mu\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\right)\right]^2, \qquad (29)
\end{aligned}
$$

where A, B are arbitrary real constants and $A^2+B^2>0$.

• *When* $(\Omega = \lambda^2 - 4\lambda\mu + 4\mu < 0)$ *and* $(\lambda(\nu-1) \neq 0)$ *or* $(\mu(\nu-1) \neq 0)$*:*

$$P_{112}(x,t) = \Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d+\frac{1}{2(\nu-1)}\left(-\lambda+\sqrt{-\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \quad (30)$$

$$Q_{112}(x,t) = \frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d+\frac{1}{2(\nu-1)}\left(-\lambda+\sqrt{-\Omega}\tan\left(\frac{\sqrt{-\Omega}}{2}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\Bigg]^2. \quad (31)$$

$$P_{113}(x,t) = \Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{-\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \quad (32)$$

$$Q_{113}(x,t) = \frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{-\Omega}\cot\left(\frac{\sqrt{-\Omega}}{2}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\Bigg]^2. \quad (33)$$

$$P_{114}(x,t) = \Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d+\frac{1}{2(\nu-1)}\left(-\lambda+\sqrt{-\Omega}\left(\tan\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right) \pm \sec\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \quad (34)$$

$$Q_{114}(x,t) = \frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d+\frac{1}{2(\nu-1)}\left(-\lambda+\sqrt{-\Omega}\left(\tan\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right) \pm \sec\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\Bigg]^2. \quad (35)$$

$$P_{115}(x,t) = \Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d-\frac{1}{2(\nu-1)}\left(\lambda+\sqrt{-\Omega}\left(\cot\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right) \pm \csc\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \quad (36)$$

$$Q_{115}(x,t) = \frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d - \frac{1}{2(\nu-1)}\left(\lambda + \sqrt{-\Omega}\left(\cot\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right) \pm \csc\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{37}$$

$$P_{116}(x,t) = \left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d + \frac{1}{4(\nu-1)}\left(-2\lambda + \sqrt{-\Omega}\left(\tan\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right) - \cot\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{38}$$

$$Q_{116}(x,t) = \frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d + \frac{1}{4(\nu-1)}\left(-2\lambda + \sqrt{-\Omega}\left(\tan\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right) - \cot\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{39}$$

$$P_{117}(x,t) = \left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d + \frac{1}{2(\nu-1)}\left(-\lambda + \frac{\pm\sqrt{-\Omega(A^2-B^2)} - A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{40}$$

$$Q_{117}(x,t) = \frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d + \frac{1}{2(\nu-1)}\left(-\lambda + \frac{\pm\sqrt{-\Omega(A^2-B^2)} - A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\right)\right)\right]^2. \tag{41}$$

$$P_{118}(x,t) = \left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}} - \frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}} \times \left(d + \frac{1}{2(\nu-1)}\left(-\lambda + \frac{\pm\sqrt{-\Omega(A^2-B^2)} + A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{42}$$

$$Q_{118}(x,t)=\frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}$$
$$\times\Bigg(d+\frac{1}{2(\nu-1)}\Bigg(-\lambda$$
$$+\frac{\pm\sqrt{-\Omega(A^2-B^2)}+A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\Bigg)\Bigg)\Bigg]^2. \tag{43}$$

$$P_{119}(x,t)=\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}$$
$$\times\Bigg(d-\frac{2\mu\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\Bigg)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{44}$$

$$Q_{119}(x,t)=\frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}$$
$$\times\Bigg(d-\frac{2\mu\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\Bigg)\Bigg]^2. \tag{45}$$

$$P_{120}(x,t)=\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}$$
$$\times\Bigg(d+\frac{2\mu\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\Bigg)\Bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{46}$$

$$Q_{120}(x,t)=\frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}$$
$$\times\Bigg(d+\frac{2\mu\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\Bigg)\Bigg]^2. \tag{47}$$

$$P_{121}(x,t)=\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}$$
$$\times\Bigg(d-\frac{2\mu\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm\sqrt{-\Omega}}\Bigg)\Bigg]$$
$$\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{48}$$

$$Q_{121}(x,t)=\frac{1}{1-4k^2}\Bigg[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}$$
$$\times\Bigg(d-\frac{2\mu\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm\sqrt{-\Omega}}\Bigg)\Bigg]^2. \tag{49}$$

$$P_{122}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d+\frac{2\mu\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm\sqrt{-\Omega}}\right)\right]$$
$$\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{50}$$

$$Q_{122}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d+\frac{2\mu\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm\sqrt{-\Omega}}\right)\right]$$
$$\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{51}$$

where A, B are arbitrary real constants and $A^2-B^2>0$.

