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A new result on stability analysis for discrete system with interval time-varying delays

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Abstract

This paper is concerned with the stability criteria for a discrete-time linear system with interval time-varying delays. By using the time delay division we construct an augmented Lyapunov–Krasovskii functional for two delay subintervals. Moreover, we use a new summation inequality to estimate the derivatives of LKFs more accurately and derive less conservative criteria. Finally, we present two numerical examples to demonstrate that the obtained results are less conservative.

Keywords: Discrete system; Stability; Lyapunov–Krasovskii functional; Interval time-varying delay

1 Introduction

During the transmission and processing of signal or data, time delay inevitably occurs in such practical systems as neural networks and networked control systems. It is well known that the presence of time delay often affects the dynamic behavior of the system, resulting in deterioration or even instability in system performance. Therefore, the research on the stability of time delay systems has been the focus of many scholars [1–5].

One essential research focus on a system with time delay is how to reduce the conservatism of the stability condition. In view of this, many researchers choose to approach this from two perspectives, selecting an appropriate Lyapunov–Krasovskii functional (LKF) and making a more accurate estimation of its derivative. With regard to the former aspect, many significant results have been achieved such as constructing an amplified LKF [6], increasing its integral multiplicity [7], or dividing the time delay interval [8]. With regard to the latter, the Jensen inequality [9, 10] is the first powerful tool used to estimate the derivative of LKF in the stability theory of time-delay systems. Later on, the Wirtinger-based integral inequality [11, 12], the free-matrix-based integral inequality [13–15], and a series of integral inequalities [16] have been proposed. These inequalities are widely applied in time delay systems [17–28]. Of course, there are other methods or combinations of different methods, such as the free-weighting matrix approach proposed in the literature [29], the basic idea is introducing some free weighting matrix through the Newton–Leibniz formula and/or the equation of state of the system, and the less conservative stability results can also be obtained.

It is worth noting that not only continuous-time systems, but also discrete-time systems play an important role and have received extensive attention in the literature [30–47]. In

[12], by using the Wirtinger-based inequality an improved stability condition is obtained. However, the allowable delay upper bound can be further lifted. This inspires our research in this paper.

Based on the above discussions, by means of a new summation inequality and time delay division, less conservative criteria than some existing ones are obtained, and two examples are presented to demonstrate the effectiveness and superiority of the proposed approach. The main contributions of this paper are as follows:

   (i) A new summation inequality is obtained by the Schmidt orthogonalization method, and the corresponding result is less conservative.
   (ii) The proof of Corollary 1 in [15] can be simplified by the substitution of variables, which is demonstrated by Lemma 2.1 in this paper.

Notation: Throughout this paper, $\Re^n$ denotes the n-dimensional Euclidean space, and $\Re^{n \times m}$ is the set of all $n \times m$ real matrices; I is used to denote the identity matrix of proper dimension; diag$\{\cdots\}$ denotes block diagonal matrix, and $\langle f, g \rangle$ denotes the inner product of f and g; $P > 0 \ (\geq 0)$ means that P is a real symmetric and positive definite (semipositive definite) matrix; the symmetric term in a matrix is denoted by $*$, and sym$[A] = A + A^T$; $N^+(N)$ is the set of all positive (nonnegative) integers.

2 Preliminaries

Consider the following discrete time-varying delay system:

$$
\begin{cases}
x(k+1) = Ax(k) + Bx(k - d(k)), & k \in N, \\
x(k) = \phi(k), & k = -d_2, -d_2 + 1, \dots, 0,
\end{cases}
\tag{1}
$$

where $x(k) = (x_1(k), x_2(k), \dots, x_n(k))^T \in \Re^n$ is the state vector, $\phi(k) \in \Re^n$ is the initial condition, $n \in N^+$, A and $B \in \Re^{n \times n}$ are constant matrices, and $d(k)$ is the discrete time-varying delay satisfying $d_1 \leq d(k) \leq d_2$, where d_1 and d_2 are nonnegative integers.

