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A pest control model with birth pulse and residual and delay effects of pesticides

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Abstract

Pesticides often cause residual and delayed effects on pests. Considering these effects, we use a pollution emission model to simulate the process of spraying pesticides. Many pests reproduce only at a fixed time in a year. So a pest control model with birth pulse and spraying pesticides is proposed. Using the limit system of the developed model, we analyze the dynamics of the system. The stability of the trivial equilibrium and the positive equilibrium of the model is analyzed, and the threshold conditions of pest eradication and permanence of the system are given. We obtain the optimal frequency of spraying pesticides by numerical simulations. The important parameters related to the pest eradication or permanence of the system are given by analyzing the sensitivity of the parameters. Finally, biological explanations are provided.

Keywords: Pesticide function; Birth pulse; Locally asymptotic stability; Threshold conditions; Sensitivity analysis

1 Introduction

Recently, the economic development of China has been advancing rapidly. Methods to increase crop yield are highly desired and are critical to the economic growth of the nation. Every year, crops such as corn and fruit trees are severely damaged by pests, resulting in tremendous financial losses to the farmers. How to control pests quickly and effectively without a negative impact on the environment and the growth of crops is becoming even more important. Now the common pest control strategies include chemical control (spraying pesticides) and biological control (releasing natural enemies). Many scholars [1–10] used mathematical models to simulate pest control strategies. Some of them [1–4] analyzed the dynamics of the pest control models based on biological control. Whereas some studies [5–10] built pest management models only based on chemical controls. Liu and Teng [5] studied a pest management model with spraying pesticides at a fixed time of the pest reproductive cycles. They assumed that pesticides only affected adult pests. The optimal time of pest control was obtained if the pesticides were applied just before each birth pulse of the cycle. Wei [9] analyzed the dynamics of the pest control models with birth pulse under the assumptions that pesticides killed adult pests or larvae or both of them, respectively. By mathematical simulations, it was found that with the different elimination rates for larvae and adults, the corresponding optimal time for pesticide applications was also different. To more efficiently control pests, some authors es-

established mathematical models with integrated pest management to control pests, which combined the spraying of pesticides with the release of natural enemies at the fixed time [11–17]. However, the release of natural enemies is difficult to manage, so chemical control remains the most common and convenient pest management method. It is noted that the long-term repetitive use of the same pesticide could result in resistance to pests [18, 19], causing relapse of pest and severe crop losses. Therefore, when applying chemical control to manage pest, critical factors influencing pest management must be well understood, and we should use as little pesticides as possible. In nature, pests generally go through two stages: larval pests and adult pests. Assuming adults only lay eggs at a fixed time in a year, then the increase in the number of larvae occurs instantly. Based on the above biological background, in this paper, we study a pest management model with stage structure and birth pulse, and only chemical control is applied. In our study, an impulsive differential equation [20–22] is used to better describe the birth pulse of pests [6, 9, 23]. In the past, scholars often assumed that the effects of pesticides on pests were instantaneous and the pests were killed proportionally. In fact, the effects of pesticides will gradually weaken with time and will last for a long period of time [24, 25]. When pesticides are applied, they first affect the growth environment, then are absorbed by the pests, which cause their death. This mechanism is similar to that of pollution emission [26]. Therefore, we adopt the pollution emission model to simulate the spraying process of pesticides and construct the pesticide function with residual and delayed effects of pesticide. The pest management model built in this paper is more consistent with the reality. To the best of our knowledge, there have been no results on dynamics for such a system.

This paper is organized as follows. In Sect. 2, we assume that pesticides are applied q times periodically at the fixed time during a birth cycle. The pollution emission model is used to obtain the pesticide function. Then a pest control model with stage structure and birth pulse is set up. In Sect. 3, the dynamics is analyzed by using the limit system of the model. By numerical simulations, the optimal frequency of spraying pesticides is discussed. The important parameters related to the pest eradication or permanence of the system are also given. In the last section, a discussion of our results is given.

2 Model formulation

Without a staged structure in the growth cycle, we assume that the population size satisfies the following birth function equation:

$$\dot{N} = B(N)N - dN, \quad (2.1)$$

where $d > 0$ indicates the population mortality rate, $B(N)$ is the birth function of the population and $B(N)$ ($N \in (0, \infty)$) satisfies the following assumptions:

(H_1): $B(N) > 0$;

(H_2): $B(N)$ is a continuously differentiable function and $B'(N) < 0$;

(H_3): $B(0^+) > d > B(\infty)$.

Based on assumptions (H_2) and (H_3), we know there exists an inverse function $B^{-1}(N)$ of $B(N)$ for $N \in (0, \infty)$. There are various types of $B(N)$ that satisfy assumptions (H_1)–(H_3). Here, we only discuss the following two types: (A_1) $B_1(N) = be^{-N}$, where b is the population

birth rate, $b > d$, called the Ricker function. (A_2) $B_2(N) = \frac{p}{\beta + N^n}$, where $p, \beta, n > 0$ and $\frac{p}{\beta} > d$, called the Beverton–Holt function. Considering the stage structure of the pest population, we divide the pest population $N(t)$ into the larval stage $x(t)$ and the adult stage $y(t)$. So $N(t) = x(t) + y(t)$, and only the adult population can reproduce. Thus, the pest population model with stage structure is as follows:

$$\begin{cases} \dot{x}(t) = B(N(t))y(t) - dx(t) - \delta x(t), \\ \dot{y}(t) = \delta x(t) - dy(t), \end{cases} \tag{2.2}$$

where δ ($\delta > 0$) is the maturity rate, which determines the mean length of the juvenile period.

