Lee and Park *Advances in Difference Equations* (2019) 2019:184
https://doi.org/10.1186/s13662-019-2124-2

Advances in Difference Equations
a SpringerOpen Journal

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Design of sampled-data controllers for the synchronization of complex dynamical networks under controller attacks

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Abstract

This paper addresses the design problem of sampled-data controllers for the synchronization of complex dynamical networks under controller attack. Some discontinuous Lyapunov functionals and zero equation are employed to deal with a sampled-data pattern. Due to the limited access of networks to hackers, cyber-attacks on the controller are considered randomly occurring and are described as an attack function which is nonlinear but assumed to be certain known conditions. The designing conditions for sampled-data controllers against cyber-attack are developed in terms of linear matrix inequality (LMI) by using Lyapunov theory and some novel inequalities. Finally, a numerical example is given to prove the usefulness of the proposed method.

Keywords: Complex network; Synchronization; Sampled-data control; Controller attack

1 Introduction

Nowadays, most of systems, components, plants are connected complicatedly to each other thanks to high level of communication technology, which can be called complex dynamic networks (CDNs). CDNs consist of many nodes with complicated interconnections among them, so each node is affected and affects the others whether they are willing or not; for example, World Wide Web (WWW), social media, metabolic systems, Internet of Things (IoT), transportation networks, power grids, food chain, and so on. By considering the high potential of CDNs, therefore, it is natural that the research on CDNs has become a large research area, and many researchers have discovered the nature of CDNs [1–3]. The synchronization is one of the popular applications of CDNs, which makes all states of each node in a CDN follow the same trajectories, and it has been widely studied [4–20]. To solve the synchronization problem for CDNs, various control schemes, such as feedback control [4–6], adaptive control [7, 8], sliding-mode control [9, 10], dynamic control [11, 12], event-triggered control [13], impulsive control [14, 15], $\mathcal{H}_\infty$ control [16], and pinning control [17–20], have been employed.

In [17], the problem of both exponential synchronization and generalized synchronization of CDNs with/without time-varying coupling delays was investigated by designing an intermittent periodically adaptive pinning controller, and the results were applied to the nearest-neighbor network and the Barabasi–Albert network to show the effectiveness. The paper [7] proposed a concept of consecutive synchronization and designed a distributed adaptive controller for consecutive synchronization of CDNs with special topology structures. Synchronization of CDNs with N-coupled fractional-order chaotic system oscillators and regular/irregular topologies was studied in [6], and their results were applied to various fractional-order chaotic systems, Lorenz, Volta, Duffing, and financial chaotic oscillators. [8] dealt with traffic road network as an application of CDNs, in which an adaptive controller and adjustable coupling strength were designed to reduce the impact of uncertainties, and pinning control method was employed as well for the smooth flow of traffic.

Rapidly growing communication technology opened a research field on sampled-data control. Sampled-data control has many benefits such as easy installation and maintenance, low operation fee, and so on. Therefore, it is natural to have a meteoric rise of research field on the sampled-data control scheme [21–32]. In [21], an aperiodic sampled-data controller was designed for controlling the two-wheel inverted pendulum in the T-S fuzzy model with disturbances on actuators where the proposed controller satisfies very-strict passivity. In [22], $\mathcal{H}_\infty$ state estimator for genetic regulatory networks with random delays, uncertainties, and disturbances was designed using sampled-data of the concentrations of mRNAs and proteins. In [23], an observer-based sampled-data controller was designed for a class of scalar nonlinear affine systems where the discrete-time states were firstly estimated by a nonlinear state observer, and then they were used for designing the sampled-data control. And there are a number of papers on the synchronization of CDNs by a sampled-data controller. The synchronization of CDNs with coupling time-varying delays was achieved via a sampled-data controller in [24], in which sampled-data had a constant time delay, and a discontinuous Lyapunov functional based on the extended Wirtinger inequality was used. In [25], the synchronizability analysis for CDNs with sampled-data coupling signals was conducted by employing multiple-integral Lyapunov functionals, and then a sampling period-dependent criterion was derived in terms of linear matrix inequalities (LMIs). The main feature of the sampled-data controller is that it uses discontinuous signals because the control signals can be updated at sampling instants and are held a constant value for sampling periods. To make the efficient use of this feature, several techniques have been reported, such as input delay approach [26, 27], discontinuous Lyapunov functional based on extended Wirtinger inequality [28], free-matrix-based time-dependent discontinuous Lyapunov approach [30], looped-functional-based approach [29], sampling-instant-to-present-time fragmentation approach [31, 32], and so on.

On the other hand, from the recent situation of growing communication technology, it can be understandably expected that the importance of security against cyber-attack has been emphasized [33–42]. Adversaries maliciously modified the data by exploiting the vulnerabilities of networks, which has degraded or destroyed the stability and performance of the system. Mainly two kinds of cyber-attacks exist [33, 34]: one is denial of service (DoS), another is deception attacks. DoS attacks attempt to block traffic from the actuator and sensor and as a result bring the absence of data for the related components; this situation is well known as the packet dropout. On the other hand, the data from components can be substituted secretly for the data that adversaries want by hackers which is called deception attacks. Thus, to develop criteria with prevention measure against cyber-attacks is a major concern of scholars [38, 39]. In [40], $\mathcal{H}_\infty$ observer-based periodic event-triggered controller for the control of a class of cyber-physical systems was designed under consideration of DoS attacks where the designed controller maximized the frequency and duration of the DoS attacks. In [41], distributed event-triggered $\mathcal{H}_\infty$ filters were designed on sensor networks in the presence of sensor saturations and randomly occurring cyber-attacks. In [42], the consensus problem of nonlinear multi-agent systems was studied by using sampled agents' states and considering cyber-attacks on connectivity of the network topology, but it was recoverable. Authors in [43] proposed both event- and self-triggered control schemes for the leader-following consensus problem of multi-agent systems under consideration of DoS attacks, in which both synchronous and asynchronous updated strategies for control protocols were derived. Obviously, cyber-attacks would become a hotter issue for CDNs because CDNs consist of numerous nodes connected through communication networks. Nevertheless, research on CDNs with the consideration of cyber-attacks deserves much interest from scholars, only several works have been reported, which motivates this research.