Lemma 2.1 *Let $x(i)$ be a sequence of discrete-time variables in $[a, a+n] \to \Re^n$. For symmetric matrices $R \in \Re^{n \times n}$ and $Z_{11}, Z_{22}, Z_{33} \in \Re^{4n \times 4n}$ and for any matrices $Z_{12}, Z_{13}, Z_{23} \in \Re^{4n \times 4n}$ and $N_1, N_2, N_3 \in \Re^{4n \times n}$ satisfying*

$$
\Xi = \begin{pmatrix}
Z_{11} & Z_{12} & Z_{13} & N_1 \\
* & Z_{22} & Z_{23} & N_2 \\
* & * & Z_{33} & N_3 \\
* & * & * & R
\end{pmatrix} \geq 0,
$$

we have the following inequality:

$$
- \sum_{i=a}^{a+n-1} \eta^T(i) R \eta(i) \leq \omega^T \Omega \omega,
\tag{2}
$$

where

$$
\eta(i) = x(i+1) - x(i),
$$

$$\omega = \left(x^T(a+n), x^T(a), \frac{1}{n+1} \sum_{i=a}^{a+n} x^T(i), \frac{2}{(n+1)(n+2)} \sum_{i=a}^{a+n} \sum_{j=i}^{a+n} x^T(j) \right)^T,$$

$$\Omega = n\left(Z_{11} + \frac{1}{3}Z_{22} + \frac{1}{5}Z_{33} \right) + \mathrm{sym}[N_1\Pi_1 + N_2\Pi_2 + N_3\Pi_3],$$

$$\Pi_1 = \varepsilon_1 - \varepsilon_2, \qquad \Pi_2 = \varepsilon_1 + \varepsilon_2 - 2\varepsilon_3, \qquad \Pi_3 = \varepsilon_1 - \varepsilon_2 + 6\varepsilon_3 - 6\varepsilon_4,$$

$$\varepsilon_1 = \begin{pmatrix} I & 0 & 0 & 0 \end{pmatrix}, \qquad \varepsilon_2 = \begin{pmatrix} 0 & I & 0 & 0 \end{pmatrix},$$

$$\varepsilon_3 = \begin{pmatrix} 0 & 0 & I & 0 \end{pmatrix}, \qquad \varepsilon_4 = \begin{pmatrix} 0 & 0 & 0 & I \end{pmatrix}.$$

Proof Define

$$f_0(i) = 1, \qquad f_1(i) = i, \qquad f_2(i) = i^2.$$

By the Schmidt orthogonalization method, where $\langle f, g \rangle = \sum_{i=1}^{n} f(i)g(i)$, we obtain the following orthogonal polynomials $g_k(i)$, $k = 0, 1, 2$:

$$g_0(i) = 1, \qquad g_1(i) = i - \frac{n+1}{2}, \qquad g_2(i) = i^2 - (n+1)i + \frac{(n+1)(n+2)}{6}.$$

Then converting the coefficient of constant term to 1 or -1 by multiplying by a positive number, we have

$$h_0(i) = 1, \qquad h_1(i) = -1 + \frac{2}{n+1}i, \qquad h_2(i) = 1 - \frac{6}{n+2}i + \frac{6}{(n+1)(n+2)}i^2,$$

and $h_k(i)$, $k = 0, 1, 2$, satisfy

$$\langle h_0, h_0 \rangle = n, \qquad \langle h_1, h_1 \rangle = n\frac{n-1}{3(n+1)}, \qquad \langle h_2, h_2 \rangle = n\frac{(n-1)(n-2)}{5(n+1)(n+2)}.$$

Let $\varsigma(i) = (h_0(i)u^T, h_1(i)u^T, h_2(i)u^T, \xi^T(i))^T$, where $u = (y^T(n+1), y^T(1), \frac{1}{n+1}\sum_{i=1}^{n+1} y^T(i),$ $\frac{2}{(n+1)(n+2)}\sum_{i=1}^{n+1}\sum_{j=i}^{n+1} y^T(j))^T$, $y(i)$ is a sequence of discrete-time variable in $[1, 1+n] \to \Re^n$, and $\xi(i) = y(i+1) - y(i)$. Noticing that $\sum_{i=1}^{n+1} iy(i) = \sum_{i=1}^{n+1}\sum_{j=i}^{n+1} y(j)$, we can obtain

$$\sum_{i=1}^{n} h_k(i)\xi(i) = \Pi_{k+1}u, \qquad k = 0, 1, 2.$$

It is easy to see that $\varsigma^T(i)\varXi\varsigma(i) \geq 0$. Then summing both sides of the inequality before from 1 to n yields

$$-\sum_{i=1}^{n} \xi^T(i)R\xi(i) \leq u^T\tilde{\Omega}u,$$

where $\tilde{\Omega} = n(Z_{11} + \frac{n-1}{3(n+1)}Z_{22} + \frac{(n-1)(n-2)}{5(n+1)(n+2)}Z_{33}) + \mathrm{sym}[N_1\Pi_1 + N_2\Pi_2 + N_3\Pi_3]$.

