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Semianalytical solutions by homotopy analysis method for EIAV infection with stability analysis

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Abstract

Equine Infectious Anemia Virus (EIAV) is a bovine lentivirus group that creates equine infectious anemia in horses around the globe. In this paper, we propose a basic nonlinear differential equation for EIAV infection. Moreover, we obtain a semianalytic approximate solution by the homotopy analysis method (HAM). The gain of this method allows providing a direct scheme for solving the problem. Furthermore, we find the local and global stability of disease-free and endemic equilibrium using a Lyapunov functional. We use Mathematica to carry out the calculations. Graphical outcomes are carried out, and six terms are sufficient to show the effectiveness and power of the method.

MSC: 34G20; 34A34

Keywords: EIAV; HAM; Stability

1 Introduction

Equine Infectious Anemia (EIA) is a worldwide infection of lentivirus subfamily of retroviruses, producing fever and anemia. Human immunodeficiency virus (HIV) is also a similar lentivirus subgroup. The resemblances of these infections inspire the investigation of the immune response to EIAV appropriate to study going on HIV. In 1904, the infectious creature that produced EIA was recognized as a filterable agent, assuming that EIA is the animal disease that is allotted a viral etiology [1]. In the 1970s the virus continues lifelong of infected horses in the white blood cells. This lentivirus can be spread by blood sucking biting flies and by the transmission from contaminated needles and improper sterilization of instruments between horses. EIAV is found all through the world with the maximum occurrence in environmental areas with warm atmospheres; henceforth the disease is called a swamp fever [2].

EIAV is the simplest genome of characterized lentiviruses. To control the infection and disease, exposing the working rule of these lentivirus systems has a significant part of persistence and pathogenesis. For the past 30 years, vaccine research for AIDS enduring virus-specific immune responses occuring in inapparent carriers looks for an immunologic regulator of disease by the basic model of EIAV [3].

At first, Nowak and Bangham [4] proposed a mathematical model that includes only immune response CTL. Wodarz [5] suggested a model for cell-mediated and humoral responses in Hepatitis C virus. Schwartz et al. [6] created a model in which antibody creation is subsidiary amount to virus to regulate the EIAV infection. Ciupe et al. [7] studied a model for EIAV strains and neutralizing antibodies. Schwartz et al. [8] derived a model for EIAV to predict the condition to eradicate the wild-type infection with antibody infusions.

Mathematical model is an essential tool for improvement of our understanding of virus dynamics [9]. Based on review of the literature, till now, there is dearth of knowledge on stability analysis and approximate analytic solutions of EIAV infection. Li and his group analyzed the oscillation criteria for various differential equations [10–13]. Moreover, some approximate solutions are also searched by HAM, which is a powerful method based on the concept of basic topology and shows how to regulate the rate of convergence by allowing an auxiliary parameter h to vary. A suitable choice of the auxiliary linear operator, initial condition, and h ensures the convergence of the HAM solution series. This method was devised by Shijun Liao to solve nonlinear problems. Recently, HAM was used to get the solutions of nonlinear problems of HIV [14–17]. Also, this method gives the solutions of the Klein–Gordon and modified Kawahara equations [18–23]. In the present study, we examine the transmission dynamics of EIAV infection using a mathematical model. We examine the stability of a nonlinear model; also, we find the basic reproduction number and an approximate analytic solution by using HAM.

2 Basic knowledge of HAM

Consider the equation

$$M\big[v(t)\big] = 0.$$

According to the definitions by Liao [14, 15], we give the following zero-order deformation equation:

$$(1-s)L\big[\phi(t;s) - v_0(t)\big] = shH(t)M\big[\phi(t;s)\big], \quad s \in [0,1], h \neq 0,$$

where L is an auxiliary linear operator such that $L[c_i] = 0$ for integral constants c_i ($i = 1,2,3$). When $s = 0$ and $s = 1$, the zero-order deformation equation becomes

$$\phi(t;0) = v_0(t), \qquad \phi(t;1) = v(t).$$

By Taylor series expansion of $\phi(t;s)$ with respect to s we have

$$\phi(t,s) = v_0(t) + \sum_{i=1}^{\infty} v_i(t)s^i,$$

where

$$v_i = \frac{1}{i!}\frac{\partial^i \phi(t;s)}{\partial s^i}\bigg|_{s=0}.$$

Differentiating the equation i times with respect to s, then fixing $s = 0$, and finally dividing them by $i!$, we get the ith-order deformation equations

$$L\big[v_i(t) - \chi_i v_{i-1}(t)\big] = hH(t)R_i\big[\vec{v}_{i-1}(t)\big],$$

where

$$R_i\big[\vec{v}_{i-1}(t)\big] = \frac{1}{(i-1)!}\frac{\partial^{i-1}N[\phi(t;s)]}{\partial s^{m-1}}\Big|_{s=0}$$

and

$$\chi_i = \begin{Bmatrix} 0 & i \le 1 \\ 1 & i > 1 \end{Bmatrix}.$$

These equations can be easily solved using software such as Maple, Matlab, and so on.