This paper is about the synchronization of CDNs using sampled-data information under controller attacks. The main contribution of the paper is laid on mainly two streams: (i) Development of novel discontinuous Lyapunov functionals and zero inequality. Novel discontinuous Lyapunov functional would help the utilization of sampling characteristic. Combining with the novel discontinuous Lyapunov functionals, a zero inequality is needed to deal with a rest term of the time derivative of the novel discontinuous Lyapunov functionals. These may lead to less conservative conditions for designing a sampled-data controller. (ii) Consideration of cyber-attacks on controllers. Recently, the concept of cyber-attacks is the hottest issue in control engineering, and many works on cyber-attacks have been done. However, only few works cover controllers under cyber-attacks, even controllers of CDNs with cyber-attacks are few or nonexistent. Therefore, this paper considers deception attacks on controllers and designs the suitable controller against the attacks.

2 Problem formulation

Consider the following a CDN with N linearly coupled identical nodes:

$$\dot{x}_i(t) = Ax(t) + Bf\big(x_i(t)\big) + \sum_{j=1}^{N} c_{ij}x_j(t) + u_i(t), \quad i = 1,\dots,N, \tag{1}$$

where $x_i = (x_{i1}, x_{i2}, \dots, x_{in})^T \in \mathbb{R}^n$ is the state vector of the ith node, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times n}$ are known constant matrices, $f(t) = (f_1(t), f_2(t), \dots, f_n(t))^T : \mathbb{R}^n \to \mathbb{R}^n$ is a smooth nonlinear vector field, and $u_i(t) = (u_{i1}, u_{i2}, \dots, u_{in})^T$ is the control input of ith node. $C = (c_{ij})_{N\times N}$ is the coupling matrix of the network, where each element of the matrix c_{ij} is defined as follows: if there is a connection from node i to node j ($i \neq j$), then $c_{ij} = 1$; otherwise $c_{ij} = 0$ ($i \neq j$), and the diagonal elements of matrix C are assumed by

$$c_{ii} = -\sum_{j=1, j\neq i}^{N} c_{ij} = -\sum_{j=1, j\neq i}^{N} c_{ji}, \quad i = 1,\dots,N.$$

Assumption 1 The nonlinear function $f_i(\cdot)$ $(i = 1, 2, \dots, n)$ satisfies the following condition:

$$0 \le \frac{f_i(a) - f_i(b)}{a - b} \le l_i, \qquad f_i(0) = 0,$$

where l_i is a positive constant.

Let us consider a target note as $\dot{s}(t) = As(t) + Bf(s(t))$. Then the aim of this paper is synchronizing all nodes of a CDN up to the target node, i.e., $\lim_{t\to\infty} \|s(t) - x_i(t)\| = 0$ for $i = 1, 2, \dots, N$. To this end, we define error vectors as $e_i(t) = s(t) - x_i(t)$. Then the error dynamics is given as follows:

$$\dot{e}_i(t) = Ae(t) + B\bar{f}_i(t) - \sum_{j=i}^{N} c_{ij}e_j(t) - u_i(t), \quad i = 1, \dots, N, \tag{2}$$

where $\bar{f}_i(t) = f(s(t)) - f(x_i(t))$.

In this paper, the controllers are designed using sampled-data signals as follows:

$$u_i^F(t) = K_i e_i(t_k), \quad t_k \le t < t_{k+1}, \tag{3}$$

where K_i is the gain matrices for ith node to be determined, t_k is the updating instant time of the Zero-Order-Hold (ZOH) satisfying $t_{k+1} - t_k = h$, h is a positive scalar.

Recently, the controller has become a target of hackers to alter the transmitted information. So, this paper is concerned with designing a sampled-data controller under cyber-attack. Because of the limited communication capacity of the network resources for hackers, the attacks might randomly destroy the control signals, and the maliciously modified signals would not be far away from the original one. To reflect this situation, we introduce Bernoulli random variable $\gamma_i(t) \in \{0, 1\}$, satisfying $\Pr\{\gamma_i(t) = 1\} = \gamma_i$, where $\Pr\{x\}$ implies the occurrence probability of the event x, and γ_i is a known positive constant less than 1. Here, $\gamma_i(t) = 1$ means the cyber-attacks are launched to the controller $u_i^F(t)$. The specific controller model under cyber-attack is as follows:

$$u_i(t) = \bigl(1 - \gamma_i(t)\bigr)K_i e_i(t_k) + \gamma_i(t)K_i g_i\bigl(e_i\bigl(t - d(t)\bigr)\bigr), \tag{4}$$

where $g_i(t) = (g_{i1}(t), g_{i2}(t), \dots, g_{in}(t))^T : \mathbb{R}^n \to \mathbb{R}^n$ is the function of controller attacks, $d(t)$ is time-varying delay satisfying $0 \le d(t) \le d$ and $\dot{d}(t) \le \mu$, and d and μ are known positive constants.

Assumption 2 The cyber-attack function $g_i(\cdot)$ is satisfied $g_i(0) = 0$ and $\forall i \in \{1, 2, \dots, n\}$, $a \ne b$

$$0 \le \frac{g_{ij}(a) - g_{ij}(b)}{a - b} \le m_{ij},$$

where m_{ij} is a positive constant.