- *When* $\mu=0$ *and* $\lambda(\nu-1)\neq 0$, *we have:*

$$P_{123}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d-\frac{\lambda k}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha}))-\sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{52}$$

$$Q_{123}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d-\frac{\lambda k}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha}))-\sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right]^2. \tag{53}$$

$$P_{124}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d-\frac{\lambda(\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha}))+\sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha}))+\sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{54}$$

$$Q_{124}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d-\frac{\lambda(\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha}))+\sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha}))+\sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right]^2. \tag{55}$$

$$P_{125}(x,t)=\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d-\frac{1}{(\nu-1)(c-\frac{2kt^\alpha}{\alpha})+C}\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{56}$$

$$Q_{125}(x,t)=\frac{1}{1-4k^2}\left[\frac{1}{2}\big((2\nu-2)d-\lambda\big)\sqrt{\frac{4k^2-1}{4\delta k^2-\delta+1}}-\frac{4k^2\nu-4k^2-\nu+1}{4\delta k^2-\delta+1}\frac{1}{\sqrt{\frac{4k^2+1}{4\delta k^2-\delta+1}}}\right.$$
$$\left.\times\left(d-\frac{1}{(\nu-1)(c-\frac{2kt^\alpha}{\alpha})+C}\right)\right]^2. \tag{57}$$

Family 2

$$k = \sqrt{\frac{\delta - 1}{4\delta}}, \qquad \nu = 1, \qquad a_{-1} = 0, \qquad a_0 = -\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1}.$$

So, the solitary traveling wave solutions are as follows:

- *When* $(\Omega = \lambda^2 - 4\lambda\mu + 4\mu > 0)$ *and* $(\lambda(\nu - 1) \neq 0)$ *or* $(\mu(\nu - 1) \neq 0)$*:*

$$P_{21}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right]e^{i(kx + \frac{ct^\alpha}{\alpha})}, \tag{58}$$

$$Q_{21}(x,t) = \frac{1}{1 - 4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\tanh\left(\frac{\sqrt{\Omega}}{2}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right]^2. \tag{59}$$

$$P_{22}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right]e^{i(kx + \frac{ct^\alpha}{\alpha})}, \tag{60}$$

$$Q_{22}(x,t) = \frac{1}{1 - 4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\coth\left(\frac{\sqrt{\Omega}}{2}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right]^2. \tag{61}$$

$$P_{23}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\left(\tanh\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right) \pm i\,\mathrm{sech}\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx + \frac{ct^\alpha}{\alpha})}, \tag{62}$$

$$Q_{23}(x,t) = \frac{1}{1 - 4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\left(\tanh\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right) \pm i\,\mathrm{sech}\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{63}$$

$$P_{24}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\left(\coth\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right) \pm \mathrm{csch}\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx + \frac{ct^\alpha}{\alpha})}, \tag{64}$$

$$Q_{24}(x,t) = \frac{1}{1 - 4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda - \mu)}{4c\delta + \delta - 1} + a_1\left(d - \frac{1}{2(\nu - 1)}\left(\lambda + \sqrt{\Omega}\left(\coth\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right) \pm \mathrm{csch}\left(\sqrt{\Omega}\left(c - \frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{65}$$

$$P_{25}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{1}{4(\nu-1)}\left(2\lambda + \sqrt{\Omega}\left(\tanh\left(\frac{\sqrt{\Omega}}{4}\left(c - \frac{2kt^{\alpha}}{\alpha}\right)\right)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\pm\coth\left(\frac{\sqrt{\Omega}}{4}\left(c - \frac{2kt^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^{\alpha}}{\alpha})}, \tag{66}$$

