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Dynamical behaviors of a predator-prey system with prey impulsive diffusion and dispersal delay between two patches

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Abstract
In this paper, we consider a predator-prey model with prey impulsive diffusion and dispersal delay. By utilizing the dynamical properties of a single-species model with diffusion and dispersal delay between two patches and the comparison principle of impulsive differential equations, we establish the sufficient conditions on the global attractivity of predator-extinction periodic solution and the permanence of species for the model.

MSC: 34A37; 34C55; 34C25; 34D23

Keywords: Predator-prey system; Impulsive diffusion; Dispersal delay; Permanence; Global attractivity

1 Introduction

Ecosystems are characterized by the interaction between different species and natural environment. One of the important types of interaction, which has effect on population dynamics, is predation. Thus, predator-prey models have been the focus of ecological science since the early days of this discipline [1]. Since the great work of Lotka (in 1925) and Volterra (in 1926), modeling predator-prey interaction has been one of the central themes in mathematical ecology [2, 3].

Owing to severe competition, natural enemy, or deterioration of the patch environment, the migration phenomena of biological species can often occur between heterogeneous spatial environments and patches. More recently, increasing attention has been paid to the dynamics of a large number of mathematical models with diffusion, and many nice results have been obtained. The persistence and extinction for ordinary differential equation and delayed differential equation models were investigated in [4–6]. Global stability of periodic solution for the model with diffusion was studied in [7–12]. Particularly, the predator-prey system with the prey dispersal was also studied in [13–17]. Regretfully, in all of the above population dispersing systems, they always assumed that the dispersal occurs at every time. For example, Zhang and Teng investigated the following periodic predator-prey Lotka–Volterra type system with prey dispersal in n patches in [14]:

$$\begin{cases} \dot{x}_1(t) = x_1(t)[a_1(t) - b_1(t)x_1(t) - c(t)y(t)] \\ \qquad + \sum_{j=1}^{n} d_{1j}(t)(x_j(t) - x_1(t)), \\ \dot{x}_i(t) = x_i(t)[a_i(t) - b_i(t)x_i(t)] + \sum_{j=1}^{n} d_{ij}(t)(x_j(t) - x_i(t)), \\ \dot{y}(t) = y(t)[-e(t) + f(t)x_1(t)], \quad i = 2,3,\ldots,n, \end{cases} \tag{1.1}$$

where $e(t)$ denotes the death rate of the predator, $d_{ij}(t)$ $(i,j \in I, i \neq j)$ represents the dispersal rate of the prey species from the ith patch to the jth patch. Sufficient conditions on the boundedness, permanence, and existence of a positive periodic solution for system (1.1) are established.

Actually, many man-made factors (e.g., drought, hunting, harvesting, breeding, fire, etc.) always lead to rapid increase or decrease of population number at some transitory time slots. These short-term perturbations were often assumed to be in the form of impulses. For example, birds often migrate between patches in winter to find suitable environments. Impulsive differential equations [18] have attracted the interest of researchers, and many important studies have been performed [19–23].

It is well known that time delay is quite common for a natural population. Therefore, it is necessary to take the effect of time delay into account in forming a biologically meaningful mathematical model. Recently, many impulsive predator-prey models with dispersion and time delay have been investigated in [24–28]. For example in [24], Li and Zhang proposed and studied the following delayed predator-prey system with impulsive diffusion:

$$\begin{cases} \dot{x}_1(t) = x_1(t)[r_1 - a_1x_1(t)] - \beta x_1(t)y(t), \\ \dot{x}_2(t) = -r_2x_2(t), & t \neq nT, \\ \dot{y}(t) = y(t)[-d_1 + k\beta x_1(t-\tau_1) - a_2y(t-\tau_2)], \\ \Delta x_1(t) = d_{21}x_2(t) - d_{12}x_1(t), \\ \Delta x_2(t) = d_{12}x_1(t) - d_{21}x_2(t)], & t = nT, \\ \Delta y(t) = 0, \end{cases} \tag{1.2}$$

with the initial conditions

$$\begin{aligned} &x_1(s) = \phi_1(s), \qquad x_2(s) = \phi_2(s), \\ &y(s) = \phi_3(s), \qquad \tau = \max\{\tau_1, \tau_2\}, \\ &\phi = \bigl(\phi_1(s), \phi_2(s), \phi_3(s)\bigr)^T \in C\bigl([-\tau, 0], R_+^3\bigr), \qquad \phi_i(0) > 0, \quad i = 1,2,3. \end{aligned}$$