Remark 1 It can be strictly said that the cyber-attacks considered in this paper are randomly occurring deception attacks because, when a controller is under cyber-attacks, the real control inputs are replaced with the incorrect ones which adversaries want. And the cyber-attacks would be launched at any time, this introduces randomly occurring sense. Since adversaries try to adjust real control signals and transmit them through communication networks, the considered cyber-attacks signals contain time-delayed information.

Then the closed-loop error systems with controller (4) can be rewritten in the following vector-matrix form:

$$\begin{aligned}\dot{e}(t) &= A_N e(t) + B_N F\bigl(e(t)\bigr) - C_N e(t) - \bar{\Gamma}(t) K e(t_k) \\ &\quad - \Gamma(t) G\bigl(e\bigl(t-d(t)\bigr)\bigr),\end{aligned} \tag{5}$$

where $e(t) = [e_1^T(t), e_2^T(t), \ldots, e_N^T(t)]^T$, $F(e(t)) = [\bar{f}_1^T(e_1(t)), \bar{f}_2^T(e_2(t)), \ldots, \bar{f}_N^T(e_N(t))]^T$, $G(e(t)) = [g_1^T(e_1(t)), g_2^T(e_2(t)), \ldots, g_N^T(e_N(t))]^T$, $K = \operatorname{diag}\{K_1, K_2, \ldots, K_N\}$, $\Gamma(t) = \operatorname{diag}\{\gamma_1(t), \gamma_2(t), \ldots, \gamma_N(t)\} \otimes I_n$, $\bar{\Gamma}(t) = (I_N - \Gamma(t)) \otimes I_n$, $A_N = I_N \otimes A$, $B_N = I_N \otimes B$, $C_N = C \otimes I_n$, and $\otimes$ stands for the notation of Kronecker product.

3 Main results

This section proposes a design scheme for the sampled-data controller under cyber-attack synchronizing a CDN. Before proceeding further, the following lemma is given.

Lemma 1 *For given three scalars a, b, c where $a \le b \le c$, a positive definite matrix $W \in \mathbb{R}^{n\times n}$, and any matrix $S \in \mathbb{R}^{2n\times 2n}$ and all continuous function η in $[a,c] \to \mathbb{R}^n$ the following inequality holds*:

$$\int_a^c \dot{\eta}^T(s) W \dot{\eta}(s)\,ds \ge \beta^T(t) \mathcal{W} \beta(t),$$

subject to

$$\begin{bmatrix} \operatorname{diag}\{W, 3W\} & S \\ \star & \operatorname{diag}\{W, 3W\} \end{bmatrix} > 0,$$

where

$$\begin{aligned}\beta(t) &= \bigl[\Omega_1^T(a,b), \Omega_2^T(a,b), \Omega_1^T(b,c), \Omega_2^T(b,c)\bigr]^T, \\ \Omega_1(a,b) &= \eta(b) - \eta(a), \\ \Omega_2(a,b) &= \eta(b) + \eta(a) - \frac{2}{b-a}\int_a^b \eta(s)\,ds.\end{aligned}$$

Proof It is easy to derive Lemma 1 when we combined with Wirtinger-based integral inequality [44] and reciprocal convex lemma [45].

The following notations are used in the paper. The block entry matrices and a vector are defined as b_i $(i = 1,\dots,10) \in \mathbb{R}^{10nN\times nN}$ (for example, $b_3 = [0,0,I,\underbrace{0,\dots,0}_{7}]$) and

$$\zeta(t) = \Big[e^T(t), e^T\big(t-d(t)\big), e^T(t-d), \frac{1}{d(t)}\int_{t-d(t)}^{t} e^T(s)\,ds, \frac{1}{d-d(t)}\int_{t-d}^{t-d(t)} e^T(s)\,ds, e^T(t_k), \int_{t_k}^{t} e^T(s)\,ds, \dot{e}^T(t), F^T(t), G^T\big(t-d(t)\big)\Big]^T. \qquad \square$$

Now, the main result is given by the following theorem.

Theorem 1 *For given scalars d, h, μ, α, γ_i $(i = 1,2,\dots,N)$, l_i $(i = 1,2,\dots,n)$, m_{ij} $(i = 1,2,\dots,N; j = 1,2,\dots,n)$, there exists a sampled-data controller with cyber-attack (4) for the synchronization of CDN (1) if there exist positive-definite matrices $P, Q_1, Q_2, Q_3 \in \mathbb{R}^{nN\times nN}$,*

$$R = \left[\begin{array}{c|cc} R_1 & R_2 & R_3 \\ \hline \star & \multicolumn{2}{c}{\multirow{2}{*}{R_4}} \\ \star & & \end{array}\right] \in \mathbb{R}^{3nN\times 3nN},$$

positive diagonal matrices $U_1, U_2, X_1, X_2 \in \mathbb{R}^{nN\times nN}$, a symmetric matrix $H_5 \in \mathbb{R}^{nN\times nN}$, a diagonal matrix $D \in \mathbb{R}^{nN\times nN}$, any matrices $H_1, H_2, H_3, H_4, \bar{K} \in \mathbb{R}^{nN\times nN}$, $S \in \mathbb{R}^{2nN\times 2nN}$, $Y \in \mathbb{R}^{10nN\times nN}$ satisfying the following LMIs:

$$\mathcal{P} = \operatorname{diag}\{P,0,0\} + hH > 0, \tag{6}$$

$$\mathcal{Q} = \begin{bmatrix} \operatorname{diag}\{Q_3, 3Q_3\} & S \\ \star & \operatorname{diag}\{Q_3, 3Q_3\} \end{bmatrix} > 0, \tag{7}$$