Let $x(i) = y(i-a+1)$ be a sequence of discrete-time variables in $[a, a+n] \to \Re^n$ satisfying

$$-\sum_{i=a}^{a+n-1} \eta^T(i)R\eta(i) = -\sum_{i=1}^{n} \xi^T(i)R\xi(i) \leq u^T\tilde{\Omega}u = \omega^T\tilde{\Omega}\omega.$$

Table 1 Allowable delay upper bound d_2 for various d_1 for Example 1

Method	d_1	2	4	6	10	15	20	25	30
[32]	d_2	13	13	14	15	18	22	26	30
[33]	d_2	13	13	14	17	20	24	29	33
[20]	d_2	14	15	16	18	21	25	30	34
[21]	d_2	17	17	18	20	23	27	31	35
[34]	d_2	18	18	19	20	23	26	30	35
[40]	d_2	20	21	21	22	24	27	29	34
[43] (Theorem 2)	d_2	22	22	22	23	25	28	32	36
[12] (Remark 4)	d_2	20	21	21	23	25	29	32	36
[12] (Theorem 1)	d_2	22	22	22	23	26	29	32	36
Remark 3	d_2	21	22	22	23	25	29	32	36
Theorem 3.1	d_2	22	22	22	23	26	29	32	36

Table 2 Allowable delay upper bound d_2 for various d_1 for Example 2

Method	d_1	2	5	7	10	20
[33]	d_2	7	9	11	14	24
[21]	d_2	9	11	13	16	26
[40]	d_2	13	14	16	19	29
[12] (Theorem 1)	d_2	14	17	18	21	31
Theorem 3.1	d_2	15	17	18	21	31

Define $\Omega = \tilde{\Omega} + \frac{2n}{3(n+1)}Z_{22} + \frac{6n^2}{5(n+1)(n+2)}Z_{33}$. Then Ω is affinely dependent on n, and we obtain (2). This completes the proof. □

Remark 1 Define $f_3(i) = i^3$. Then $h_3(i) = -1 + \lambda\frac{12}{n+3}i - \frac{30}{(n+2)(n+3)}i^2 + \frac{20}{(n+1)(n+2)(n+3)}i^3$, where $\lambda = \frac{6n^2+15n+11}{6(n+1)(n+2)}$, and it is not the fourth Legendre polynomial anymore [16]. Thus deriving a general form of the discrete inequality (2) is still an interesting and challenging task.

Remark 2 By the substitution of variables demonstrated in Lemma 2.1, the proof of Corollary 1 in [15] can be simplified. By means of the new summation inequality the results calculated based on the criteria given in this paper are less conservative than those reported in the existing literature from Tables 1 and 2.

3 Main results

For reducing the conservatism, we further present a delay partitioning method, where the time delay interval is divided into two segments: $[d_1, d_2] = [d_1, d] \cup [d, d_2]$.

Theorem 3.1 *For given positive integers d_1 and d_2 satisfying $d_1 \le d(k) \le d_2$, system (1) is asymptotically stable if there exist matrices $P > 0$ ($\in \Re^{4n \times 4n}$), $Q_i > 0$ ($\in \Re^{n \times n}$), symmetric matrices $R \in \Re^{n \times n}$ and $Z_{j11}, Z_{j22}, Z_{j33} \in \Re^{4n \times 4n}$, any matrices $Z_{j12}, Z_{j13}, Z_{j23} \in \Re^{4n \times 4n}$ and $N_{ji} \in \Re^{4n \times n}$ ($i = 1, 2, 3; j = 1, 2, \ldots, 7$) such that the following LMIs hold*:

$$\Psi_1\big(d(k)\big) = D_1 + D_3 + D_4 + D_5 + D_6 + G_1\big(d(k)\big) + G_3\big(d(k)\big) + G_4\big(d(k)\big) + H_1 + H_2$$
$$< 0, \quad d(k) \in [d_1, d] \cap N, \tag{3}$$

$$\Psi_2\big(d(k)\big) = D_2 + D_3 + D_4 + D_7 + D_8 + G_2\big(d(k)\big) + G_5\big(d(k)\big) + G_6\big(d(k)\big) + H_1 + H_3$$
$$< 0, \quad d(k) \in [d, d_2] \cap N, \tag{4}$$

$$\Xi_j = \begin{pmatrix} Z_{j11} & Z_{j12} & Z_{j13} & N_{j1} \\ * & Z_{j22} & Z_{j23} & N_{j2} \\ * & * & Z_{j33} & N_{j3} \\ * & * & * & R_i \end{pmatrix} \geq 0, \quad and \quad i = \begin{cases} 1 & if\, j = 1, \\ 2 & if\, j = 2,3,5, \\ 3 & if\, j = 4,6,7, \end{cases} \tag{5}$$