$$Q_{25}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$+ a_1\left(d - \frac{1}{4(\nu-1)}\left(2\lambda + \sqrt{\Omega}\left(\tanh\left(\frac{\sqrt{\Omega}}{4}\left(c - \frac{2kt^{\alpha}}{\alpha}\right)\right)\right.\right.\right.$$
$$\left.\left.\left.\left.\pm\coth\left(\frac{\sqrt{\Omega}}{4}\left(c - \frac{2kt^{\alpha}}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{67}$$

$$P_{26}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda\right.\right.\right.$$
$$\left.\left.\left.+\frac{\pm\sqrt{\Omega(A^2+B^2)} - A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))+B}\right)\right)\right]e^{i(kx+\frac{ct^{\alpha}}{\alpha})}, \tag{68}$$

$$Q_{26}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda\right.\right.\right.$$
$$\left.\left.\left.+\frac{\pm\sqrt{\Omega(A^2+B^2)} - A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))+B}\right)\right)\right]^2. \tag{69}$$

$$P_{27}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda\right.\right.\right.$$
$$\left.\left.\left.+\frac{\pm\sqrt{\Omega(A^2+B^2)} + A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))+B}\right)\right)\right]e^{i(kx+\frac{ct^{\alpha}}{\alpha})}, \tag{70}$$

$$Q_{27}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda\right.\right.\right.$$
$$\left.\left.\left.+\frac{\pm\sqrt{\Omega(A^2+B^2)} + A\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))}{A\sinh(\sqrt{\Omega}(c-\frac{2kt^{\alpha}}{\alpha}))+B}\right)\right)\right]^2. \tag{71}$$

$$P_{28}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$\left.+ a_1\left(d + \frac{2\mu\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha}))}{\sqrt{\Omega}\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha})) - \lambda\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha}))}\right)\right]e^{i(kx+\frac{ct^{\alpha}}{\alpha})}, \tag{72}$$

$$Q_{28}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$\left.+ a_1\left(d + \frac{2\mu\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha}))}{\sqrt{\Omega}\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha})) - \lambda\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha}))}\right)\right]^2. \tag{73}$$

$$P_{29}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$\left.+ a_1\left(d + \frac{2\mu\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha}))}{\sqrt{\Omega}\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha})) - \lambda\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^{\alpha}}{\alpha}))}\right)\right]e^{i(kx+\frac{ct^{\alpha}}{\alpha})}, \tag{74}$$

$$
\begin{aligned}
Q_{19}(x,t) = \frac{1}{1-4k^2}\bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\\
&+a_1\bigg(d+\frac{2\mu\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\cosh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sinh(\frac{\sqrt{\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\bigg)\bigg]^2. \qquad (75)
\end{aligned}
$$

$$
\begin{aligned}
P_{210}(x,t) = \bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\\
&+a_1\bigg(d+\frac{2\mu\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\bigg)\bigg]\\
&\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (76)
\end{aligned}
$$

$$
\begin{aligned}
Q_{210}(x,t) = \frac{1}{1-4k^2}\bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\bigg(d\\
&+\frac{2\mu\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\bigg)\bigg]^2. \qquad (77)
\end{aligned}
$$

$$
\begin{aligned}
P_{211}(x,t) = \bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\\
&+a_1\bigg(d+\frac{2\mu\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\bigg)\bigg]\\
&\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (78)
\end{aligned}
$$

$$
\begin{aligned}
Q_{211}(x,t) = \frac{1}{1-4k^2}\bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\bigg(d\\
&+\frac{2\mu\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{\Omega}\cosh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sinh(\sqrt{\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm i\sqrt{\Omega}}\bigg)\bigg]^2, \qquad (79)
\end{aligned}
$$

where A, B are arbitrary real constants and $A^2+B^2>0$.