$$R_4 > 0, \tag{8}$$

$$\Upsilon_1 + h\Upsilon_2 < 0, \tag{9}$$

$$\begin{bmatrix} \Upsilon_1 - hb_6R_1b_6^T & Y \\ \star & -\frac{1}{h}R_4 \end{bmatrix} < 0, \tag{10}$$

where

$$\begin{aligned} \Upsilon_1 = {} & \operatorname{Sym}\big\{b_1Pb_8^T - b_6R_2(b_1-b_6)^T - b_6R_3b_7^T + Y\Pi_5^T + b_1LU_1b_9^T \\ & + b_2MX_1b_{10}^T + (b_1+\alpha b_8)\big(-Db_8^T + D(A_N - C_N)b_1^T + DB_Nb_9^T - \bar{\Gamma}\bar{K}b_6^T \\ & - \Gamma\bar{K}b_{10}^T\big)\big\} - \Pi_1H\Pi_1^T + b_1(Q_1+Q_2)b_1^T - (1-\mu)b_2Q_1b_2^T - b_3Q_2b_3^T \\ & + db_8Q_3b_8^T - \Pi_3\mathcal{Q}\Pi_3^T + b_1LU_2Lb_1^T - b_9(2U_1+U_2)b_9^T + b_2MX_2Mb_2^T \\ & - b_{10}(2X_1+X_2)b_{10}^T, \\ \Upsilon_2 = {} & \operatorname{Sym}\big\{\Pi_1H\Pi_2^T\big\} + \Pi_4R\Pi_4^T, \end{aligned}$$

$$H = \begin{bmatrix} H_1 + H_1^T & -H_1 - H_2 & H_3 \\ \star & H_2 + H_2^T & H_4 \\ \star & \star & H_5 \end{bmatrix},$$

$$\Pi_1 = [b_1, \quad b_6, \quad b_7],$$

$$\Pi_2 = [b_8, \quad 0, \quad b_1 - b_6],$$

$$\Pi_3 = [b_1 - b_2, \quad b_1 + b_2 - 2b_4, \quad b_2 - b_3, \quad b_2 + b_3 - 2b_5],$$

$$\Pi_4 = [b_6, \quad b_8, \quad b_1],$$

$$\Pi_5 = [b_1 - b_6, \quad b_7],$$

$$L = I_N \otimes \operatorname{diag}\{l_1, l_2, \dots, l_n\},$$

$$M = \operatorname{diag}\{M_1, M_2, \dots, M_N\},$$

$$M_i = \operatorname{diag}\{m_{i1}, m_{i2}, \dots, m_{in}\},$$

$$\Gamma = \operatorname{diag}\{\gamma_1, \gamma_2, \dots, \gamma_N\} \otimes I_n,$$

$$\bar{\Gamma} = (I_N - \Gamma) \otimes I_n,$$

with the notation Sym$\{X\}$ *indicating* $X + X^T$. *Also, the desired control gain matrices* (4) *can be given by* $K = D^{-1}\bar{K}$.

Proof Consider the following discontinuous Lyapunov functional for the error system (5):

$$V(t) = V_1(t) + V_2(t) + V_3(t), \quad t \in [t_k, t_{k+1}), \tag{11}$$

where

$$V_1(t) = e^T(t)Pe(t) + (t_{k+1} - t)\nu_1^T(t)H\nu_1(t),$$

$$V_2(t) = \int_{t-d(t)}^{t} e^T(s)Q_1e(s)\,ds + \int_{t-d}^{t} e^T(s)Q_2e(s)\,ds + \int_{-d}^{0}\int_{t+\theta}^{t} \dot{e}^T(s)Q_3\dot{e}(s)\,ds\,d\theta,$$

$$V_3(t) = (t_{k+1} - t)\int_{t_k}^{t} \nu_2^T(s)R\nu_2(s)\,ds,$$

with

$$\nu_1(t) = \left[e^T(t), \quad e^T(t_k), \quad \int_{t_k}^{t} e^T(s)\,ds\right]^T,$$

$$\nu_2(t) = \left[e^T(t_k), \quad \dot{e}^T(t), \quad e^T(t)\right]^T.$$

The matrix H is not defined as positive definite, so we firstly show the positiveness of $V_1(t)$.

$$V_1(t) = \left(\frac{t_{k+1} - t}{h} + \frac{t - t_k}{h}\right)e^T(t)Pe(t) + (t_{k+1} - t)\nu_1^T(t)H\nu_1(t) = \frac{t_{k+1} - t}{h}\nu_1^T(t)\mathcal{P}\nu_1(t) + \frac{t - t_k}{h}e^T(t)Pe(t).$$

From LMI (6), we can know $V_1(t)$ is positive definite. And next, it is obvious that (i) $V_3(t)$ is discontinuous at $t = t_k$, i.e., $V_3(t_k) = \lim_{t\to t_k^+} V_3(t) \neq \lim_{t\to t_k^-} V_3(t)$; (ii) $V_3(t)$ is positive during $t \in (t_{k-1}, t_k)$; (iii) $V_3(t_k) = 0$. Therefore, we can conclude $\lim_{t\to t_k^-} V(t) \geq V(t_k)$.

The infinitesimal operator $\mathcal{L}$ of $V(x_t)$ is defined as follows:

$$\mathcal{L}V(x_t) = \lim_{c\to 0^+} \frac{1}{c}\bigl\{\mathbb{E}\bigl\{V(x_{t+h})|x_t\bigr\}\bigr\},$$

where $\mathbb{E}\{x\}$ means the expectation of the stochastic variable x.