Obviously, system (2.2) has a trivial equilibrium $E_0(0, 0)$. If

$$B^-\left(\frac{d(d + \delta)}{\delta}\right) > 0, \tag{2.3}$$

there exists a unique positive equilibrium $E^*(x^*, y^*)$ in system (2.2), where

$$E^*(x^*, y^*) = \left(\frac{d}{\delta + d} B^-\left(\frac{d(\delta + d)}{\delta}\right), \frac{\delta}{\delta + d} B^-\left(\frac{d(\delta + d)}{\delta}\right) \right).$$

Lemma 2.1 ([23]) *Assume (H_1) and (H_2) hold. If inequality (2.3) is reversed, then the trivial equilibrium $E_0(0, 0)$ is locally asymptotically stable. If inequality (2.2) holds, then the trivial equilibrium $E_0(0, 0)$ is unstable, and the positive equilibrium $E^*(x^*, y^*)$ is locally asymptotically stable.*

If $B(\infty) < d$, then it is easy to see that system (2.2) is dissipative, that is, there exists a positive constant $M > 0$ such that the set $\Omega = \{(x, y) | 0 \leq x \leq M, 0 \leq y \leq M\}$ is a positive invariant set of system (2.2). Then, the global stability of the system on the equilibria is as follows.

Lemma 2.2 ([23]) *Assume (H_1) , (H_2) and $B(\infty) < d$ hold. If inequality (2.3) is reversed, then the set Ω is the asymptotically stable region of the trivial equilibrium $E_0(0, 0)$. If inequality (2.3) holds, then the set Ω is the asymptotically stable region of the positive equilibrium $E^*(x^*, y^*)$.*

Assuming the adult population reproduce at the fixed time in a year, the model considering stage structure and birth pulse constructed by Tang [23] is as follows:

$$\begin{cases} \dot{x} = -dx(t) - \delta x(t), \\ \dot{y} = \delta x(t) - dy(t), \end{cases} \quad t \neq m, \tag{2.4}$$

$$x(m^+) = x(m^-) + B(N(m^-))y(m^-), \quad t = m, m \in N.$$

Based on the above birth pulse model and the assumption that the pesticide is applied at fixed time in a birth cycle, Liu [6] built the following pest management model:

$$\begin{cases} \dot{x} = -dx(t) - \delta x(t), \\ \dot{y} = \delta x(t) - dy(t), \end{cases} \quad t \neq m, t \neq m + l - 1, \\ \begin{cases} \Delta x(t) = 0, \\ \Delta y(t) = -ky(t), \end{cases} \quad t = m + l - 1, \\ x(m^+) = x(m^-) + B(N(m^-))y(m^-), \quad t = m, m \in N, \\ y(m^+) = 0, \end{cases} \tag{2.5}$$

where k is the mortality rate of adult pest caused by the pesticide and $m + l - 1$ ($0 < l < T$, $m \in N$) is the time for pesticide application during each birth cycle.

For the above pest management model, the pest mortality rate caused by the pesticide was assumed to be instant and proportional. In fact, the effects of pesticides on pests gradually weaken with time and could last for a long time. In this study, we adopt the pollution emission model to simulate the function of pesticide effects. The following pest management model with stage structure and birth pulse is constructed:

$$\begin{cases} \dot{x} = -dx(t) - \delta x(t) - c(t)x(t), \\ \dot{y} = \delta x(t) - dy(t) - c(t)y(t), \end{cases} \quad t \neq m, \\ x(t^+) = x(t^-) + B(N(t^-))y(t^-), \quad t = m, m \in N, \end{cases} \tag{2.6}$$

where $c(t)$ is the pesticide function which satisfies

$$\begin{cases} \frac{dc(t)}{dt} = rf(t) - gc(t), \\ \frac{df(t)}{dt} = -hf(t), \end{cases} \quad t \neq m + nT, \\ f(t^+) = f(t) + \mu, \quad t = m + nT, \end{cases} \tag{2.7}$$

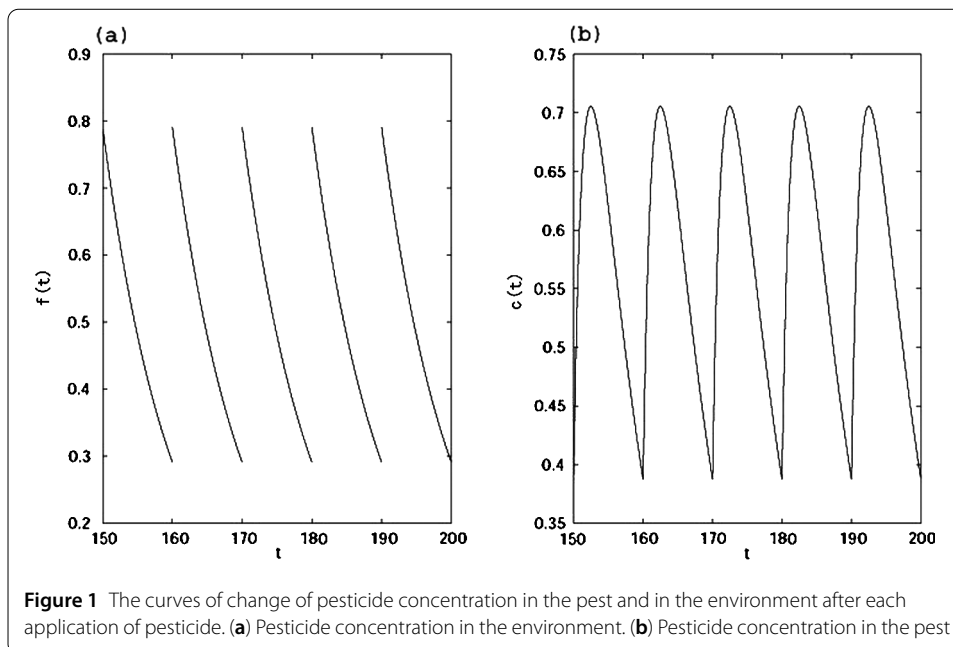
where $c(t)$ is the pesticide concentration in the pests at time t ; $f(t)$ is the pesticide concentration in the environment at time t ; $rf(t)$ ($r > 0$) represents the pest's net uptake of pesticide from the environment; $gc(t)$ is the egestion and depuration rates of pesticide in the pest population; $hf(t)$ represents the pesticide loss from the environment itself due to photosynthesis, evaporation, bacterial degradation and so on; μ ($\mu > 0$) is the dose of pesticide applied each time. For convenience, assume that there are q times of spraying pesticides at regular intervals during each birth cycle $(m, m + 1]$, and let the time interval of pesticide application be T . Then $T = \frac{1}{q}$.