In system (1.2), they assumed that the ecosystem was composed of two isolated patches and the breeding area was damaged in patch 2. By using comparison theorem of impulsive differential equation and some analysis techniques, they got the global attractivity of predator-extinction periodic solution and permanence of the system.

Many single species models with impulsive diffusion and dispersal delay have been investigated, too. In [20], the authors studied a single species model with symmetric bidirectional impulsive diffusion and dispersal delay:

$$\begin{cases} \dot{N}_1(t) = r_1 N_1(t) \ln \frac{k_1}{N_1(t)}, & \\ \dot{N}_2(t) = r_2 N_2(t) \ln \frac{k_2}{N_2(t)}, & t \neq nT, \\ \Delta N_1(t) = d_1[N_2(t-\tau_0) - N_1(t)], & \\ \Delta N_2(t) = d_2[N_1(t-\tau_0) - N_2(t)], & t = nT, \end{cases} \tag{1.3}$$

where r_i $(i = 1, 2)$ stands for the intrinsic growth rate of the population N_i, and d_i represents the dispersal rate in the ith patch. τ_0 is the time delay, that is, a period of time of species N_i disperse between patches ($\tau_0 < T$). Sufficient criteria were obtained for the permanence, existence, uniqueness, and global stability of positive periodic solutions by using discrete dynamical system theory.

Motivated by the above analysis, in this paper, based on system (1.3), we consider a predator-prey model with prey symmetric bidirectional impulsive diffusion and dispersal delay between two patches:

$$\begin{cases} \dot{N}_1(t) = r_1 N_1(t) \ln \frac{k_1}{N_1(t)}, & \\ \dot{N}_2(t) = N_2(t)[r_2 \ln \frac{k_2}{N_2(t)} - c_1 y(t)], & t \neq nT, \\ \dot{y}(t) = y(t)[-r_3 + c_2 N_2(t-\tau_1) - c_3 y(t-\tau_2)], & \\ \Delta N_1(t) = d_1[N_2(t-\tau_0) - N_1(t)], & \\ \Delta N_2(t) = d_2[N_1(t-\tau_0) - N_2(t)], & t = nT, n = 1, 2, \ldots, \\ \Delta y(t) = 0, & \end{cases} \tag{1.4}$$

with the initial conditions

$$\begin{aligned} &N_1(s) = \phi_1(s), \qquad N_2(s) = \phi_2(s), \\ &y(s) = \phi_3(s), \qquad \tau = \max\{\tau_0, \tau_1, \tau_2\}, \\ &\phi = \bigl(\phi_1(s), \phi_2(s), \phi_3(s)\bigr)^T \in C\bigl([-\tau, 0], R_+^3\bigr), \qquad \phi_i(0) > 0, \quad i = 1, 2, 3, \end{aligned}$$

where $N_i(t)$ $(i = 1, 2)$ denotes the density of the prey species in the ith patch at time t; $y(t)$ denotes the density of the predator species at time t. Predator species is confined to the second patch while the prey species can disperse between two patches. τ_0 is a positive constant ($\tau_0 < T$), which represents the time for the species to disperse between patches. $\tau_1 \geq 0$ is a constant delay due to the gestation of the predator. The term $-c_3 y(t-\tau_2)$ is the negative feedback of predator crowding. We will use methods similar to those of [24] to analyze our predator-prey model with prey symmetric bidirectional impulsive diffusion and dispersal delay.