Then we can have

$$\begin{aligned}\mathcal{L}V_1(t) &= 2e^T(t)P\dot{e}(t) + 2(t_{k+1} - t)\nu_1^T(t)H\nu_3(t) - \nu_1^T(t)H\nu_1(t)\\ &= \zeta^T(t)\bigl(\operatorname{Sym}\bigl\{b_1Pb_8^T + (t_{k+1} - t)\Pi_1H\Pi_2^T\bigr\} - \Pi_1H\Pi_1^T\bigr)\zeta(t), \end{aligned} \tag{12}$$

where Π_1 and Π_2 are defined in Theorem 1.

Also, we have

$$\begin{aligned}\mathcal{L}V_2(t) &= e^T(t)(Q_1 + Q_2)e(t) - \bigl(1 - \dot{h}(t)\bigr)e^T\bigl(t - d(t)\bigr)Q_1e\bigl(t - d(t)\bigr)\\ &\quad - e^T(t - d)Q_2e(t - d) + d\dot{e}^T(t)Q_3\dot{e}(t) - \int_{t-d}^{t} \dot{e}^T(s)Q_3\dot{e}(s)\,ds\\ &\leq \zeta^T(t)\bigl(b_1(Q_1 + Q_2)b_1^T - (1 - \mu)b_2Q_1b_2^T - b_3Q_2b_3^T + db_8Q_3b_8^T\bigr)\zeta(t)\\ &\quad - \int_{t-d}^{t} \dot{e}^T(s)Q_3\dot{e}(s)\,ds\\ &\leq \zeta^T(t)\bigl(b_1(Q_1 + Q_2)b_1^T - (1 - \mu)b_2Q_1b_2^T - b_3Q_2b_3^T + db_8Q_3b_8^T\\ &\quad - \Pi_3\mathcal{Q}\Pi_3^T\bigr)\zeta(t), \end{aligned} \tag{13}$$

where Π_3 and $\mathcal{Q}$ are defined in Theorem 1, and Lemma 1 with LMI (7) is used to obtain the last equation.

Also,

$$\begin{aligned}\mathcal{L}V_3(t) &= (t_{k+1} - t)\nu_2^T(t)R\nu_2(t) - \int_{t_k}^{t} \nu_2^T(s)R\nu_2(s)\,ds\\ &= (t_{k+1} - t)\nu_2^T(t)R\nu_2(t) - (t - t_k)e^T(t_k)R_1e(t_k)\\ &\quad - 2e^T(t_k)R_2\bigl(e(t) - e(t_k)\bigr) - 2e^T(t_k)R_3\int_{t_k}^{t} e(s)\,ds\\ &\quad - \int_{t_k}^{t} \nu_4^T(s)R_4\nu_4(s)\,ds\\ &= \zeta^T(t)\bigl((t_{k+1} - t)\Pi_4R\Pi_4^T - (t - t_k)b_6R_1b_6^T\\ &\quad - \operatorname{Sym}\bigl\{b_6R_2(b_1 - b_6)^T + b_6R_3b_7^T\bigr\}\bigr)\zeta(t)\\ &\quad - \int_{t_k}^{t} \nu_4^T(s)R_4\nu_4(s)\,ds, \end{aligned} \tag{14}$$

where Π_4 is defined in Theorem 1 and

$$\nu_4(t) = \begin{bmatrix} \dot{e}^T(t), & e^T(t) \end{bmatrix}^T.$$

If condition (8) holds, then we have

$$\begin{aligned}
0 &= 2\zeta^T(t)Y\left(\begin{bmatrix} e(t)-e(t_k) \\ \int_{t_k}^t e(s)\,ds \end{bmatrix} - \int_{t_k}^t \nu_4(s)\,ds\right) \\
&\leq (t-t_k)\zeta^T(t)YR_4^{-1}Y^T\zeta(t) + 2\zeta^T(t)Y\begin{bmatrix} e(t)-e(t_k) \\ \int_{t_k}^t e(s)\,ds \end{bmatrix} \\
&\quad + \int_{t_k}^t \nu_4^T(s)\,ds \frac{R_4}{(t-t_k)} \int_{t_k}^t \nu_4(s)\,ds \\
&\leq (t-t_k)\zeta^T(t)YR_4^{-1}Y^T\zeta(t) + 2\zeta^T(t)Y\begin{bmatrix} e(t)-e(t_k) \\ \int_{t_k}^t e(s)\,ds \end{bmatrix} \\
&\quad + \int_{t_k}^t \nu_4^T(s)R_4\nu_4(s)\,ds,
\end{aligned}$$

where Jensen's inequality [46] is used to obtain the last equation.

Therefore, the following inequality can be derived:

$$\begin{aligned}
&-\int_{t_k}^t \nu_4^T(s)R_4\nu_4(s)\,ds \\
&\quad\leq 2\zeta^T(t)Y\begin{bmatrix} e(t)-e(t_k) \\ \int_{t_k}^t e(s)\,ds \end{bmatrix} + (t-t_k)\zeta^T(t)YR_4^{-1}Y^T\zeta(t) \\
&\quad= \zeta^T(t)\bigl(\mathrm{Sym}\{Y\Pi_5^T\} + (t-t_k)YR_4^{-1}Y^T\bigr)\zeta(t),
\end{aligned} \tag{15}$$

where Π_5 is defined in Theorem 1.