- *When* $(\Omega=\lambda^2-4\lambda\mu+4\mu<0)$ *and* $(\lambda(\nu-1)\neq 0)$ *or* $(\mu(\nu-1)\neq 0)$*:*

$$
\begin{aligned}
P_{212}(x,t) = \bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\\
&+a_1\bigg(d+\frac{1}{2(\nu-1)}\bigg(-\lambda+\sqrt{-\Omega}\tan\bigg(\frac{\sqrt{-\Omega}}{2}\bigg(c-\frac{2kt^\alpha}{\alpha}\bigg)\bigg)\bigg)\bigg)\bigg]\\
&\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (80)
\end{aligned}
$$

$$
\begin{aligned}
Q_{212}(x,t) = \frac{1}{1-4k^2}\bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\\
&+a_1\bigg(d+\frac{1}{2(\nu-1)}\bigg(-\lambda+\sqrt{-\Omega}\tan\bigg(\frac{\sqrt{-\Omega}}{2}\bigg(c-\frac{2kt^\alpha}{\alpha}\bigg)\bigg)\bigg)\bigg)\bigg]^2. \qquad (81)
\end{aligned}
$$

$$
\begin{aligned}
P_{213}(x,t) = \bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\\
&+a_1\bigg(d-\frac{1}{2(\nu-1)}\bigg(\lambda+\sqrt{-\Omega}\cot\bigg(\frac{\sqrt{-\Omega}}{2}\bigg(c-\frac{2kt^\alpha}{\alpha}\bigg)\bigg)\bigg)\bigg)\bigg]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \qquad (82)
\end{aligned}
$$

$$
\begin{aligned}
Q_{213}(x,t) = \frac{1}{1-4k^2}\bigg[&-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\\
&+a_1\bigg(d-\frac{1}{2(\nu-1)}\bigg(\lambda+\sqrt{-\Omega}\cot\bigg(\frac{\sqrt{-\Omega}}{2}\bigg(c-\frac{2kt^\alpha}{\alpha}\bigg)\bigg)\bigg)\bigg)\bigg]^2. \qquad (83)
\end{aligned}
$$

$$P_{214}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda + \sqrt{-\Omega}\left(\tan\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\pm\sec\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{84}$$

$$Q_{214}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$+ a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda + \sqrt{-\Omega}\left(\tan\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right.\right.\right.$$
$$\left.\left.\left.\left.\pm\sec\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{85}$$

$$P_{215}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{1}{2(\nu-1)}\left(\lambda + \sqrt{-\Omega}\left(\cot\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right.\right.\right.\right.$$
$$\left.\left.\left.\left.\pm\csc\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{86}$$

$$Q_{215}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$+ a_1\left(d - \frac{1}{2(\nu-1)}\left(\lambda + \sqrt{-\Omega}\left(\cot\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right.\right.\right.$$
$$\left.\left.\left.\left.\pm\csc\left(\sqrt{-\Omega}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{87}$$

$$P_{216}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$+ a_1\left(d + \frac{1}{4(\nu-1)}\left(-2\lambda + \sqrt{-\Omega}\left(\tan\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right.\right.\right.$$
$$\left.\left.\left.\left.-\cot\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{88}$$

$$Q_{216}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}\right.$$
$$+ a_1\left(d + \frac{1}{4(\nu-1)}\left(-2\lambda + \sqrt{-\Omega}\left(\tan\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right.\right.\right.$$
$$\left.\left.\left.\left.-\cot\left(\frac{\sqrt{-\Omega}}{4}\left(c-\frac{2kt^\alpha}{\alpha}\right)\right)\right)\right)\right)\right]^2. \tag{89}$$

$$P_{217}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda\right.\right.\right.$$
$$\left.\left.\left.+\frac{\pm\sqrt{-\Omega(A^2-B^2)} - A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\right)\right)\right]$$
$$\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{90}$$

$$Q_{217}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{1}{2(\nu-1)}\left(-\lambda\right.\right.\right.$$
$$\left.\left.\left.+\frac{\pm\sqrt{-\Omega(A^2-B^2)} - A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\right)\right)\right]^2. \tag{91}$$

$$P_{218}(x,t)=\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d+\frac{1}{2(\nu-1)}\left(-\lambda+\frac{\pm\sqrt{-\Omega(A^2-B^2)}+A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\right)\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{92}$$

$$Q_{218}(x,t)=\frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d+\frac{1}{2(\nu-1)}\left(-\lambda+\frac{\pm\sqrt{-\Omega(A^2-B^2)}+A\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{A\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+B}\right)\right)\right]^2. \tag{93}$$