3 Mathematical formulation

In this paper, we consider the following viral infection models proposed in [4, 24–26]:

$$\begin{aligned} \frac{dE}{dt} &= \Lambda - \beta EG - \mu E, \\ \frac{dF}{dt} &= \beta EG - (\omega + \rho)F, \\ \frac{dG}{dt} &= aF - (\psi + \alpha)G. \end{aligned} \tag{1}$$

The initial conditions associated with system (1) are

$$E(0) = E_0, \qquad F(0) = F_0 \quad \& \quad G(0) = G_0.$$

A schematic figure is presented in Fig. 1. The parameters and variables are presented in Table 1.

Figure 1 Schematic figure of EIAV infection

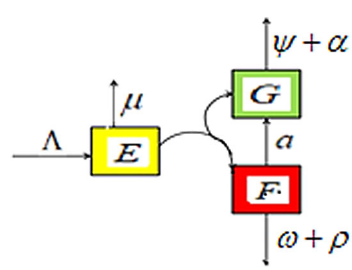


Table 1 Parameter meanings of EIAV infection

Parameters and variables meaning	
Dependent variables	
E	susceptible
F	infected
G	Virus
Parameters and constants	
Λ	recruitment rate of susceptible cells
β	rate of infection
μ	decay rate of susceptible cells
a	rate of virus production
ψ	clearance rate of virus
α	virus cleared by neutralizing antibodies
ω	death rate of infected cells
ρ	effect of infected cell killing by CTLS

4 Mathematical analysis of the EIAV model

4.1 Boundedness and positivity

Theorem 4.1 *Let* $P : [0, +\infty] \to R^3$. *If* $P(0) \in R^3_+$ *then* $P(t) \in R^3_+$ *for all* $t \in [0, +\infty]$, *and the solution is* $P(t) = (E, F, G)$.

To prove this, note that $\dot{E}|_{E=0}$, $\dot{F}|_{F=0}$ and $\dot{G}|_{G=0}$, are positive for $t > 0$. Therefore solutions of model (1) are positive.

Theorem 4.2 *Let* $P = \{(E, F, G) \in R^3_+ | 0 \leq E + F + G \leq \bar{E}_0, E, F, G \geq 0\}$, *where* $\bar{E}_0 = \frac{\Lambda}{\mu}$.

Solutions of system (1) are bounded on compact sets. Obviously, solutions of (1) are bounded in the region P.

4.2 Equilibrium analysis

Disease-free equilibrium and endemic equilibrium:

$$\begin{aligned}
D^0 &= (E_0, F_0, G_0) = (\bar{E}_0, 0, 0), \\
D^* &= \left(E^*, F^*, G^*\right), \\
E^* &= \frac{1}{\beta a}(\omega + \rho)(\psi + \alpha), \\
F^* &= \frac{\lambda}{\omega + \rho} - \frac{\mu(\psi + \alpha)}{\beta a}, \\
G^* &= \frac{1}{\beta(\omega + \rho)(\psi + \alpha)}\left[\beta a \Lambda - \mu(\omega + \rho)(\psi + \alpha)\right].
\end{aligned}$$

4.3 Basic reproduction number

We find R_0 using the next generation matrix [27]. Let us take matrices A and B that represent new infection and transfer at disease-free equilibrium,

$$\begin{aligned}
A\left(D^0\right) &= \begin{bmatrix} 0 & \frac{\beta\Lambda}{\mu} \\ 0 & 0 \end{bmatrix} \quad \text{and} \\
B\left(D^0\right) &= \begin{bmatrix} \omega + \rho & 0 \\ -a & \psi + \alpha \end{bmatrix}. \\
A\left(D^0\right)B\left(D^0\right)^{-1} &= \begin{bmatrix} \frac{\beta\Lambda a}{\mu(\omega+\rho)(\psi+\alpha)} & \frac{\beta\Lambda}{\psi+\alpha} \\ 0 & 0 \end{bmatrix}.
\end{aligned}$$

Hence

$$R_0 = \frac{\beta\Lambda a}{\mu(\omega + \rho)(\psi + \alpha)}.$$

4.4 Stability analysis

Proposition 4.3 D^0 *is locally asymptotically stable for* $R_0 < 1$ *and unstable otherwise*.