According to Assumptions 1 and 2, for positive diagonal matrices U_1, U_2, X_1, X_2, we can obtain the following inequalities:

$$\begin{aligned}
0 &\leq 2\bigl(e^T(t)L - F^T\bigl(e(t)\bigr)\bigr)U_1F\bigl(e(t)\bigr) + e^T(t)LU_2Le(t) - F^T\bigl(e(t)\bigr)U_2F\bigl(e(t)\bigr) \\
&= \zeta^T(t)\bigl(b_1LU_2Lb_1^T - b_9(2U_1+U_2)\bigr)b_9^T + \mathrm{Sym}\{b_1LU_1b_9^T\}\bigr)\zeta(t),
\end{aligned} \tag{16}$$

$$\begin{aligned}
0 &\leq 2\bigl(e^T\bigl(t-d(t)\bigr)M - G^T\bigl(e\bigl(t-d(t)\bigr)\bigr)\bigr)X_1G\bigl(e\bigl(t-d(t)\bigr)\bigr) \\
&\quad + e^T\bigl(t-d(t)\bigr)MX_2Me\bigl(t-d(t)\bigr) - G^T\bigl(e\bigl(t-d(t)\bigr)\bigr)X_2G\bigl(e\bigl(t-d(t)\bigr)\bigr) \\
&= \zeta^T(t)\bigl(b_2MX_2Mb_2^T - b_{10}(2X_1+X_2)\bigr)b_{10}^T + \mathrm{Sym}\{b_2MX_1b_{10}^T\}\bigr)\zeta(t).
\end{aligned} \tag{17}$$

Also, according to the error system (5), for a scalar α and a diagonal matrix D, the following equation holds:

$$\begin{aligned}
0 &= \mathbb{E}\bigl\{2\bigl[e^T(t)D + \alpha\dot{e}^T(t)D\bigr]\bigl[-\dot{e}(t) + (A_N - C_N)e(t) + B_NF(t) \\
&\quad - \bar{\Gamma}Ke(t_k) - \Gamma KG\bigl(t-d(t)\bigr)\bigr]\bigr\}
\end{aligned}$$

$$= \mathbb{E}\big\{\zeta^T(t)\big(\mathrm{Sym}\big\{(b_1 + \alpha b_8)\big(-Db_8^T + D(A_N - C_N)b_1^T + DB_N b_9^T - \bar{\Gamma}\bar{K}b_6^T - \Gamma\bar{K}b_{10}^T\big)\big\}\big)\zeta(t)\big\}, \tag{18}$$

where $\bar{K} = DK$.

By adding Eqs. (15)–(18) to $\mathbb{E}\{\mathcal{L}V(t)\}$, the new upper bound of $\mathbb{E}\{\mathcal{L}V(t)\}$ can be obtained:

$$\mathbb{E}\big\{\mathcal{L}V(t)\big\} \le \mathbb{E}\left\{\zeta^T(t)\left(\frac{t_{k+1} - t}{h}(\Upsilon_1 + h\Upsilon_2) + \frac{t - t_k}{h}(\Upsilon_1 + h\Upsilon_3)\right)\zeta(t)\right\}, \tag{19}$$

where

$$\Upsilon_3 = YR_4^{-1}Y^T - b_6 R_1 b_6^T.$$

By Schur complement, it is clear that LMIs (9) and (10) and $\mathbb{E}\{\mathcal{L}V(t)\} < 0$ are equivalent. In other words, the designed controller (4) makes the error system asymptotically stable against controller attacks. This completes the proof. □

4 Numerical examples

This section considers CDNs (1) consisting of five Chua's chaotic circuits in which the parameters are given as follows:

$$A = \begin{bmatrix} -am_1 & a & 0 \\ 1 & -1 & 1 \\ 0 & -b & 0 \end{bmatrix}, \qquad B = \begin{bmatrix} -a(m_0 - m_1) & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix},$$

$$C = 0.2 \times \begin{bmatrix} -3 & 1 & 1 & 0 & 1 \\ 1 & -4 & 1 & 1 & 1 \\ 1 & 1 & -3 & 1 & 0 \\ 0 & 1 & 1 & -3 & 1 \\ 1 & 1 & 0 & 1 & -3 \end{bmatrix},$$

$$f_j\big(x_{ij}(t)\big) = \frac{1}{2}\big(\big|x_{ij}(t) + c\big| - \big|x_{ij}(t) - c\big|\big), \quad j = 1, \ldots, n; i = 1, \ldots, N,$$

$$a = 9, \qquad b = 14.28, \qquad c = 1,$$

$$m_0 = -1/7, \qquad m_1 = 2/7, \qquad l_i = 1 \quad (i = 1, \ldots, n).$$

For the simulation, we choose the following parameters and initial conditions: $d(t) = 0.4 + 0.1\sin(t)$, $d = 0.5$, $h = 0.4$, $\alpha = 0.1$, $\gamma_{ij} = 0.5$ ($i = 1, \ldots, N$; $j = 1, \ldots, n$), $g_{ij}(a) = \tanh(0.04a)$ ($i = 1, \ldots, N$; $j = 1, \ldots, n$), $x_1(0) = [-0.1\ -0.5\ -0.7]$, $x_2(0) = [-0.1\ -0.4\ 0.3]$, $x_3(0) = [0.6\ -1.5\ 0]$, $x_4(0) = [0.1\ 0.1\ 0.1]$, $x_5(0) = [0\ 0.5\ -0.4]$, and $s(0) = [0.1\ 0.5\ -0.7]$.