3 Dynamics of system (2.6)

3.1 Model solving and the equilibrium of stroboscopic maps

As shown by Ref. [26], there existed a globally asymptotically stable periodic solution in the pollution emission model (2.7) as follows.

Lemma 3.1 ([26]) *System (2.7) has a unique globally asymptotically stable T -period solution $(\tilde{c}(t), \tilde{f}(t))$ and for every solution $(c(t), f(t))$ of system (2.7), we have $c(t) \rightarrow \tilde{c}(t)$ and*



$f(t) \rightarrow \tilde{f}(t)$ as $t \rightarrow \infty$, where $t \in (m + nT, m + (n + 1)T)$, $n = 0, 1, \dots, q - 1$, $m \in N$,

$$\tilde{c}(t) = \tilde{c}(m + nT^+)e^{-g(t-(m+nT))} + \frac{r\mu(e^{-g(t-(m+nT))} - e^{-h(t-(m+nT))})}{(h - g)(1 - e^{-hT})},$$

$$\tilde{f}(t) = \tilde{f}(m + nT^+)e^{-h(t-(m+nT))},$$

and

$$\tilde{c}(m + nT^+) = \frac{r\mu(e^{-gT} - e^{-hT})}{(h - g)(1 - e^{-gT})(1 - e^{-hT})}, \quad \tilde{f}(m + nT^+) = \frac{\mu}{1 - e^{-hT}}.$$

Figure 1 illustrates the curves of change of pesticide concentration in the pest and in the environment after each application, respectively. As seen in Fig. 1(a), after each application, the pesticide concentration in the environment $f(t)$ reaches its maximum and then gradually decreases. However, the pesticide concentration in the pest $c(t)$ first increases and then decreases as illustrated in Fig. 1(b). These curves suggest that the effects of pesticide on pests do not immediately work but delay to a certain extent and can last for a time. Thus, pesticides have residual and delayed effects on pest.

As seen from Lemma 3.1, the arbitrary solution $(c(t), f(t))$ of system (2.7) eventually tends to a globally asymptotically stable periodic solution $(\tilde{c}(t), \tilde{f}(t))$. Therefore, the dynamics of system (2.6) is equivalent to the ones of the following limit system of system (2.6):

$$\begin{cases} \dot{x} = -dx(t) - \delta x(t) - \tilde{c}(t)x(t), \\ \dot{y} = \delta x(t) - dy(t) - \tilde{c}(t)y(t), \end{cases} \quad t \neq m, \tag{3.1}$$

$$x(t^+) = x(t^-) + B(N(t^-))y(t^-), \quad t = m, m \in N.$$

Denote $\tilde{c}(m^+) = c_m, x(m^+) = x_m, y(m^+) = y_m$. When $m < t \leq m + T$, we integrate and solve for the larval pests population in system (3.1),

$$\begin{aligned}
 x(t) &= x_m \exp \left[-(d + \delta)(t - m) - \frac{1}{g}(1 - e^{-g(t-m)})c_m \right. \\
 &\quad \left. - \frac{r\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-g(t-m)}) - \frac{1}{h}(1 - e^{-h(t-m)}) \right) \right], \\
 x(t) + y(t) &= (x_m + y_m) \exp \left[-d(t - m) - \frac{1}{g}(1 - e^{-g(t-m)})c_m \right. \\
 &\quad \left. - \frac{r\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-g(t-m)}) - \frac{1}{h}(1 - e^{-h(t-m)}) \right) \right].
 \end{aligned}$$

Therefore,

$$\begin{aligned}
 y(t) &= x_m(1 - e^{-\delta(t-m)}) \exp \left[-d(t - m) - \frac{1}{g}(1 - e^{-g(t-m)})c_m \right. \\
 &\quad \left. - \frac{r\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-g(t-m)}) - \frac{1}{h}(1 - e^{-h(t-m)}) \right) \right] \\
 &\quad + y_m \exp \left[-d(t - m) - \frac{1}{g}(1 - e^{-g(t-m)})c_m \right. \\
 &\quad \left. - \frac{r\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-g(t-m)}) - \frac{1}{h}(1 - e^{-h(t-m)}) \right) \right].
 \end{aligned}$$

When $t = m + T$, we have

$$\begin{aligned}
 x(m + T) &= x_m \exp \left[-(d + \delta)T - \frac{1}{g}(1 - e^{-gT})c_m \right. \\
 &\quad \left. - \frac{r\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-gT}) - \frac{1}{h}(1 - e^{-hT}) \right) \right], \\
 y(m + T) &= x_m(1 - e^{-\delta T}) \exp \left[-dT - \frac{1}{g}(1 - e^{-gT})c_m \right. \\
 &\quad \left. - \frac{r\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-gT}) - \frac{1}{h}(1 - e^{-hT}) \right) \right] \\
 &\quad + y_m \exp \left[-dT - \frac{1}{g}(1 - e^{-gT})c_m \right. \\
 &\quad \left. - \frac{r\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-gT}) - \frac{1}{h}(1 - e^{-hT}) \right) \right], \\
 &\vdots
 \end{aligned}$$