Proof

$$J = \begin{bmatrix} -\mu - \beta G & 0 & -\beta E \\ \beta G & -(\omega + \rho) & \beta E \\ 0 & a & -(\psi + \alpha) \end{bmatrix},$$

$$J(D^0) = \begin{bmatrix} -\mu & 0 & -\frac{\beta\Lambda}{\mu} \\ 0 & -(\omega + \rho) & \frac{\beta\Lambda}{\mu} \\ 0 & a & -(\psi + \alpha) \end{bmatrix}.$$

Therefore the eigenvalues of the disease-free equilibrium are$-\mu$, $-(\omega+\rho)$, and $-(\psi+\alpha)$. The eigenvalues at disease-free equilibrium are all negative, and hence it is stable. Therefore if $R_0 < 1$, then D^0 is locally asymptotically stable. □

Proposition 4.4 *D^0 is globally asymptotically stable for $R_0 \leq 1$ and unstable otherwise.*

Proof Let the Lyapunov function $L(E,F,G) : R^3_+ \to R^3_+$ be defined as

$$L(E,F,G) = \frac{aF}{\omega + \rho} + G. \tag{2}$$

Differentiating (2) with respect to t, we have

$$\begin{aligned} \dot{L} &= \frac{a}{\omega + \rho}\big[\beta EG - (\omega + \rho)F\big] + aF - (\psi + \alpha)G \\ &\leq \frac{\beta\Lambda aG}{\mu(\omega + \rho)} - (\psi + \alpha)G, \qquad E \leq \bar{E}_0 = \frac{\Lambda}{\mu}, \\ &\leq (\psi + \alpha)G(R_0 - 1). \end{aligned}$$

This implies $\dot{L} \leq (R_0 - 1)G \leq 0 \because \dot{L} = 0$ only when $G = 0$, using $G = 0$ in (1) such that $E \to \frac{\Lambda}{\mu}$ and $F \to 0$ as $t \to \infty$. Hence by [28, 29] D^0 is globally asymptotically stable when $R_0 > 1$. □

Proposition 4.5 *$D^* = (E^*, F^*, G^*)$ is locally asymptotically stable for $R_0 > 1$ and unstable otherwise.*

Proof We have

$$J(D^*) = \begin{bmatrix} \frac{\beta a\Lambda}{(\omega+\rho)(\psi+\alpha)} & 0 & -\frac{(\omega+\rho)(\psi+\alpha)}{a} \\ \frac{\beta a\Lambda}{(\omega+\rho)(\psi+\alpha)} - \mu & -(\omega + \rho) & \frac{(\omega+\rho)(\psi+\alpha)}{a} \\ 0 & a & -(\psi + \alpha) \end{bmatrix},$$

$$J(D^*) = \begin{bmatrix} -\mu R_0 & 0 & -\frac{(\omega+\rho)(\psi+\alpha)}{a} \\ \mu(R_0 - 1) & -(\omega + \rho) & \frac{(\omega+\rho)(\psi+\alpha)}{a} \\ 0 & a & -(\psi + \alpha) \end{bmatrix}.$$

The characteristic equation of equilibrium D^* is

$$\lambda^3 + \lambda^2 M_1 + \lambda M_2 + M_3 = 0,$$

where

$$M_1 = \omega + \rho + \alpha + \psi + \frac{(\omega + \rho)(\psi + \alpha)}{a} - \mu R_0 > 0,$$
$$M_2 = \frac{(\omega + \rho)(\psi + \alpha)}{a}[\mu R_0 + \omega + \rho] - \mu R_0 > 0,$$
$$M_3 = \frac{(\omega + \rho)^2(\psi + \alpha)}{a}\mu R_0.$$

By the Routh–Hurwitz criterion

$$M_1 > 0, M_3 > 0, \quad \text{and} \quad M_1 M_2 - M_3 > 0.$$

Finally, D^* is locally asymptotically stable. □

Proposition 4.6 *If $R_0 > 1$, then D^* is globally asymptotically stable and unstable otherwise.*