$$P_{219}(x,t)=\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d-\frac{2\mu\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{94}$$

$$Q_{219}(x,t)=\frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d-\frac{2\mu\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]^2. \tag{95}$$

$$P_{220}(x,t)=\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d+\frac{2\mu\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{96}$$

$$Q_{220}(x,t)=\frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d+\frac{2\mu\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sin(\frac{\sqrt{-\Omega}}{2}(c-\frac{2kt^\alpha}{\alpha}))}\right)\right]^2. \tag{97}$$

$$P_{221}(x,t)=\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d-\frac{2\mu\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm\sqrt{-\Omega}}\right)\right]\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{98}$$

$$Q_{221}(x,t)=\frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d-\frac{2\mu\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))+\lambda\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm\sqrt{-\Omega}}\right)\right]^2. \tag{99}$$

$$P_{222}(x,t)=\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1}+a_1\left(d+\frac{2\mu\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))-\lambda\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))\pm\sqrt{-\Omega}}\right)\right]\times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{100}$$

$$Q_{222}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d + \frac{2\mu\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha}))}{\sqrt{-\Omega}\cos(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha})) - \lambda\sin(\sqrt{-\Omega}(c-\frac{2kt^\alpha}{\alpha})) \pm \sqrt{-\Omega}}\right)\right] \times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{101}$$

where A, B are arbitrary real constants and $A^2 - B^2 > 0$.

- *When* $\mu = 0$ *and* $\lambda(\nu-1) \neq 0$*, we have:*

$$P_{223}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{\lambda k}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha})) - \sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right] \times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{102}$$

$$Q_{223}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{\lambda k}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha})) - \sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right]^2. \tag{103}$$

$$P_{224}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{\lambda(\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha})) + \sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha})) + \sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right] \times e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{104}$$

$$Q_{224}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{\lambda(\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha})) + \sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}{(\nu-1)(k+\cosh(\lambda(c-\frac{2kt^\alpha}{\alpha})) + \sinh(\lambda(c-\frac{2kt^\alpha}{\alpha})))}\right)\right]^2. \tag{105}$$

$$P_{225}(x,t) = \left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{1}{(\nu-1)(c-\frac{2kt^\alpha}{\alpha})+C}\right)\right]e^{i(kx+\frac{ct^\alpha}{\alpha})}, \tag{106}$$

$$Q_{225}(x,t) = \frac{1}{1-4k^2}\left[-\frac{4\delta\lambda a_1(d\lambda-\mu)}{4c\delta+\delta-1} + a_1\left(d - \frac{1}{(\nu-1)(c-\frac{2kt^\alpha}{\alpha})+C}\right)\right]^2, \tag{107}$$

where C, k are arbitrary constants.

3 Conclusion

We implement two recent methods called a generalized Kudryashov method and a novel $(\frac{G'}{G})$-expansion method. We obtain many forms of solutions that cover all previous solutions obtained by implementation of different methods [1–20]. We get new forms of analytical wave solutions for the nonlinear complex fractional generalized-Zakharov equations. Studying the physical properties of these kinds of models is very motivating and interesting. As we see, both of these methods are very direct, effective, and powerful, and we also showed the ability of these methods to be applied to different kinds of nonlinear partial differential equations whether they are of the fractional order or of the integer order.

Acknowledgements
The authors thank the referees for their suggestions and comments.

Funding
Funding information is not applicable / No funding was received.

Competing interests
This research received no specific grant from any funding agency in the public, commercial, or not-for-profit sectors. The authors did not have any competing interests in this research.

Authors' contributions
All parts contained in the research were carried out by the authors through hard work and review of the various references and contributions in the field of mathematics and applied physics. All authors read and approved the final manuscript.