When $t = m + qT$, we have

$$\begin{aligned}
 x(m + qT) &= x_m \exp \left[-q(d + \delta)T - \frac{q}{g}(1 - e^{-gT})c_m \right. \\
 &\quad \left. - \frac{qr\mu}{(h - g)(1 - e^{-hT})} \left(\frac{1}{g}(1 - e^{-gT}) - \frac{1}{h}(1 - e^{-hT}) \right) \right],
 \end{aligned}$$

$$\begin{aligned}
 y(m + qT) &= x_m(1 - e^{-q\delta T}) \exp\left[-q dT - \frac{q}{g}(1 - e^{-gT})c_m\right. \\
 &\quad \left. - \frac{qr\mu}{(h-g)(1 - e^{-hT})}\left(\frac{1}{g}(1 - e^{-gT}) - \frac{1}{h}(1 - e^{-hT})\right)\right] \\
 &\quad + y_m \exp\left[-q dT - \frac{q}{g}(1 - e^{-gT})c_m\right. \\
 &\quad \left. - \frac{qr\mu}{(h-g)(1 - e^{-hT})}\left(\frac{1}{g}(1 - e^{-gT}) - \frac{1}{h}(1 - e^{-hT})\right)\right].
 \end{aligned}$$

Denote $H = \frac{r\mu}{(h-g)(1 - e^{-hT})}\left(\frac{1}{g}(1 - e^{-gT}) - \frac{1}{h}(1 - e^{-hT})\right)$, we get

$$\begin{cases}
 x(m + qT) = x((m + 1)^-) = x_m \exp[-q(d + \delta)T - \frac{q}{g}(1 - e^{-gT})c_m - qH], \\
 y(m + qT) = y((m + 1)^-) = x_m(1 - e^{-q\delta T}) \exp[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH] \\
 \quad + y_m \exp[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH].
 \end{cases} \tag{3.2}$$

Thus,

$$x((m - 1)^-) + y((m - 1)^-) = (x_m + y_m) \exp\left[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH\right].$$

When $t = m + 1$, the birth pulse occurs, and the number of larval pests increases instantly. If the birth function $B(N) = be^{-N}$, then the stroboscopic map is as follows:

$$\begin{cases}
 x_{m+1} = x_m \exp[-q(d + \delta)T - \frac{q}{g}(1 - e^{-gT})c_m - qH] + b[y_m + x_m(1 - e^{-q\delta T}) \\
 \quad \cdot \exp[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH - (x_m + y_m)e^{-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH}], \\
 y_{m+1} = x_m(1 - e^{-q\delta T}) \exp[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH] \\
 \quad + y_m \exp[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH].
 \end{cases} \tag{3.3}$$

If the birth function $B(N) = \frac{p}{\beta + N^n}$, we have

$$\begin{cases}
 x_{m+1} = x_m \exp[-q(d + \delta)T - \frac{q}{g}(1 - e^{-gT})c_m - qH] \\
 \quad + \frac{pe^{-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH} [y_m + x_m(1 - e^{-q\delta T})]}{\beta + e^{-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH} (y_m + x_m)^n}, \\
 y_{m+1} = x_m(1 - e^{-q\delta T}) \exp[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH] \\
 \quad + y_m \exp[-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH].
 \end{cases} \tag{3.4}$$

Obviously, systems (3.3) and (3.4) have trivial equilibria $E_0^b(0, 0)$ and $E_0^p(0, 0)$.

Let

$$\begin{aligned}
 R_0^b &\triangleq \frac{b(1 - e^{-q dT})e^{-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH}}{(1 - e^{-q(d+\delta)T - \frac{q}{g}(1 - e^{-gT})c_m - qH})(1 - e^{-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH})}, \\
 R_0^p &\triangleq \frac{p(1 - e^{-q dT})e^{-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH}}{\beta(1 - e^{-q(d+\delta)T - \frac{q}{g}(1 - e^{-gT})c_m - qH})(1 - e^{-q dT - \frac{q}{g}(1 - e^{-gT})c_m - qH})}.
 \end{aligned}$$

Table 1 The positive equilibrium and the threshold of model (3.2) and (3.3)

Birth function	Equilibrium	R_0
Ricker	$x^* = \frac{(1-e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH) \ln R_0^b}{(1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH)e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}$ $y^* = \frac{1-e^{-q\delta T}}{1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH} \ln R_0^b$	$R_0^b \triangleq \frac{b(1-e^{-qdT})e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}{1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH}$ $\cdot \frac{1}{1-e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}$
Beverton–Holt	$x^* = \frac{(1-e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH) \sqrt[n]{\beta(R_0^p-1)}}{(1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH)e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}$ $y^* = \frac{1-e^{-q\delta T}}{1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH} \sqrt[n]{\beta(R_0^p-1)}$	$R_0^p \triangleq \frac{p(1-e^{-qdT})e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}{\beta(1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH)}$ $\cdot \frac{1}{1-e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}$

If $R_0^b > 1$ ($R_0^p > 1$), system (3.3) ((3.4)) has a positive equilibrium $E_b^*(x^*, y^*)$ ($E_p^*(x^*, y^*)$) which satisfies the following equations:

$$\begin{cases} x^* = x^* \exp[-q(d + \delta)T - \frac{q}{g}(1 - e^{-gT})c_m - qH] + b[y^* + x^*(1 - e^{-q\delta T})] \\ \quad \cdot \exp[-qdT - \frac{q}{g}(1 - e^{-gT})c_m - qH - (x^* + y^*)e^{-qdT-\frac{q}{g}(1-e^{-gT})c_m-qH}], \\ y^* = x^*(1 - e^{-q\delta T}) \exp[-qdT - \frac{q}{g}(1 - e^{-gT})c_m - qH] \\ \quad + y^* \exp[-q\delta T - \frac{q}{g}(1 - e^{-gT})c_m - qH], \end{cases} \tag{3.5}$$

or

$$\begin{cases} x^* = x^* \exp[-q(d + \delta)T - \frac{q}{g}(1 - e^{-gT})c_m - qH] \\ \quad + \frac{pe^{-qdT-\frac{q}{g}(1-e^{-gT})c_m-qH} [y^* + x^*(1-e^{-q\delta T})]}{\beta + e^{-qn dT - \frac{qn}{g}(1-e^{-gT})c_m - qnH} (y^* + x^*)^n}, \\ y^* = x^*(1 - e^{-q\delta T}) \exp[-qdT - \frac{q}{g}(1 - e^{-gT})c_m - qH] \\ \quad + y^* \exp[-q\delta T - \frac{q}{g}(1 - e^{-gT})c_m - qH]. \end{cases} \tag{3.6}$$