Proof Consider the Lyapunov function

$$M(E^*, F^*, G^*) = -\left(E^* \log \frac{E}{E^*} + E^* - E\right) - \left(F^* \log \frac{F}{F^*} + F^* - F\right) - \left(G^* \log \frac{G}{G^*} + G^* - G\right). \tag{3}$$

Differentiating (3) with respect to t, we have

$$\dot{M} = A\dot{E} + B\dot{F} + C\dot{G},$$

where

$$AE = E^* - E, \qquad BF = F^* - F, \qquad CG = G^* - G, \qquad QE = E^*,$$
$$\dot{M} = \Lambda + A^2 E\beta G^* + B\beta GE + B\beta G^* E^* + C[aF + \alpha G^* + \psi G^*] - \Lambda Q - A^2 E\beta G - A^2 E\mu - B\beta G^* G - B\beta GE^* - B^2 F(\omega + \rho) - CaF^* - (\alpha + \psi)CG,$$
$$\dot{M} = U - V, \tag{4}$$

where

$$U = \Lambda + A^2 E\beta G^* + B\beta GE + B\beta G^* E^* + C[aF + \alpha G^* + \psi G^*],$$
$$V = \Lambda Q + A^2 E\beta G + A^2 E\mu + B\beta G^* G + B\beta GE^* + B^2 F(\omega + \rho) + CaF^* + (\alpha + \psi)CG.$$

From (4) we have that if $U < V$, then $\dot{M} \le 0$. Also, $\dot{M} = 0$ iff $E^* = E$, $F^* = F$, $G^* = G$. Hence by [28, 29] D^* is globally asymptotically stable when $R_0 > 1$. □

5 Application

To construct the solutions of (1) by HAM, we first choose

$$E(0) = E_0, \qquad F(0) = F_0, \qquad G(0) = G_0. \tag{5}$$

We choose the auxiliary linear operators L_1, L_2, and L_3 as

$$L_1\bigl[E(t,s)\bigr] = \frac{dE(t,s)}{dt},$$
$$L_2\bigl[F(t,s)\bigr] = \frac{dF(t,s)}{dt},$$
$$L_3\bigl[G(t,s)\bigr] = \frac{dG(t,s)}{dt}.$$

They satisfy the following properties:

$L_i(C_i) = 0$, where C_i $(i = 1, 2, 3)$ are integral constants. Define the nonlinear operators

$$N_1[E,F,G] = \dot{E} - \Lambda + \beta EG + \mu E,$$
$$N_2[E,F,G] = \dot{F} - \beta EG + (\omega + \rho)F,$$
$$N_3[E,F,G] = \dot{G} - aF + (\psi + \alpha)G.$$

By the definitions of Liao [14, 15] introduce the nonzero auxiliary parameter h, nonzero auxiliary function H (t), and the embedding parameter s in $[0, 1]$. Then using this, we form the zero-order deformation equations

$$(1 - s)L_1\bigl[E(t;s) - E_0(t)\bigr] = sh_1H_1(t)N_1[E,F,G], \tag{6}$$
$$(1 - s)L_2\bigl[F(t;s) - F_0(t)\bigr] = sh_2H_2(t)N_2[E,F,G], \tag{7}$$
$$(1 - s)L_3\bigl[G(t;s) - G_0(t)\bigr] = sh_3H_3(t)N_3[E,F,G], \tag{8}$$

clearly, when $s = 0$ and $s = 1$, we have

$$E(t;0) = E_0(t), \qquad E(t;1) = E(t),$$
$$F(t;0) = F_0(t), \qquad F(t;1) = F(t),$$
$$G(t;0) = G_0(t), \qquad G(t;1) = G(t).$$

Thus the embedding parameter s rises from zero to one, the solutions $E(t;s)$, $F(t;s)$, and $G(t;s)$ vary continuously from $E_0(t)$, $F_0(t)$, and $G_0(t)$ to the exact solution $E(t)$, $F(t)$, and $G(t)$. Expanding $E(t;s)$ by Taylor's series, we have

$$E(t;s) = E_0(t) + \sum_{i=1}^{\infty} E_i(t)s^i, \tag{9}$$

$$F(t;s) = F_0(t) + \sum_{i=1}^{\infty} F_i(t)s^i, \tag{10}$$

$$G(t;s) = G_0(t) + \sum_{i=1}^{\infty} G_i(t)s^i, \tag{11}$$

where

$$X_i = \frac{1}{i!}\frac{\partial^i E(t;s)}{\partial s^i}\bigg|_{s=0}, \tag{12}$$

$$F_i = \frac{1}{i!}\frac{\partial^i F(t;s)}{\partial s^s}\bigg|_{s=0}, \tag{13}$$

$$G_i = \frac{1}{i!}\frac{\partial^i G(t;s)}{\partial s^i}\bigg|_{s=0}. \tag{14}$$