Author details
[1]Department of Mathematics, Faculty of Science, Jiangsu University, Zhenjiang, China. [2]Mathematics Department, Faculty of Science, Taibah University, Al-Madinah Al-Munawarah, Saudi Arabia. [3]Mathematics Department, Faculty of Science, Beni-Suef University, Beni-Suef, Egypt.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 11 April 2018 Accepted: 27 July 2018 Published online: 03 August 2018

References

1. Malfliet, W., Hereman, W.: The tanh method: I. Exact solutions of nonlinear evolution and wave equations. Phys. Scr. **54**(6), 563 (1996)
2. Borhanifar, A., Kabir, M.M., Vahdat, L.M.: New periodic and soliton wave solutions for the generalized Zakharov system and (2 + 1)-dimensional Nizhnik–Novikov–Veselov system. Chaos Solitons Fractals **42**(3), 1646–1654 (2009)
3. Li, B., Chen, Y., Zhang, H.: Exact travelling wave solutions for a generalized Zakharov–Kuznetsov equation. Appl. Math. Comput. **146**(2), 653–666 (2003)
4. Yong, C., Biao, L.: New exact travelling wave solutions for generalized Zakharov–Kuznetsov equations using general projective Riccati equation method. Commun. Theor. Phys. **41**, 1 (2004)
5. Naher, H., Abdullah, F.A., Akbar, M.A.: New traveling wave solutions of the higher dimensional nonlinear partial differential equation by the Exp-function method. J. Appl. Math. **2012**, Article ID 575387 (2012)
6. Abdou, M.A.: The extended tanh method and its applications for solving nonlinear physical models. Appl. Math. Comput. **190**(1), 988–996 (2007)
7. Tuluce Demiray, S., Bulut, H.: Some exact solutions of generalized Zakharov system. Waves Random Complex Media **25**(1), 75–90 (2015)
8. Zhang, H.: New exact travelling wave solutions of the generalized Zakharov equations. Rep. Math. Phys. **60**(1), 97–106 (2007)
9. Seadawy, A.: Modulation instability analysis for the generalized derivative higher order nonlinear Schrödinger equation and its the bright and dark soliton solutions. J. Electromagn. Waves Appl. **31**(14), 1353–1362 (2017)
10. Wazwaz, A.-M.: The extended tanh method for the Zakharov–Kuznetsov (ZK) equation, the modified ZK equation, and its generalized forms. Commun. Nonlinear Sci. Numer. Simul. **13**(6), 1039–1047 (2008)
11. Zhang, L.-H.: Travelling wave solutions for the generalized Zakharov–Kuznetsov equation with higher-order nonlinear terms. Appl. Math. Comput. **208**(1), 144–155 (2009)
12. Wazwaz, A.-M.: Exact solutions with solitons and periodic structures for the Zakharov–Kuznetsov (ZK) equation and its modified form. Commun. Nonlinear Sci. Numer. Simul. **10**(6), 597–606 (2005)
13. Kabir, M.M., Khajeh, A., Abdi Aghdam, E., Yousefi Koma, A.: Modified Kudryashov method for finding exact solitary wave solutions of higher-order nonlinear equations. Math. Methods Appl. Sci. **34**(2), 213–219 (2011)
14. Layeni, O.P.: A new rational auxiliary equation method and exact solutions of a generalized Zakharov system. Appl. Math. Comput. **215**(8), 2901–2907 (2009)
15. Betchewe, G., Bouetou, T.B., Victor, K.K., Kofane, T.C.: Dynamical survey of a generalized-Zakharov equation and its exact travelling wave solutions. Appl. Math. Comput. **217**(1), 203–211 (2010)
16. Shang, Y., Huang, Y., Yuan, W.: New exact traveling wave solutions for the Klein–Gordon–Zakharov equations. Comput. Math. Appl. **56**(5), 1441–1450 (2008)
17. Jin, S., Markowich, P.A., Zheng, C.: Numerical simulation of a generalized Zakharov system. J. Comput. Phys. **201**(1), 376–395 (2004)
18. Al-Muhiameed, Z.I.A., Abdel-Salam, E.A.-B.: Generalized Jacobi elliptic function solution to a class of nonlinear Schrödinger-type equations. Math. Probl. Eng. **2011**, Article ID 575679 (2011)
19. El-Wakil, S.A., Degheidy, A.R., Abulwafa, E.M., Madkour, M.A., Attia, M.T., Abdou, M.A.: Exact travelling wave solutions of generalized Zakharov equations with arbitrary power nonlinearities. Int. J. Nonlinear Sci. **7**(4), 455–461 (2009)
20. Huang, D.-J., Zhang, H.-Q.: New exact travelling waves solutions to the combined KdV-MKdV and generalized Zakharov equations. Rep. Math. Phys. **57**(2), 257–269 (2006)