The solution for the positive equilibrium is

$$\begin{cases} x^* = \frac{1-e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}{(1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH)e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH} \ln R_0^b, \\ y^* = \frac{1-e^{-q\delta T}}{1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH} \ln R_0^b, \end{cases} \tag{3.7}$$

or

$$\begin{cases} x^* = \frac{1-e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH}{(1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH)e^{-qdT-\frac{q}{g}(1-e^{-gT})}c_m-qH} \sqrt[n]{\beta(R_0^p-1)}, \\ y^* = \frac{1-e^{-q\delta T}}{1-e^{-q(d+\delta)T-\frac{q}{g}(1-e^{-gT})}c_m-qH} \sqrt[n]{\beta(R_0^p-1)}. \end{cases} \tag{3.8}$$

In summary, we have the results shown in Table 1.

3.2 Stability of equilibrium

Lemma 3.2 ([27]; Jury criterion) *The linearization equation of a differential equation system is*

$$X_m = AX_{m-1}, \tag{3.9}$$

where A is the linear segment of the system and $X = (x, y)^T$. If the modulus of all eigenvalues of the matrix A is less than 1, then the equilibrium of the system is stable, that is, the matrix A satisfies the following three Jury criteria:

- (a) $1 - \text{tr} A + \det A > 0$,
- (b) $1 + \text{tr} A + \det A > 0$,
- (c) $1 - \det A > 0$.

These three conditions correspond to the three ways that an eigenvalue may exceed the unit circle in the complex plane. If inequality (3.10a) is violated, then matrix A has an eigenvalue which is greater than 1. If the inequality (3.10b) is violated, then the matrix A has an eigenvalue which is less than -1 . If the inequality (3.10c) is not true, then the matrix A has a pair of complex conjugated eigenvalues which exceed the unit circle.

Let

$$b_0 = \frac{(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})}{(1 - e^{-q dT})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}},$$

$$p_0 = \frac{\beta(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})}{(1 - e^{-q dT})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}}.$$

Theorem 3.1 *If $0 < b < b_0$ ($0 < p < p_0$), then the trivial equilibrium of (3.2) ((3.3)) is locally asymptotically stable.*

Proof For trivial equilibrium $E_0^b(0, 0)$ and $E_0^p(0, 0)$, the linearization matrix of (3.2) ((3.3)) is

$$A_{E_0^b} = \begin{pmatrix} e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH} + b(1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} & b e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} \\ (1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} & e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} \end{pmatrix} \tag{3.11}$$

or

$$A_{E_0^p} = \begin{pmatrix} e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH} + \frac{p(1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}}{\beta} & \frac{p e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}}{\beta} \\ (1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} & e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} \end{pmatrix}. \tag{3.12}$$

With matrix A defined in Eq. (3.11) or (3.12), inequalities (3.10b) and (3.10c) always hold. If inequality (3.10a) is true, then the following inequalities should be satisfied:

$$1 - \text{tr} A_{E_0^b} + \det A_{E_0^b} = (1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}) - b(1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} > 0,$$

or

$$1 - \text{tr} A_{E_0^p} + \det A_{E_0^p} = \left(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}\right) \left(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}\right) - \frac{p(1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}}{\beta} > 0.$$

Based on the above two inequalities, we have

$$0 < b < \frac{(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})}{(1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}} \triangleq b_0, \tag{3.13}$$

or

$$0 < p < \frac{\beta(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})}{(1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}} \triangleq p_0. \tag{3.14}$$

By Lemma 3.2, we know that if $0 < b < b_0$ ($0 < p < p_0$), then the trivial equilibrium of system (3.3) ((3.4)) is locally asymptotically stable.

The locally asymptotical stability of the trivial equilibrium of system (3.3) ((3.4)) suggests that system (2.6) has a pest-eradication periodic solution which is locally asymptotically stable. The inequality (3.13) ((3.14)) is equivalent to $R_0^b < 1$ ($R_0^p < 1$). Thus, if $R_0^b < 1$ ($R_0^p < 1$), then the pest population will be extinct. Define

$$b_c \triangleq b_0 \cdot \exp\left\{ \frac{2 + 2e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} \cdot e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}}{(1 + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \cdot (1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})} \right\},$$

$$p_c \triangleq p_0 \cdot \left(n(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \right) / \left(n(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) - 2(1 + e^{-q(2d+\delta)T - \frac{2q}{g}(1-e^{-gT})c_m - 2qH}) \right). \quad \square$$

Theorem 3.2 *If $b_0 < b < b_c$ ($p_0 < p < p_c$), then the positive equilibrium of (3.3)((3.4)) is locally asymptotically stable.*

Proof For the positive equilibrium $E_b^*(x^*, y^*)$, the linearization matrix of (3.3) is

$$W_{E_b^*} = \begin{pmatrix} w_{11} & w_{12} \\ (1 - e^{-q\delta T})e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} & e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH} \end{pmatrix},$$

where

$$w_{11} = e^{-q(d+\delta)T - \frac{qc}{\alpha}(1-e^{-\alpha T})} + \left(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH}\right) \cdot \left(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}\right) - \left(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}\right) \ln R_0^b,$$

$$w_{12} = \frac{(1 - e^{-q dT - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})}{1 - e^{-q\delta T}} - \left(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}\right) \ln R_0^b.$$

Obviously, the inequalities (3.10a) and (3.10b) always hold. If inequality (3.10c) is true, then the following inequality should be satisfied:

$$b_0 < b < b_c,$$

where

$$b_c \triangleq b_0 \cdot \exp \left\{ \frac{2 + 2e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH} \cdot e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH}}{(1 + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \cdot (1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})} \right\}.$$

From Lemma 3.2, we know that if $b_0 < b < b_c$, then the positive equilibrium $E_b^*(x^*, y^*)$ of (3.3) is locally asymptotically stable.