If h_1, h_2, h_3, $H_1(t)$, $H_2(t)$, and $H_3(t)$ are chosen, then the series is convergent at $p = 1$.

$$E(t) = E_0(t) + \sum_{i=1}^{\infty} E_i(t), \tag{15}$$

$$F(t) = F_0(t) + \sum_{i=1}^{\infty} F_i(t), \tag{16}$$

$$Z(t) = Z_0(t) + \sum_{i=1}^{\infty} Z_i(t). \tag{17}$$

Differentiating (6)–(8) i times with respect to s, allocating by $i!$, and fixing $s = 0$, the ith-order deformation equations are

$$L_1\bigl[E_i(t) - \chi_i E_{i-1}(t)\bigr] = hR_{1,i}\bigl(E_{i-1}(t)\bigr), \tag{18}$$

$$L_2\bigl[F_i(t) - \chi_i F_{i-1}(t)\bigr] = hR_{2,i}\bigl(F_{i-1}(t)\bigr), \tag{19}$$

$$L_3\bigl[G_i(t) - \chi_i G_{i-1}(t)\bigr] = hR_{3,i}\bigl(G_{i-1}(t)\bigr), \tag{20}$$

where

$$R_{1,i}(t) = \frac{dE_{i-1}(t)}{dt} + \beta \sum_{j=0}^{i-1} E_j(t)G_{i-1-j}(t) + \mu E_{i-1}(t) - (1 - \chi_i)\Lambda,$$

$$R_{2,i}(t) = \frac{dF_{i-1}(t)}{dt} - \beta \sum_{j=0}^{i-1} G_j(t)E_{i-1-j}(t) + (\omega + \rho)F_{i-1}(t),$$

$$R_{3,i}(t) = \frac{dG_{i-1}(t)}{dt} - aF_{i-1}(t) + (\psi + \alpha)Z_{i-1}(t),$$

and

$$\chi_i = \begin{Bmatrix} 0 & i \le 1 \\ 1 & i > 1 \end{Bmatrix}.$$

Then the ith-order deformation (18)–(20) for i greater than or equal to 1 becomes

$$E_i(t) = \chi_i E_{i-1}(t) + h\int_0^t R_{1,i}(\tau)\,d\tau,$$

$$F_i(t) = \chi_i F_{i-1}(t) + h\int_0^t R_{2,i}(\tau)\,d\tau,$$

$$G_i(t) = \chi_i G_{i-1}(t) + h\int_0^t R_{3,i}(\tau)\,d\tau.$$

6 Numerical results

Consider the following values for numerical results [30]:

$$E_0 = 42{,}390, \qquad F_0 = 0, \qquad G_0 = 0.001,$$
$$\Lambda = 2019, \qquad \beta = 0.00000094, \qquad \mu = 0.048,$$
$$a = 101, \qquad \psi = 2.33, \qquad \alpha = 3, \qquad \omega = 0.057, \qquad \rho = 0.01.$$

We use Mathematica software to get the sixth-order expansions for $E(t)$, $F(t)$, and $G(t)$:

$$\begin{aligned} E(t) ={}& 42{,}390 + 94.32ht + 235.8h^2t + 314.4h^3t + 235.8h^4t \\ &+ 94.32h^5t + 15.72h^6t + 5.65936h^2t^2 + 15.0916h^3t^2 + 16.97814h^4t^2 \\ &+ 9.05498h^5t^2 + 1.88645h^6t^2 + 0.121116h^3t^3 + 0.27251h^4t^3 + \cdots \end{aligned} \tag{21}$$

$$\begin{aligned} F(t) ={}& -0.000023908ht - 0.0000597699h^2t - 0.000079632h^3t \\ &- 0.0000597699h^4t - 0.000023908h^5t - 0.0000098466h^6t \\ &- 0.0001613h^2t^2 - 0.000430134h^3t^2 - 0.0004839h^4t^2 \\ &- 0.00025808h^5t^2 - 0.0000537667h^6t^2 - 0.000387534h^3t^3 \\ &- 0.0000871952h^4t^3 - 0.000697561h^5t^3 - 0.000193767h^6t^3 - \cdots \end{aligned} \tag{22}$$