where

$$D_1 = E_2^T P E_2 - E_1^T P E_1,$$

$$D_2 = E_4^T P E_4 - E_3^T P E_3,$$

$$D_3 = e_1^T Q_1 e_1 - e_2^T (Q_1 - Q_2) e_2 - e_4^T (Q_2 - Q_3) e_4 - e_5^T Q_3 e_5,$$

$$D_4 = d_1 (e_0 - e_1)^T R_1 (e_0 - e_1) + \alpha_1 \operatorname{sym}[N_{11} \Pi_1 + N_{12} \Pi_2 + N_{13} \Pi_3] \alpha_1^T,$$

$$D_5 = (d - d_1)(e_0 - e_1)^T R_2 (e_0 - e_1) + \alpha_2 \operatorname{sym}[N_{21} \Pi_1 + N_{22} \Pi_2 + N_{23} \Pi_3] \alpha_2^T$$
$$\qquad + \alpha_3 \operatorname{sym}[N_{31} \Pi_1 + N_{32} \Pi_2 + N_{33} \Pi_3] \alpha_3^T,$$

$$D_6 = (d_2 - d)(e_0 - e_1)^T R_3 (e_0 - e_1) + \alpha_4 \operatorname{sym}[N_{41} \Pi_1 + N_{42} \Pi_2 + N_{43} \Pi_3] \alpha_4^T,$$

$$D_7 = (d - d_1)(e_0 - e_1)^T R_2 (e_0 - e_1) + \alpha_5 \operatorname{sym}[N_{51} \Pi_1 + N_{52} \Pi_2 + N_{53} \Pi_3] \alpha_5^T,$$

$$D_8 = (d_2 - d)(e_0 - e_1)^T R_3 (e_0 - e_1) + \alpha_6 \operatorname{sym}[N_{61} \Pi_1 + N_{62} \Pi_2 + N_{63} \Pi_3] \alpha_6^T$$
$$\qquad + \alpha_7 \operatorname{sym}[N_{71} \Pi_1 + N_{72} \Pi_2 + N_{73} \Pi_3] \alpha_7^T,$$

$$G_1(d(k)) = \operatorname{sym}[(E_2 - E_1)^T P F_1(d(k))],$$

$$G_2(d(k)) = \operatorname{sym}[(E_4 - E_3)^T P F_2(d(k))],$$

$$G_3(d(k)) = \alpha_2 \left[(d(k) - d_1) \left(Z_{211} + \frac{1}{3} Z_{222} + \frac{1}{5} Z_{233} \right) \right] \alpha_2^T,$$

$$G_4(d(k)) = \alpha_3 \left[(d - d(k)) \left(Z_{311} + \frac{1}{3} Z_{322} + \frac{1}{5} Z_{333} \right) \right] \alpha_3^T,$$

$$G_5(d(k)) = \alpha_6 \left[(d(k) - d) \left(Z_{611} + \frac{1}{3} Z_{622} + \frac{1}{5} Z_{633} \right) \right] \alpha_6^T,$$

$$G_6(d(k)) = \alpha_7 \left[(d_2 - d(k)) \left(Z_{711} + \frac{1}{3} Z_{722} + \frac{1}{5} Z_{733} \right) \right] \alpha_7^T,$$

$$F_1(d(k)) = (0, 0, (d(k) - d_1 + 1)e_7^T + (d - d(k) + 1)e_8^T, 0)^T,$$

$$F_2(d(k)) = (0, 0, (d(k) - d + 1)e_8^T + (d_2 - d(k) + 1)e_9^T, 0)^T,$$

$$E_1 = \Big(e_1^T, (d_1 + 1)e_6^T - e_1^T, (d_2 - d + 1)e_9^T - e_2^T - e_3^T - e_4^T,$$
$$\qquad \frac{1}{2}(d_1 + 1)(d_1 + 2)e_{10}^T - (d_1 + 1)e_1^T \Big)^T,$$

$$E_2 = \Big(e_0^T, (d_1 + 1)e_6^T - e_2^T, (d_2 - d + 1)e_9^T - e_3^T - e_4^T - e_5^T,$$
$$\qquad \frac{1}{2}(d_1 + 1)(d_1 + 2)e_{10}^T - (d_1 + 1)e_6^T \Big)^T,$$

$$E_3 = \left(e_1^T, (d_1 + 1)e_6^T - e_1^T, (d - d_1 + 1)e_7^T - e_2^T - e_3^T - e_4^T, \right.$$

$$\left. \frac{1}{2}(d_1 + 1)(d_1 + 2)e_{10}^T - (d_1 + 1)e_1^T \right)^T,$$

$$E_4 = \left(e_0^T, (d_1 + 1)e_6^T - e_2^T, (d - d_1 + 1)e_7^T - e_3^T - e_4^T - e_5^T, \right.$$