21. Lu, D., Seadawy, A.R., Ali, A.: Applications of exact traveling wave solutions of modified Liouville and the symmetric regularized long wave equations via two new techniques. Results Phys. **9**, 1403–1410 (2018)
22. Khater, M., Seadawy, A.R., Lu, D.: New optical soliton solutions for nonlinear complex fractional Schrödinger equation via new auxiliary equation method and novel G'/G-expansion method. Pramana **90**, 59 (2018)
23. Seadawy, A.R., Lu, D., Khater, M.M.A.: New wave solutions for the fractional-order biological population model, time fractional burgers, Drinfel'd–Sokolov–Wilson and system of shallow water wave equations and their applications. Eur. J. Comput. Mech. **26**, 508–524 (2017)
24. Seadawy, A.R., Lu, D., Khater, M.M.A.: Bifurcations of traveling wave solutions for Dodd–Bullough–Mikhailov equation and coupled Higgs equation and their applications. Chin. J. Phys. **55**(4), 1310–1318 (2017)
25. Khater, M.M.A., Seadawy, A.R., Lu, D.: Elliptic and solitary wave solutions for Bogoyavlenskii equations system, couple Boiti–Leon–Pempinelli equations system and time-fractional Cahn–Allen equation. Results Phys. **7**, 2325–2333 (2017)
26. Lu, D., Seadawy, A.R., Khater, M.M.A.: Bifurcations of new multi soliton solutions of the van der Waals normal form for fluidized granular matter via six different methods. Results Phys. **7**, 2028–2035 (2017)
27. Hafez, M.G., Alam, M.N., Akbar, M.A.: Exact traveling wave solutions of the Klein–Gordon equation using the novel (G'/G)-expansion method. Results Phys. **4**, 177–184 (2014)
28. Alam, M.N., Akbar, M.A., Mohyud-Din, S.T.: General traveling wave solutions of the strain wave equation in microstructured solids via the new approach of generalized (G'/G)-expansion method. Alex. Eng. J. **53**(1), 233–241 (2014)
29. Alam, M.N., Akbar, M.A., Harun-Or-Roshid: Study of nonlinear evolution equations to construct traveling wave solutions via the new approach of the generalized (G'/G)-expansion method. Math. Stat. **1**(3), 102–112 (2013)
30. Islam, R., Alam, M.N., Hossain, A.K.M.K.S., Harun-Or-Roshid, Akbar, M.A.: Traveling wave solutions of nonlinear evolution equations via $\mathrm{Exp}(-\Phi(\eta))$-expansion method. Glob. J. Sci. Front. Res. **13**(11), 63–71 (2013)
31. Ravichandran, C., Jothimani, K., Baskonus, H.M., Valliammal, N.: New results on nondensely characterized integrodifferential equations with fractional order. Eur. Phys. J. Plus **133**(109), 1–10 (2018)
32. Baskonus, H.M., Hammouch, Z., Mekkaoui, T., Bulut, H.: Chaos in the fractional order logistic delay system: circuit realization and synchronization. AIP Conf. Proc. **1738**, 290005 (2016)
33. Baskonus, H.M., Mekkaoui, T., Hammouch, Z., Bulut, H.: Active control of a chaotic fractional order economic system. Entropy **17**(8), 5771–5783 (2015)
34. Baskonus, H.M., Bulut, H.: On the numerical solutions of some fractional ordinary differential equations by fractional Adams–Bashforth–Moulton method. Open Math. **13**(1), 547–556 (2015)
35. Esen, A., Sulaiman, T.A., Bulutb, H., Baskonus, H.M.: Optical solitons to the space-time fractional $(1 + 1)$-dimensional coupled nonlinear Schrödinger equation. Optik, Int. J. Light Electron Opt. **167**, 150–156 (2018)
36. Zhang, Z.: New exact traveling wave solutions for the nonlinear Klein–Gordon equation. Turk. J. Phys. **32**, 235–240 (2008)
37. Zhang, Z., Liu, Z., Miao, X., Chen, Y.: New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Appl. Math. Comput. **216**, 3064–3072 (2010)