For positive equilibrium $E_p^*(x^*, y^*)$, the linearization matrix of (3.4) is

$$W_{E_p^*} = \begin{pmatrix} z_{11} & z_{12} \\ (1 - e^{-q\delta T})e^{-q d T - \frac{qc}{\alpha}(1-e^{-\alpha T})} & e^{-q d T - \frac{qc}{\alpha}(1-e^{-\alpha T})} \end{pmatrix}, \tag{3.15}$$

where

$$\begin{aligned} z_{11} &= e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH} + (1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \\ &\quad \cdot (1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) - \frac{n(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(R_0^p - 1)}{R_0^p}, \\ z_{12} &= \frac{(1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})}{1 - e^{-q\delta T}} \\ &\quad - \frac{n(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(R_0^p - 1)}{R_0^p}. \end{aligned}$$

From matrix (3.15), the determinant value and trace of the matrix are

$$\begin{aligned} \det W_{E_p^*} &= e^{-q(2d+\delta)T - \frac{2q}{g}(1-e^{-gT})c_m - 2qH} \\ &\quad - \frac{ne^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH} (1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(R_0^p - 1)}{R_0^p}, \\ \text{tr } W_{E_p^*} &= e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH} + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH} \\ &\quad + (1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \cdot (1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \\ &\quad - \frac{n(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(R_0^p - 1)}{R_0^p}, \end{aligned}$$

respectively.

Then, the inequalities

$$\begin{aligned} 1 - \det W_{E_p^*} &= 1 - e^{-q(2d+\delta)T - \frac{2q}{g}(1-e^{-gT})c_m - 2qH} \\ &\quad + \frac{ne^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH} (1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(R_0^p - 1)}{R_0^p} > 0 \end{aligned}$$

and

$$\begin{aligned}
 & 1 - \text{tr } W_{E_p^*} + \det W_{E_p^*} \\
 &= \frac{n(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(R_0^p - 1)}{R_0^p} \cdot (1 - e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \\
 &> 0
 \end{aligned}$$

always hold. In other words, inequalities (3.10a) and (3.10c) always hold. If inequality (3.10b) is true, then the following inequality should be satisfied:

$$\begin{aligned}
 1 + \text{tr } W_{E_p^*} + \det W_{E_p^*} &= 2(1 + e^{-q(2d+\delta)T - \frac{2q}{g}(1-e^{-gT})c_m - 2qH}) \\
 &\quad - \frac{n(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(R_0^p - 1)}{R_0^p} \\
 &\quad \cdot (1 + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) > 0.
 \end{aligned}$$

Solving it, we get

$$p_0 < p < p_c,$$

where

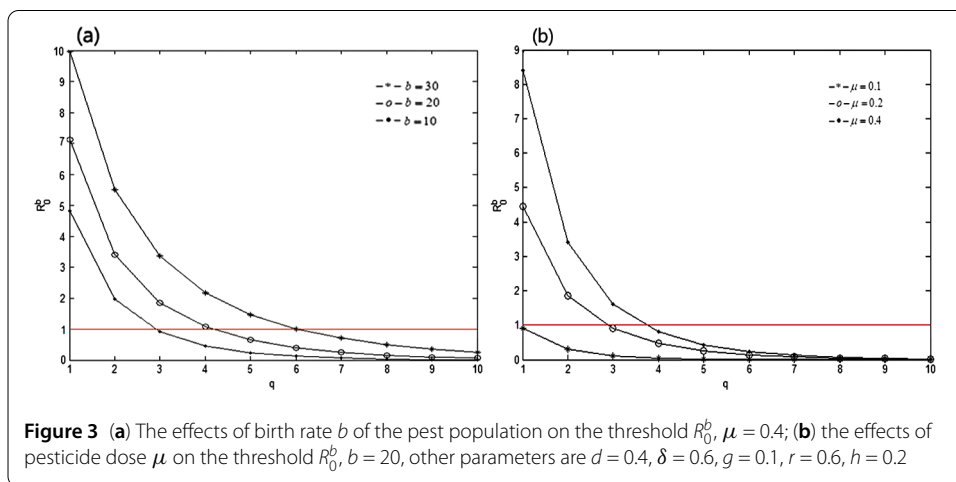
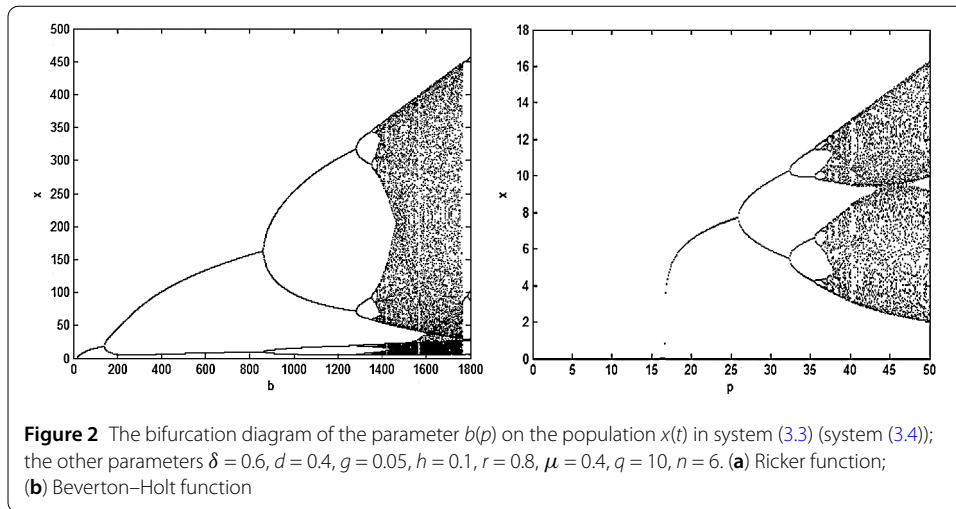
$$\begin{aligned}
 p_c \triangleq & p_0 \cdot (n(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH})) \\
 & / (n(1 - e^{-q d T - \frac{q}{g}(1-e^{-gT})c_m - qH})(1 + e^{-q(d+\delta)T - \frac{q}{g}(1-e^{-gT})c_m - qH}) \\
 & - 2(1 + e^{-q(2d+\delta)T - \frac{2q}{g}(1-e^{-gT})c_m - 2qH})). \quad \square
 \end{aligned}$$