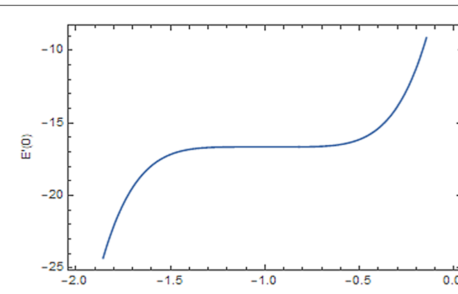
$$\begin{aligned} G(t) ={}& 0.001 + 0.03198ht + 0.07995h^2t + 0.1066h^3t + 0.07995h^4t \\ &+ 0.03198h^5t + 0.00533h^6t + 0.216085h^2t^2 + 0.576227h^3t^2 \\ &+ 0.648255h^4t^2 + 0.345736h^5t^2 + 0.0720284h^6t^2 + 0.519122h^3t^3 \\ &+ 1.16803h^4t^3 + 0.93442h^5t^3 + 0.0720284h^6t^3 + \cdots \end{aligned} \tag{23}$$

7 Discussion

We have found the solution of (18)–(19) containing h by showing an easy technique for control and adjustment of curves ensuring the convergence of solution series, as recommended by Liao. Figures 2–7 show the plots of fifth and sixth term approximation of $E'(0)$, $F'(0)$ *and* $G'(0)$.

These curves show that the valid region of h is parallel to the horizontal axis. The valid region is listed in Table 2.

Figure 2 The h-curves of $E'(0)$ obtained by 5th-order approximation of the HAM



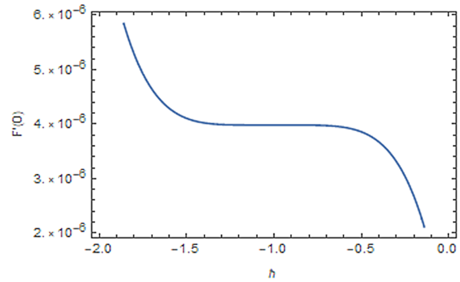


Figure 3 The h-curves of $F'(0)$ obtained by 5th-order approximation of the HAM

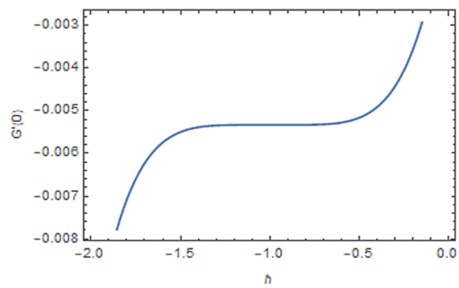


Figure 4 The h-curves of $G'(0)$ obtained by 5th-order approximation of the HAM

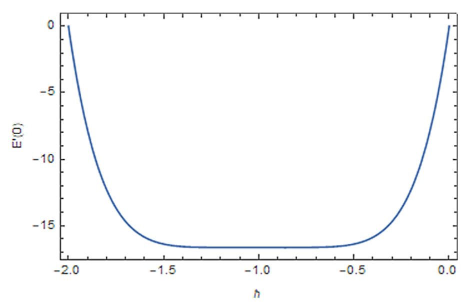


Figure 5 The h-curves of $E'(0)$ obtained by 6th-order approximation of the HAM

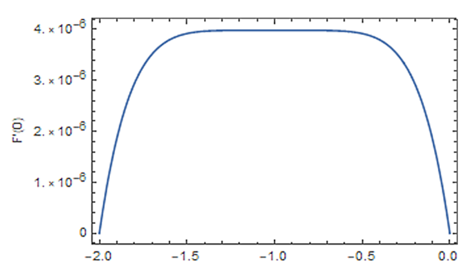


Figure 6 The h-curves of $F'(0)$ obtained by 6th-order approximation of the HAM

In this context, an error analysis is performed to get the optimal values of h. We substitute Eqs. (21)–(23) into (1) and obtain the residual functions as follows:

$$ER_1(E,F,G;h_1) = \frac{d\phi_E(t;h_1)}{dt} - \Lambda + \beta\phi_E(t;h_1)\phi_G(t;h_1) + \mu\phi_E(t;h_1), \tag{24}$$

$$ER_2(E,F,G;h_2) = \frac{d\phi_F(t;h_2)}{dt} - \beta\phi_E(t;h_2)\phi_G(t;h_2) + (\omega + \rho)\phi_F(t;h_2), \tag{25}$$

$$ER_3(E,F,G;h_3) = \frac{d\phi_G(t;h_3)}{dt} - a\phi_F(t;h_3) - (\psi + \alpha)\phi_F(t;h_3). \tag{26}$$