$$\left. \frac{1}{2}(d_1 + 1)(d_1 + 2)e_{10}^T - (d_1 + 1)e_6^T \right)^T,$$

$$H_1 = \alpha_1 \left[d_1 \left(Z_{111} + \frac{1}{3}Z_{122} + \frac{1}{5}Z_{133} \right) \right]\alpha_1^T,$$

$$H_2 = \alpha_4 \left[(d_2 - d) \left(Z_{411} + \frac{1}{3}Z_{422} + \frac{1}{5}Z_{433} \right) \right]\alpha_4^T,$$

$$H_3 = \alpha_5 \left[(d - d_1) \left(Z_{511} + \frac{1}{3}Z_{522} + \frac{1}{5}Z_{533} \right) \right]\alpha_5^T,$$

and

$$\alpha_1 = \left(e_1^T, e_2^T, e_6^T, e_{10}^T \right), \qquad \alpha_2 = \left(e_2^T, e_3^T, e_7^T, e_{11}^T \right), \qquad \alpha_3 = \left(e_3^T, e_4^T, e_8^T, e_{12}^T \right),$$

$$\alpha_4 = \left(e_4^T, e_5^T, e_9^T, e_{13}^T \right), \qquad \alpha_5 = \left(e_2^T, e_4^T, e_7^T, e_{11}^T \right), \qquad \alpha_6 = \left(e_4^T, e_3^T, e_8^T, e_{12}^T \right),$$

$$\alpha_7 = \left(e_3^T, e_5^T, e_9^T, e_{13}^T \right), \qquad e_i = (0_{n \times (i-1)n}, I, 0_{n \times (13-i)n}), \quad i = 1, 2, \ldots, 13,$$

$$e_0 = Ae_1 + Be_3.$$

Proof Construct a Lyapunov functional as follows:

$$V(k) = \sum_{i=1}^{5} V_i(k),$$

where

$$V_1(k) = \theta^T(k)P\theta(k),$$

$$V_2(k) = \sum_{i=k-d_1}^{k-1} x^T(i)Q_1x(i) + \sum_{i=k-d}^{k-d_1-1} x^T(i)Q_2x(i) + \sum_{i=k-d_2}^{k-d-1} x^T(i)Q_3x(i),$$

$$V_3(k) = \sum_{i=-d_1}^{-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_1\eta(j),$$

$$V_4(k) = \sum_{i=-d}^{-d_1-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_2\eta(j),$$

$$V_5(k) = \sum_{i=-d_2}^{-d-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_3\eta(j),$$

and

$$\theta(k) = \left(x^T(k), \sum_{i=k-d_1}^{k-1} x^T(i), \sum_{i=k-d_2}^{k-d_1-1} x^T(i), \sum_{i=k-d_1}^{k-1} \sum_{j=i}^{k-1} x^T(j) \right)^T.$$

For simplicity, we first define the following vectors:

$$\xi_1^T(k) = \begin{pmatrix} x^T(k) & x^T(k-d_1) & x^T\big(k-d(k)\big) & x^T(k-d) & x^T(k-d_2) \end{pmatrix}$$
$$U_1^T \quad U_2^T \quad U_3^T \quad U_4^T \quad W_1^T \quad W_2^T \quad W_3^T \quad W_4^T \big),$$

$$\xi_2^T(k) = \begin{pmatrix} x^T(k) & x^T(k-d_1) & x^T\big(k-d(k)\big) & x^T(k-d) & x^T(k-d_2) \end{pmatrix}$$
$$U_1^T \quad U_5^T \quad U_6^T \quad U_7^T \quad W_1^T \quad W_5^T \quad W_6^T \quad W_7^T \big),$$

where

$$U_1 = \frac{1}{d_1+1} \sum_{i=k-d_1}^{k} x(i),$$

$$U_2 = \frac{1}{d(k)-d_1+1} \sum_{i=k-d(k)}^{k-d_1} x(i),$$

$$U_3 = \frac{1}{d-d(k)+1} \sum_{i=k-d}^{k-d(k)} x(i),$$

$$U_4 = \frac{1}{d_2-d+1} \sum_{i=k-d_2}^{k-d} x(i),$$

$$U_5 = \frac{1}{d-d_1+1} \sum_{i=k-d}^{k-d_1} x(i),$$

$$U_6 = \frac{1}{d(k)-d+1} \sum_{i=k-d(k)}^{k-d} x(i),$$

$$U_7 = \frac{1}{d_2-d(k)+1} \sum_{i=k-d_2}^{k-d(k)} x(i),$$

$$W_1 = \frac{2}{(d_1+1)(d_1+2)} \sum_{i=k-d_1}^{k} \sum_{j=i}^{k} x(j),$$

$$W_2 = \frac{2}{(d(k)-d_1+1)(d(k)-d_1+2)} \sum_{i=k-d(k)}^{k-d_1} \sum_{j=i}^{k-d_1} x(j),$$