With the above parameters, Theorem 1 calculates the following control gains:

$$K = \mathrm{diag}\{K_1, \ldots, K_5\},$$

where

$$K_1 = \begin{bmatrix} 4.8871 & 0.0018 & -0.0005 \\ -0.0016 & 0.6933 & -0.0005 \\ -0.0004 & 0.0010 & 4.8007 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} 4.8307 & 0.0011 & 0.0001 \\ 0.0011 & 0.6290 & 0.0000 \\ -0.0003 & 0.0002 & 4.7882 \end{bmatrix},$$

$$K_3 = \begin{bmatrix} 4.8360 & -0.0005 & -0.0005 \\ 0.0000 & 0.6312 & 0.0006 \\ -0.0009 & 0.0002 & 4.8005 \end{bmatrix},$$

$$K_4 = \begin{bmatrix} 4.8643 & 0.0026 & -0.0004 \\ 0.0019 & 0.6111 & 0.0007 \\ 0.0001 & -0.0012 & 4.8000 \end{bmatrix},$$

$$K_5 = \begin{bmatrix} 4.8733 & 0.0014 & 0.0005 \\ 0.0008 & 0.6719 & 0.0023 \\ 0.0069 & 0.0030 & 4.8101 \end{bmatrix}.$$

Then the controlled error signals can be obtained in Fig. 1 which shows the error dynamics asymptotically converse to zero; in other words, the synchronization is achieved between $s(t)$ and $x_i(t)$ for $i = 1, \ldots, N$. Figures 2 and 3 show the applied control signals $u_i(t)$ and cyber-attack scenario $\gamma_i(t)$, respectively. Figure 4 displays the attack signals transmitted to the CDNs instead of the real control signals when cyber-attacks are launched to the controller of ith node, i.e., $\gamma_i(t) = 1$. The difference between the real control signals(without cyber-attack) $u_i^F(t)$ and the control signals under attack $u_i(t)$ is depicted in Fig. 5. As seen in Figs. 1 and 5, the controller attacks make big changes from real control

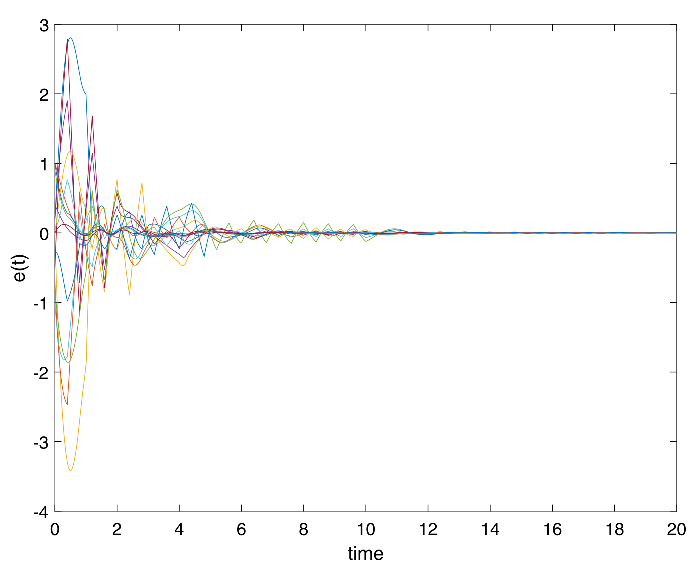

Figure 1 The error trajectories with controller (4) under cyber-attacks *e*(*t*)