Figure 7 The h-curves of $G'(0)$ obtained by 6th-order approximation of the HAM

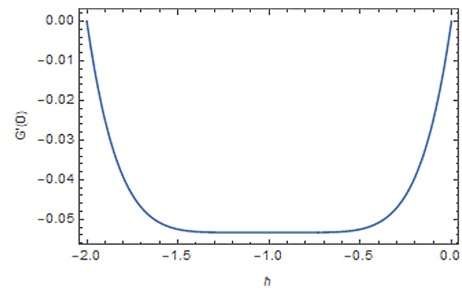


Table 2 Figures 2–7 show the values of h

$E(t)$	$-1.35 \leq h \leq -0.6$
$F(t)$	$-1.4 \leq h \leq -0.7$
$G(t)$	$-1.4 \leq h \leq -0.7$

Table 3 The minimum values of $RE(h_1^*)$, $RF(h_2^*)$, $RG(h_3^*)$ (see Figs. 11–13)

	h^*	Minimum value
$RE(h_1)$	−0.906678	2.227296×10^{-4}
$RF(h_2)$	−0.660648	1.32472×10^{-15}
$RG(h_3)$	−0.660667	2.37742×10^{-9}

Following [31, 32], we consider the square residual error for the sixth-order approximation:

$$RE(h_1) = \int_0^1 \left(ER_1(E,F,G;h_1)\right)^2 dt, \tag{27}$$

$$RF(h_2) = \int_0^1 \left(ER_2(E,F,G;h_2)\right)^2 dt, \tag{28}$$

$$RG(h_3) = \int_0^1 \left(ER_3(E,F,G;h_3)\right)^2 dt. \tag{29}$$

The values of h_1, h_2, and h_3 for which $RE(h_1)$, $RF(h_2)$, $RG(h_3)$ are minimal can be obtained. We have

$$\frac{dRE(h_1^*)}{dh_1} = 0, \qquad \frac{dRF(h_2^*)}{dh_2} = 0, \qquad \frac{dRG(h_3^*)}{dh_3} = 0.$$

The optimal values of h_1, h_2, and h_3 for all of the cases considered are

$$h_1^* = -0.906678, \qquad h_2^* = -0.660648, \qquad h_3^* = -0.660667.$$

In Table 3, the minimum values of $RE(h_1)$, $RF(h_2)$, and $RG(h_3)$ are given for optimal values of h_1, h_2, and h_3.

In Table 4, we calculated errors ER_1, ER_2, and ER_3 for various t in $(0,1)$. This shows that the HAM offers us an accurate approximate solution for EIAV model (1).

Figures 8–10 show the residual errors of ER_1, ER_2, and ER_3 for t in $(0,1)$ and various h.

By considering the following figures note the solutions obtained by using HAM.

Table 4 The residual errors ER_1, ER_2, and ER_3 for various $t \in (0, 1)$

t	$ER_1(E, F, G; h_1^*)$	$ER_2(E, F, G; h_2^*)$	$ER_3(E, F, G; h_3^*)$
0.0	1.03838×10^{-5}	6.08546×10^{-9}	8.13736×10^{-6}
0.1	7.64421×10^{-6}	1.13927×10^{-9}	1.53872×10^{-6}
0.2	5.2269×10^{-6}	5.09644×10^{-9}	6.81296×10^{-6}
0.3	3.09924×10^{-6}	1.78769×10^{-9}	2.40852×10^{-6}
0.4	1.27796×10^{-6}	1.28159×10^{-8}	1.71806×10^{-5}
0.5	1.38351×10^{-7}	9.88247×10^{-9}	1.32496×10^{-5}
0.6	8.18731×10^{-7}	1.60802×10^{-8}	2.15323×10^{-5}
0.7	1.95882×10^{-8}	5.51901×10^{-8}	7.39272×10^{-5}
0.8	3.65462×10^{-6}	7.69584×10^{-8}	1.03087×10^{-4}
0.9	1.04486×10^{-3}	3.68833×10^{-8}	4.93837×10^{-5}
1	3.02903×10^{-5}	1.08705×10^{-7}	1.45783×10^{-4}