$$W_3 = \frac{2}{(d-d(k)+1)(d-d(k)+2)} \sum_{i=k-d}^{k-d(k)} \sum_{j=i}^{k-d(k)} x(j),$$

$$W_4 = \frac{2}{(d_2-d+1)(d_2-d+2)} \sum_{i=k-d_2}^{k-d} \sum_{j=i}^{k-d} x(j),$$

$$W_5 = \frac{2}{(d-d_1+1)(d-d_1+2)} \sum_{i=k-d}^{k-d_1} \sum_{j=i}^{k-d_1} x(j),$$

$$W_6 = \frac{2}{(d(k)-d+1)(d(k)-d+2)} \sum_{i=k-d(k)}^{k-d} \sum_{j=i}^{k-d} x(j),$$

$$W_7 = \frac{2}{(d_2-d(k)+1)(d_2-d(k)+2)} \sum_{i=k-d_2}^{k-d(k)} \sum_{j=i}^{k-d(k)} x(j).$$

Case (i): $d(k) \in [d_1, d] \cap N$. The forward difference of $V_i(k)$ $(i = 1, 2, \ldots, 5)$ is given by

$$
\begin{aligned}
\triangle V_1(k) &= \theta^T(k+1)P\theta(k+1) - \theta^T(k)P\theta(k) \\
&= \xi_1^T(k)\big\{E_2^T PE_2 - E_1^T PE_1 + \mathrm{sym}\big[(E_2 - E_1)^T PF_1\big(d(k)\big)\big]\big\}\xi_1(k) \\
&= \xi_1^T(k)\big[D_1 + G_1\big(d(k)\big)\big]\xi_1(k), \tag{6} \\
\triangle V_2(k) &= x^T(k)Q_1 x(k) - x^T(k-d_1)Q_1 x(k-d_1) + x^T(k-d_1)Q_2 x(k-d_1) \\
&\quad - x^T(k-d)Q_2 x(k-d) + x^T(k-d)Q_3 x(k-d) - x^T(k-d_2)Q_3 x(k-d_2) \\
&= \xi_i^T(k)\big[e_1^T Q_1 e_1 - e_2^T(Q_1 - Q_2)e_2 - e_4^T(Q_2 - Q_3)e_4 - e_5^T Q_3 e_5\big]\xi_i(k) \\
&= \xi_i^T(k)D_3\xi_i(k), \quad i = 1, 2. \tag{7}
\end{aligned}
$$

Using Lemma 2.1, we can obtain

$$
\begin{aligned}
\triangle V_3(k) &= \sum_{i=-d_1}^{-1}\sum_{j=k+i+1}^{k}\eta^T(j)R_1\eta(j) - \sum_{i=-d_1}^{-1}\sum_{j=k+i}^{k-1}\eta^T(j)R_1\eta(j) \\
&= d_1\eta^T(k)R_1\eta(k) - \sum_{i=k-d_1}^{k-1}\eta^T(i)R_1\eta(i) \\
&\leq \xi_i^T(k)\big[d_1(e_0 - e_1)^T R_1(e_0 - e_1)\big]\xi_i(k) \\
&\quad + \xi_i^T(k)\big(e_1^T, e_2^T, e_6^T, e_{10}^T\big)\bigg\{d_1\bigg(Z_{111} + \frac{1}{3}Z_{122} + \frac{1}{5}Z_{133}\bigg) \\
&\quad + \mathrm{sym}[N_{11}\Pi_1 + N_{12}\Pi_2 + N_{13}\Pi_3]\bigg\}\big(e_1^T, e_2^T, e_6^T, e_{10}^T\big)^T\xi_i(k) \\
&= \xi_i^T(k)[D_4 + H_1]\xi_i(k), \quad i = 1, 2, \tag{8} \\
\triangle V_4(k) &= \sum_{i=-d}^{-d_1-1}\sum_{j=k+i+1}^{k}\eta^T(j)R_2\eta(j) - \sum_{i=-d}^{-d_1-1}\sum_{j=k+i}^{k-1}\eta^T(j)R_2\eta(j) \\
&= (d-d_1)\eta^T(k)R_2\eta(k) - \sum_{i=k-d(k)}^{k-d_1-1}\eta^T(i)R_2\eta(i) - \sum_{i=k-d}^{k-d(k)-1}\eta^T(i)R_2\eta(i) \\
&\leq \xi_1^T(k)\big[(d-d_1)(e_0 - e_1)^T R_2(e_0 - e_1)\big]\xi_1(k) + \xi_1^T(k)\big(e_2^T, e_3^T, e_7^T, e_{11}^T\big)\bigg\{\big(d(k) - d_1\big) \\
&\quad \times \bigg(Z_{211} + \frac{1}{3}Z_{222} + \frac{1}{5}Z_{233}\bigg) + \mathrm{sym}[N_{21}\Pi_1 + N_{22}\Pi_2 + N_{23}\Pi_3]\bigg\} \\
&\quad \times \big(e_2^T, e_3^T, e_7^T, e_{11}^T\big)^T\xi_1(k) \\
&\quad + \xi_1^T(k)\big(e_3^T, e_4^T, e_8^T, e_{12}^T\big)\bigg\{\big(d - d(k)\big)\bigg(Z_{311} + \frac{1}{3}Z_{322} + \frac{1}{5}Z_{333}\bigg) \\
&\quad + \mathrm{sym}[N_{31}\Pi_1 + N_{32}\Pi_2 + N_{33}\Pi_3]\bigg\}\big(e_3^T, e_4^T, e_8^T, e_{12}^T\big)^T\xi_1(k) \\
&= \xi_1^T(k)\big[D_5 + G_3\big(d(k)\big) + G_4\big(d(k)\big)\big]\xi_1(k), \tag{9}
\end{aligned}
$$