Therefore, if $p_0 < p < p_c$ holds, then the positive equilibrium of system (3.4) is locally asymptotically stable. The locally asymptotical stability of positive equilibrium of system (3.3) or (3.4) shows that system (2.6) has a positive periodic solution which is locally asymptotically stable. In this case, the pest population would be permanent.

3.3 Mathematical simulation and the biological significance

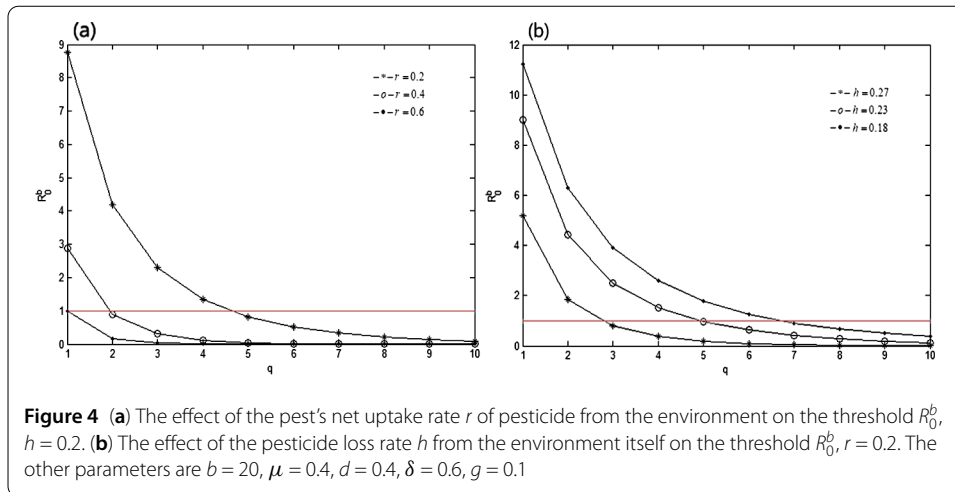
If we fix other parameters and let b or p vary, as learned from Theorems 3.1 and 3.2, then the extinction and permanence of the pest population rely on the birth rate b or p . If $0 < b < b_0$, then the trivial equilibrium of (3.3) ((3.4)) is locally asymptotically stable, which implies the pest-eradication periodic solution of system (2.6) is locally asymptotically stable. If $b_0 < b < b_c$ ($p_0 < p < p_c$), then the positive equilibrium of (3.3) ((3.4)) is locally asymptotically stable, which implies system (2.6) has a locally stable positive periodic solution, the system will be permanent, and the pest population does not go extinct. If parameter b or p continues to increase, then the stability of the positive periodic solution is destroyed and system (3.3) ((3.4)) will present very complex dynamics including period-doubling bifurcations and chaos (see Fig. 2).

Taking the Ricker function as an example, we further analyze the important factors which influence pest extinction and the optimal frequency of pesticide applications. It



is well known that $0 < b < b_0$ is equivalent to $R_0^b < 1$; under this condition, the pest population is extinct. Figures 3(a)–(b) give the effects of pest birth rate b and the dose of pesticide applied on the threshold R_0^b , respectively, which can help us determine the optimal frequency of pesticide applications when their values are different. Figure 3(a) indicates that the higher the birth rate b is, the greater R_0^b is. Thus, to keep pests under control, the pesticide applications should be more frequent. To avoid environmental pollution and financial losses due to excessive pesticide use, we consider the minimum times of pesticide applications such that $R_0^b < 1$ as the optimal frequencies of pesticide applications. So if $b = 10, b = 20, b = 30$, the optimal frequencies of pesticide applications is $q = 3, q = 4, q = 6$, respectively. Figure 3(b) suggests the smaller the pesticide dose μ is, the greater R_0^b is, which is undesired for pest control. So if $\mu = 0.1, \mu = 0.2, \mu = 0.4$, then the optimal frequencies of pesticide applications is $q = 4, q = 3, q = 1$, respectively.