Figure 8 The residual errors function Eq. (24) using the 6th-order approximation solution for various $h = -0.9$, $h = -1$, $h =$ optimal, $h = -1.1$ and $t \in (0, 1)$

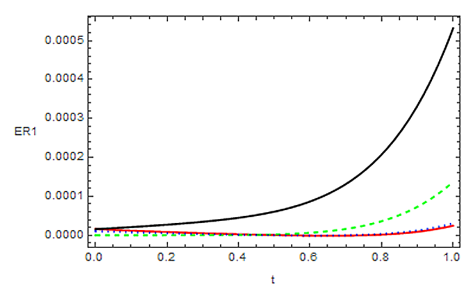


Figure 9 The residual error function Eq. (25) using the 6th-order approximation solution for various $h = -0.9$, $h = -1$, $h =$ optimal, $h = -1.1$ and $t \in (0, 1)$

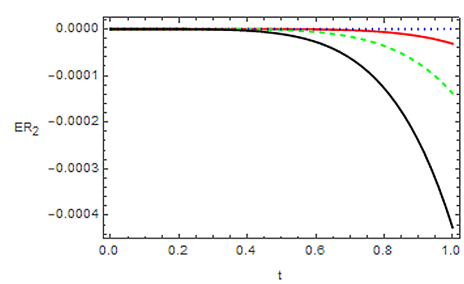
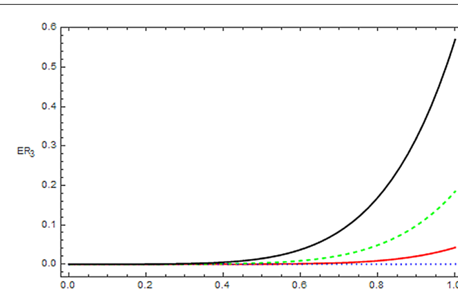


Figure 10 The residual error function Eq. (26) using the 6th-order approximation solution for various $h = -0.9$, $h = -1$, $h =$ optimal, $h = -1.1$ and $t \in (0, 1)$



8 Conclusion

In this paper, we examined the global dynamics by R_0. The disease-free equilibrium is globally asymptotically stable if $R_0 \leq 1$. Moreover, the endemic equilibrium is globally asymptotically stable if $R_0 > 1$. Also, we obtain an approximate analytical solution for this model. Moreover, we illustrate the ability of HAM to confirm the convergence of sequence

Figure 11 The optimum and minimum values of $E(t)$

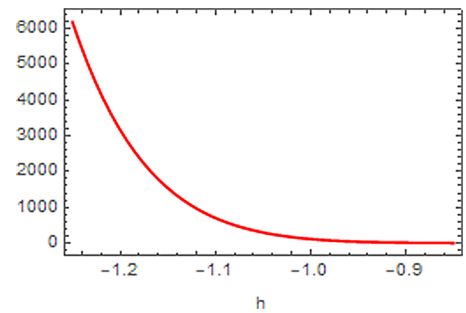


Figure 12 The optimum and minimum values of $F(t)$

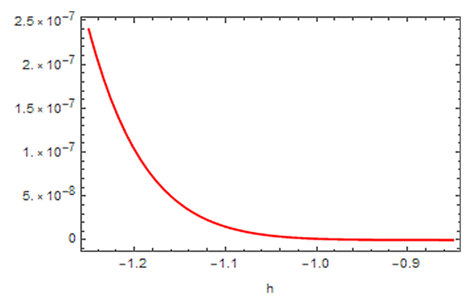
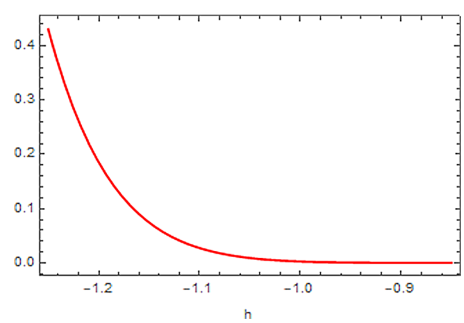


Figure 13 The optimum and minimum values of $G(t)$



solution for nonlinear differential equations and obtain approximate semianalytic solutions. In this manner, HAM is very effective and powerful technique to find the solutions.

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