38. Zhang, Z., Li, Y., Liu, Z., Miao, X.: New exact solutions to the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity via modified trigonometric function series method. Commun. Nonlinear Sci. Numer. Simul. **16**(8), 3097–3106 (2011)
39. Zhang, Z., Liu, Z., Miao, X., Chen, Y.: Qualitative analysis and traveling wave solutions for the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Phys. Lett. A **375**, 1275–1280 (2011)
40. Zhang, Z., Xia, F.-L., Li, X.-P.: Bifurcation analysis and the travelling wave solutions of the Klein–Gordon–Zakharov equations. Pramana **80**(1), 41–59 (2013)
41. Zhang, Z.-Y., Gan, X.-Y., Yu, D.-M., Zhang, Y.-H., Li, X.-P.: A note on exact traveling wave solutions of the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Commun. Theor. Phys. **57**, 764–770 (2012)
42. Miao, X., Zhang, Z.: The modified (G/G')-expansion method and traveling wave solutions of nonlinear the perturbed nonlinear Schrödinger's equation with Kerr law nonlinearity. Commun. Nonlinear Sci. Numer. Simul. **16**(11), 4259–4267 (2011)
43. Khater, A.H., Callebaut, D.K., Seadawy, A.R.: General soliton solutions of an n-dimensional complex Ginzburg–Landau equation. Phys. Scr. **62**, 353–357 (2000)
44. Seadawy, A.: The generalized nonlinear higher order of KdV equations from the higher order nonlinear Schrödinger equation and its solutions. Optik, Int. J. Light Electron Opt. **139**, 31–43 (2017)
45. Seadawy, A.R.: Travelling wave solutions of a weakly nonlinear two-dimensional higher order Kadomtsev–Petviashvili dynamical equation for dispersive shallow water waves. Eur. Phys. J. Plus **132**, 29 (2017)
46. Khater, A.H., Callebaut, D.K., Seadawy, A.R.: Nonlinear dispersive Kelvin–Helmholtz instabilities in magnetohydrodynamic flows. Phys. Scr. **67**, 340–349 (2003)
47. Seadawy, A.R.: Solitary wave solutions of two-dimensional nonlinear Kadomtsev–Petviashvili dynamic equation in a dust acoustic plasmas. Pramana **89**(3), 49 (2017)
48. Seadawy, A.R., El-Rashidy, K.: Rayleigh–Taylor instability of the cylindrical flow with mass and heat transfer. Pramana **87**, 20 (2016)
49. Kilic, B., Inc, M.: Soliton solutions for the Kundu–Eckhaus equation with the aid of unified algebraic and auxiliary equation expansion methods. J. Electromagn. Waves Appl. **30**, 871–879 (2016)
50. Inc, M., Ates, E., Tchier, F.: Optical solitons of the coupled nonlinear Schrödinger's equation with spatiotemporal dispersion. Nonlinear Dyn. **85**, 1319–1329 (2016)
51. Tchier, F., Inc, M., Kilic, B., Akgül, A.: On soliton structures of generalized resonance equation with time dependent coefficients. Optik **128**, 218–223 (2017)
52. Kilic, B., Inc, M.: Optical solitons for the Schrodinger–Hirota equation with power law nonlinearity by the Backlund transformation. Optik **138**, 64–67 (2017)
53. Al Qurashi, M., Baleanu, D., Inc, M.: Optical solitons of transmission equation of ultra-short optical pulse in parabolic law media with the aid of Backlund transformation. Optik **140**, 114–122 (2017)
54. Tchier, F., Aslan, E.C., Inc, M.: Optical solitons in parabolic law medium: Jacobi elliptic function solution. Nonlinear Dyn. **85**, 2577–2582 (2016)
55. Al Qurashi, M.M., Yusuf, A., Aliyu, A.I., Inc, M.: Optical and other solitons for the fourth-order dispersive nonlinear Schrödinger equation with dual-power law nonlinearity. Superlattices Microstruct. **105**, 183–197 (2017)