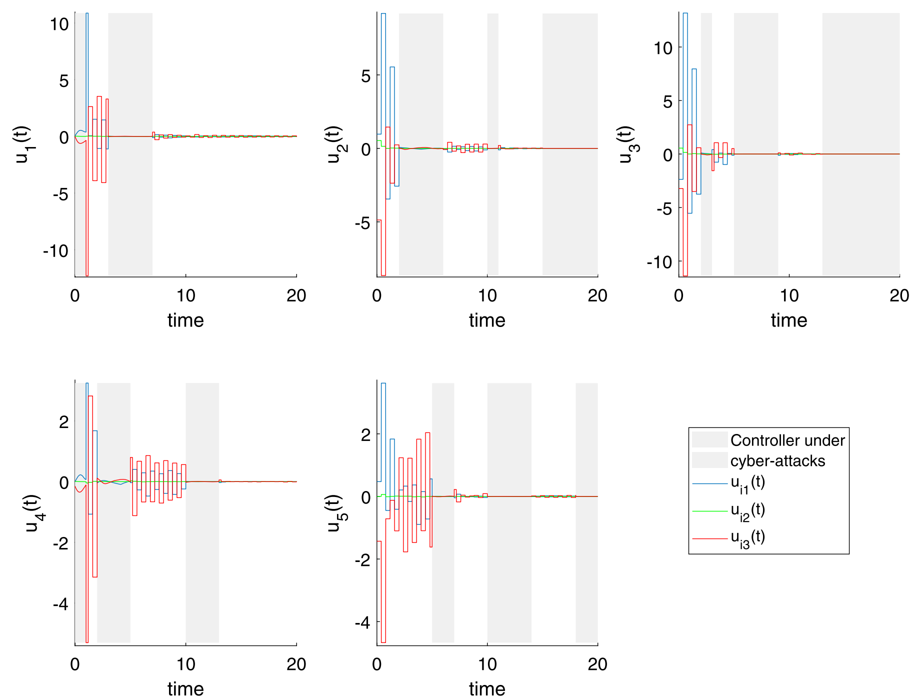

Figure 2 The applied control signals for each node under cyber-attacks $u_i(t)$

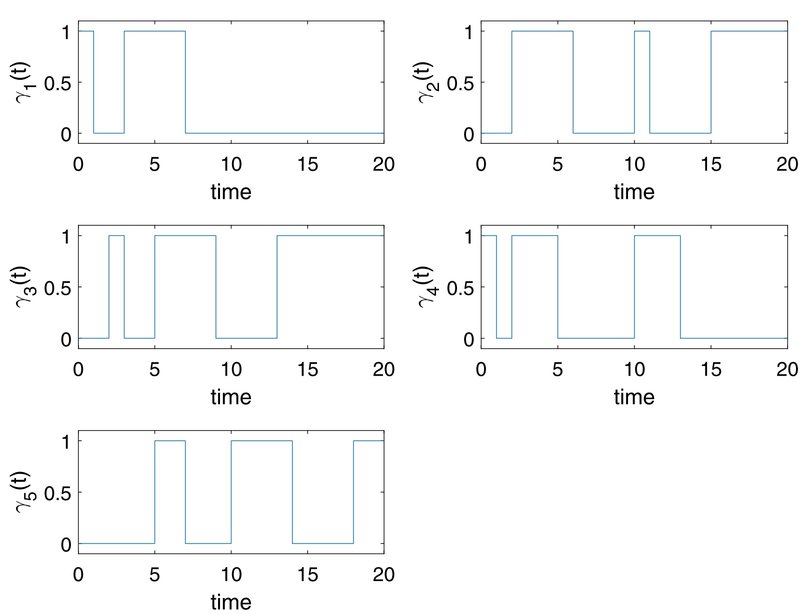

Figure 3 The random cyber-attack scenario for each node $\gamma_i(t)$

signals, but our designed controller works very well to synchronize all the states of CDNs up to the target.

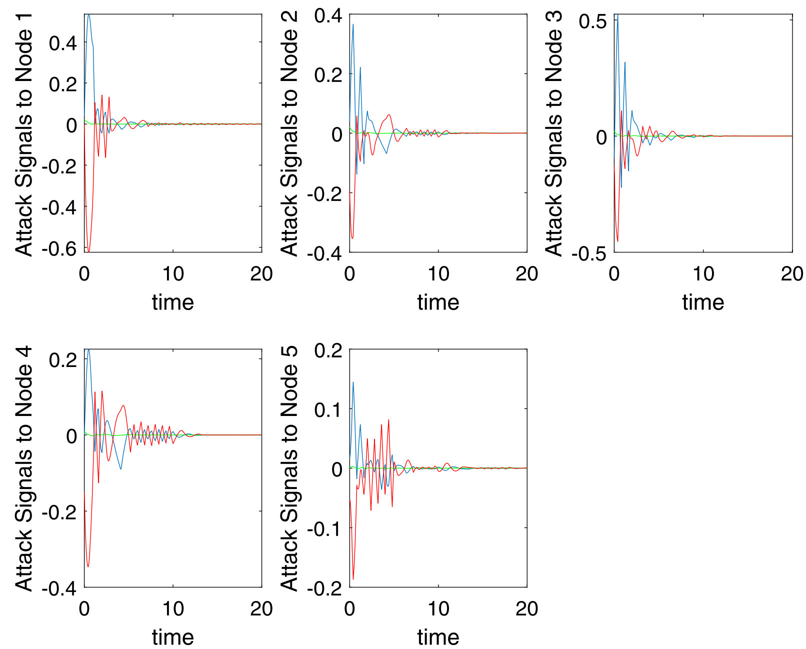

Figure 4 The attack signals to each node $K_i g_i(e_i(t-d(t))$

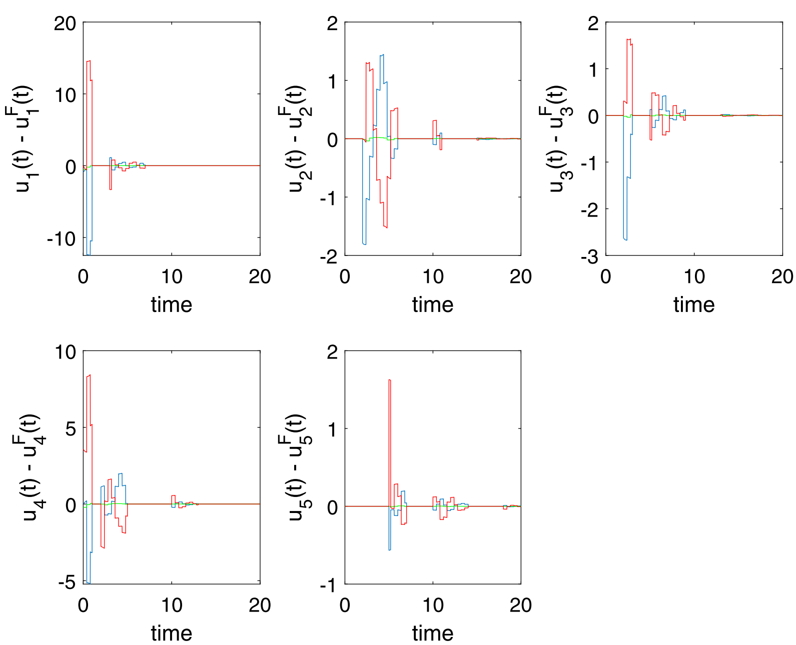

Figure 5 The difference between control signals with/without cyber-attacks, $u_i(t) - u_i^F(t)$

5 Conclusions

In this paper, the design problem of sampled-data controllers for the synchronization of CDNs was investigated. To reflect a real-world situation, we considered the controllers under cyber-attacks which are randomly occurring. The maliciously reformed control data by adversaries was assumed to be a nonlinear function including time-varying delayed control information. Using discontinuous Lyapunov functionals and a zero inequality, the designing conditions of the sampled-data controller were derived in terms of LMI. The validity of the proposed method was proven by a numerical example.

Acknowledgements
The authors would like to thank the anonymous reviewers and the editor for their valuable comments and suggestions that helped improve the manuscript.

Funding
This work was supported by the National Research Foundation of Korea (NRF) grant funded by the Korea government (MSIT) (Grant number NRF-2018R1C1B5036886) and also supported by research funds for newly appointed professors of Chonbuk National University in 2017.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 4 December 2018 Accepted: 1 May 2019 Published online: 14 May 2019

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