$$\triangle V_5(k) = \sum_{i=-d_2}^{-d-1} \sum_{j=k+i+1}^{k} \eta^T(j)R_3\eta(j) - \sum_{i=-d_2}^{-d-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_3\eta(j)$$

$$\leq \xi_1^T(k)\big[(d_2-d)(e_0-e_1)^T R_3(e_0-e_1)\big]\xi_1(k) + \xi_1^T(k)\big(e_4^T,e_5^T,e_9^T,e_{13}^T\big)\bigg\{(d_2-d)$$

$$\times \bigg(Z_{411} + \frac{1}{3}Z_{422} + \frac{1}{5}Z_{433}\bigg) + \text{sym}[N_{41}\Pi_1 + N_{42}\Pi_2 + N_{43}\Pi_3]\bigg\}$$

$$\times \big(e_4^T,e_5^T,e_9^T,e_{13}^T\big)^T \xi_1(k)$$

$$= \xi_1^T(k)[D_6 + H_2]\xi_1(k). \tag{10}$$

From (6)–(10) we have $\triangle V(k) \leq \xi_1^T(k)\Psi_1(d(k))\xi_1(k)$. According to (3), we can obtain $\triangle V(k) < 0$ for any $\xi_1(k) \neq 0$ and $d(k) \in [d_1, d] \cap N$.

Case (ii): $d(k) \in [d, d_2] \cap N$. Similarly, we have

$$\triangle V_1(k) = \theta^T(k+1)P\theta(k+1) - \theta^T(k)P\theta(k)$$

$$= \xi_2^T(k)\big\{E_4^T PE_4 - E_3^T PE_3 + \text{sym}\big[(E_4-E_3)^T PF_2\big(d(k)\big)\big]\big\}\xi_2(k)$$

$$= \xi_2^T(k)\big[D_2 + G_2\big(d(k)\big)\big]\xi_2(k), \tag{11}$$

$$\triangle V_4(k) = \sum_{i=-d}^{-d_1-1} \sum_{j=k+i+1}^{k} \eta^T(j)R_2\eta(j) - \sum_{i=-d}^{-d_1-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_2\eta(j)$$

$$\leq \xi_2^T(k)\big[(d-d_1)(e_0-e_1)^T R_2(e_0-e_1)\big]\xi_2(k) + \xi_2^T(k)\big(e_2^T,e_4^T,e_7^T,e_{11}^T\big)\bigg\{(d-d_1)$$

$$\times \bigg(Z_{511} + \frac{1}{3}Z_{522} + \frac{1}{5}Z_{533}\bigg) + \text{sym}[N_{51}\Pi_1 + N_{52}\Pi_2 + N_{53}\Pi_3]\bigg\}$$

$$\times \big(e_2^T,e_4^T,e_7^T,e_{11}^T\big)^T \xi_2(k)$$

$$= \xi_2^T(k)[D_7 + H_3]\xi_2(k), \tag{12}$$

$$\triangle V_5(k) = \sum_{i=-d_2}^{-d-1} \sum_{j=k+i+1}^{k} \eta^T(j)R_3\eta(j) - \sum_{i=-d_2}^{-d-1} \sum_{j=k+i}^{k-1} \eta^T(j)R_3\eta(j)$$