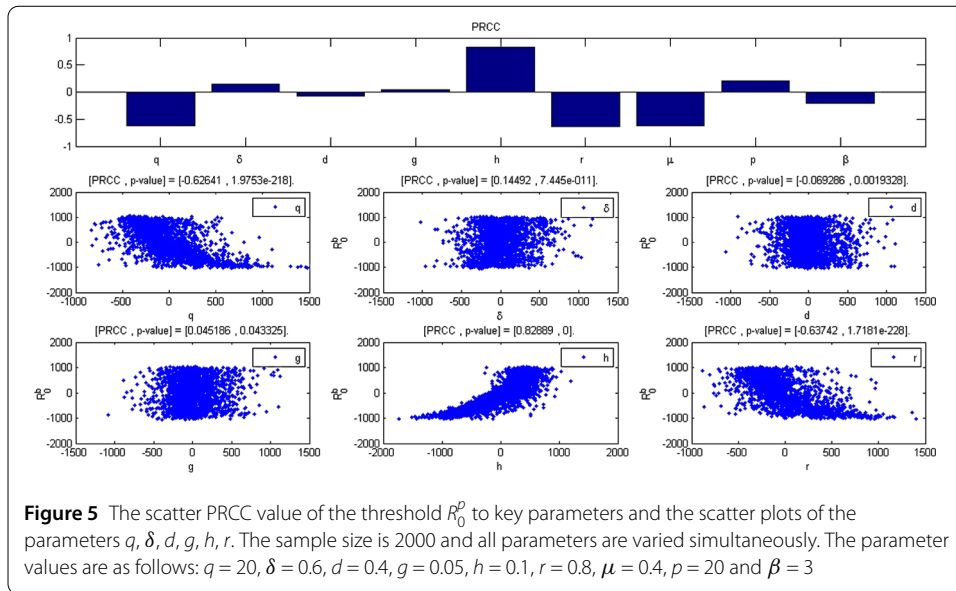
Similarly, let the parameter r vary and the other parameters be the same as in Fig. 3, it can be seen from Fig. 4(a) that the greater the pest’s net uptake rate r is, the smaller R_0^b is, which is more beneficial to pest control. If $r = 0.2, r = 0.4, r = 0.6$, then the optimal frequencies of pesticide applications is $q = 5, q = 2, q = 1$, respectively. Setting $r = 0.2$ if we let parameter h vary and fix the other parameters, from Fig. 4(b) we know that the smaller



pesticide loss rate h is more beneficial to pest control. If $h = 0.18$, $h = 0.23$, $h = 0.27$, the optimal frequencies of pesticide applications are $q = 3$, $q = 5$, $q = 7$, respectively.

With the above method, we can investigate the effects of pest mortality rate d , the maturity rate δ , and the egestion and depuration rates g of pesticide on the threshold R_0^p and their corresponding optimal frequencies of pesticide applications.

To determine critical factors which affect pest control strategy, we take Beverton–Holt function as the birth function of (2.6) and perform sensitivity analysis for the pest-eradication threshold R_0^p [28, 29]. By calculating the PRCC value of the parameters, the sensitivities of these parameters on R_0^p are analyzed, then some key parameters of affecting outbreaks of the pest population are determined. When performing the analysis, 2000 samples are included for each parameter for uncertainty sensitivity analysis. Additionally, the following rules are applied: when the PRCC values of the parameter are positive, the parameter is positively correlated to R_0^p , which means that if the parameter is increased, the threshold value increases (and vice versa). On the contrary, when the PRCC values of the parameter are negative, the parameter is negatively correlated with R_0^p , which means that if the parameter is increased, the threshold value R_0^p decreases (and vice versa). If $|PRCC| > 0.4$, it indicates a strong correlation between input parameters and output variables, and the parameter has a greater effect on R_0^p . If $0.2 < |PRCC| < 0.4$, the parameter has a moderate effect on R_0^p . If $|PRCC| < 0.2$, the parameter has weak effect on R_0^p . In Fig. 5, we set $q = 20$, $\delta = 0.6$, $d = 0.4$, $g = 0.05$, $h = 0.1$, $r = 0.8$, $\mu = 0.4$, $p = 20$ and $\beta = 3$. It can be seen that the parameters δ , g , h , p are positively correlated with the threshold R_0^p . So the increase of the parameters δ , g , h , p lead to the increase of R_0^p , then the pest population expands rapidly, which is undesired for pest management. Parameters q , d , r , μ , β are negatively correlated with the threshold R_0^p . With the increasing in q , d , r , μ , β , there is a decrease in R_0^p , and the pest population will tend to be extinct, which is advantageous for pest management. The effects of the parameters d , g on pest eradication and permanence are relatively less and even near zero. Parameters δ , p , β have moderate effects on the threshold R_0^p . Therefore, from the above analysis we see that the threshold R_0^p is most strongly affected by the frequencies q of pesticide applications, the pesticide loss rate h from the environment, the absorbance of pesticide by the pest and the dose μ of each pesticide application. Thus, these parameters are critical to pest management.



4 Conclusion

In this paper, we divide pests population into larval stage and adult stage and assume that the increase in the number of larva is instantaneous at the fixed time in a year. Spraying pesticide is applied to control pest. Considering the delayed and residual effects of the pesticide, the pollution emission model is employed to simulate the mathematical function of pesticide effects. Furthermore, a pest management model with stage structure and birth pulse is established. Ricker and Beverton–Holt birth functions are investigated in our study. This is the first study in which a mathematical function based on pesticide effects has been incorporated into a pest management model with stage structure and birth pulse. The theory of impulsive differential equations is adopted to investigate the dynamics of the pest management model. The threshold conditions for pest eradication or permanent are obtained through theoretical analysis. Additionally, by mathematical simulations, we find that the model studied here have complex dynamics including period-doubling bifurcation and chaos. The effects of the various factors on the threshold conditions for pest eradication are analyzed by numerical simulations. The optimal frequencies of pesticide applications corresponding to the factors are also confirmed. Furthermore, a sensitivity analysis is performed for the main parameters. We find that the frequency of pesticide applications q , the decay of environment pesticide h , the absorbance of pesticide by the pest population r , and the pesticide dose of each application μ are critical to the eradication and permanent of the pests. Our results could provide a theoretical base for policy-making departments.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors contributed equally to the manuscript and read and approved the final draft.

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