$$= (d_2-d)\eta^T(k)R_3\eta(k) - \sum_{i=k-d(k)}^{k-d-1} \eta^T(i)R_3\eta(i) - \sum_{i=k-d_2}^{k-d(k)-1} \eta^T(i)R_3\eta(i)$$

$$\leq \xi_2^T(k)\big[(d_2-d)(e_0-e_1)^T R_3(e_0-e_1)\big]\xi_2(k) + \xi_2^T(k)\big(e_4^T,e_3^T,e_8^T,e_{12}^T\big)\bigg\{\big(d(k)-d\big)$$

$$\times \bigg(Z_{611} + \frac{1}{3}Z_{622} + \frac{1}{5}Z_{633}\bigg) + \text{sym}[N_{61}\Pi_1 + N_{62}\Pi_2 + N_{63}\Pi_3]\bigg\}$$

$$\times \big(e_4^T,e_3^T,e_8^T,e_{12}^T\big)^T \xi_2(k)$$

$$+ \xi_2^T(k)\big(e_3^T,e_5^T,e_9^T,e_{13}^T\big)\bigg\{\big(d_2-d(k)\big)\bigg(Z_{711} + \frac{1}{3}Z_{722} + \frac{1}{5}Z_{733}\bigg)$$

$$+ \text{sym}[N_{71}\Pi_1 + N_{72}\Pi_2 + N_{73}\Pi_3]\bigg\}\big(e_3^T,e_5^T,e_9^T,e_{13}^T\big)^T \xi_2(k)$$

$$= \xi_2^T(k)\big[D_8 + G_5\big(d(k)\big) + G_6\big(d(k)\big)\big]\xi_2(k), \tag{13}$$

and $\triangle V(k) \leq \xi_2^T(k)\Psi_2(d(k))\xi_2(k)$, which under condition (4) indicates that $\triangle V(k)$ is negative definite.

Now by combining Case (i) and Case (ii), $\triangle V(k)$ is negative definite, which guarantees that system (1) is asymptotically stable. This completes the proof. $\square$

Remark 3 To show the advantage of time delay division for the stability condition without using time delay division, we omitted the proof process and only display the results in Table 1.

4 Numerical illustrations

In this section, we present some numerical examples that often appear in the literature to demonstrate the effectiveness and sophistication of the proposed approach.

Example 1 Consider system (1) with the following parameters:

$$A = \begin{pmatrix} 0.8 & 0 \\ 0.05 & 0.9 \end{pmatrix}, \qquad B = \begin{pmatrix} -0.1 & 0 \\ -0.2 & -0.1 \end{pmatrix}.$$

Example 2 Consider system (1), where

$$A = \begin{pmatrix} 0.7 & 0.1 \\ 0.05 & 0.7 \end{pmatrix}, \qquad B = \begin{pmatrix} -0.1 & 0.1 \\ -0.1 & -0.2 \end{pmatrix}.$$

The allowable delay upper bound d_2 can be found for given d_1 or vice versa. The simulation results are listed in Tables 1 and 2.

Remark 4 In [12] and this paper, the stability conditions are obtained by using the Wirtinger-based integral inequality and the free-matrix-based integral inequality, respectively. We can see from Table 1 that the latter is superior to the former in reducing the conservatism when time delay division is not used, whereas the conservatism of both is the same when time delay division is used. Therefore, on the basis of some processing techniques, such as the time delay division mentioned before, our future research is discussing the relationship between the tightness of inequalities and the conservatism of criteria.

Remark 5 From Table 2 we can see that with the lower bounds $d_1 = 2, 5, 7, 10, 20$, the corresponding $d_2 = 13, 14, 16, 19, 29$ obtained in [40], $d_2 = 14, 17, 18, 21, 31$ obtained by Theorem 1 in [12], and $d_2 = 15, 17, 18, 21, 31$ derived by Theorem 3.1 in this paper. Thus the results obtained from Theorem 3.1 in this paper are significantly better than those obtained from the existing methods.

5 Conclusion

In this paper, the problem of stability criteria of discrete systems with interval time-varying delays has been investigated. By means of the discrete form of free-matrix-based integral inequality and the time delay division, less conservative criteria than some existing ones are derived due to a tighter estimation of the new inequality. Finally, some illustrative examples are presented to demonstrate the effectiveness of the obtained method.

Availability of data and materials
Not applicable.

Competing interests
The authors declare that there is no conflict of interest regarding the publication of this paper.

Authors' contributions
All authors drafted the manuscript, and they read and approved the final version.

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