2 Preliminaries

Firstly, for simplicity and convenience, we let $x_1 = \frac{N_1}{k_1}$, $x_2 = \frac{N_2}{k_2}$, $k = \frac{k_2}{k_1}$, then system (1.4) can be written as follows:

$$\begin{cases} \dot{x_1}(t) = r_1 x_1(t) \ln \frac{1}{x_1(t)}, & \\ \dot{x_2}(t) = x_2(t)[r_2 \ln \frac{1}{x_2(t)} - c_1 y(t)], & t \neq nT, \\ \dot{y}(t) = y(t)[-r_3 + k_2 c_2 x_2(t-\tau_1) - c_3 y(t-\tau_2)], & \\ \Delta x_1(t) = d_1[k x_2(t-\tau_0) - x_1(t)], & \\ \Delta x_2(t) = d_2[\frac{1}{k} x_1(t-\tau_0) - x_2(t)], & t = nT, n = 1, 2, \ldots. \\ \Delta y(t) = 0, & \end{cases} \tag{2.1}$$

Next, we discuss the dynamical behaviors of the following single species model:

$$\begin{cases} \dot{v}_1(t) = r_1 v_1(t) \ln \frac{1}{v_1(t)}, & \\ \dot{v}_2(t) = r_2 v_2(t) \ln \frac{1}{v_2(t)}, & t \neq nT, \\ \Delta v_1(t) = d_1[k v_2(t-\tau_0) - v_1(t)], & \\ \Delta v_2(t) = d_2[\frac{1}{k} v_1(t-\tau_0) - v_2(t)], & t = nT. \end{cases} \tag{2.2}$$

We introduce the following assumptions for system (2.2):

(H_1) $0 < d_1 + d_2 < 1$,

(H_2) $b_1 + b_2 + d_1 \leq 1$,

(H_3) $1 - b_i \leq (1 - b_i e^{r_i \tau_0}) e^{(r_1 + r_2)\tau_0}$, $i = 1, 2$,

where $b_i = e^{-r_i T}$.

Lemma 2.1 ([20]) *Suppose that assumptions (H_1)–(H_3) hold, then system (2.2) has a unique globally attractive positive T-periodic solution $(v_1^*(t), v_2^*(t))$, that is,*

$$\lim_{t\to\infty} \big(v_1(t), v_2(t)\big) = \big(v_1^*(t), v_2^*(t)\big).$$

Next, we consider the following system:

$$\begin{cases} \dot{v}_{1\alpha}(t) = r_1 v_{1\alpha}(t) \ln \frac{1}{v_{1\alpha}(t)}, & \\ \dot{v}_{2\alpha}(t) = v_{2\alpha}(t)[r_2 \ln \frac{1}{v_{2\alpha}(t)} - \alpha], & t \neq nT, \\ \Delta v_{1\alpha}(t) = d_1[k v_{2\alpha}(t-\tau_0) - v_{1\alpha}(t)], & \\ \Delta v_{2\alpha}(t) = d_2[\frac{1}{k} v_{1\alpha}(t-\tau_0) - v_{2\alpha}(t)], & t = nT, \end{cases} \tag{2.3}$$

where α is a positive constant.

Let $u_1(t) = v_{1\alpha}(t)$, $u_2(t) = e^{\frac{\alpha}{r_2}} v_{2\alpha}(t)$, then system (2.3) is transformed into the following form:

$$\begin{cases} \dot{u}_1(t) = r_1 u_1(t) \ln \frac{1}{u_1(t)}, & \\ \dot{u}_2(t) = r_2 u_2(t) \ln \frac{1}{u_2(t)}, & t \neq nT, \\ \Delta u_1(t) = d_1[k^* u_2(t-\tau_0) - u_1(t)], & \\ \Delta u_2(t) = d_2[\frac{1}{k^*} u_1(t-\tau_0) - u_2(t)], & t = nT, \end{cases} \tag{2.4}$$

where $k^* = k e^{-\frac{\alpha}{r_2}}$.

Therefore system (2.3) has the following result as system (2.2).

Lemma 2.2 *Suppose that assumptions* (H_1)–(H_3) *hold, then system* (2.3) *has a unique globally attractive positive T-periodic solution* $(v_{1\alpha}^*(t), v_{2\alpha}^*(t))$, *that is,*

$$\lim_{t\to\infty}\bigl(v_{1\alpha}(t), v_{2\alpha}(t)\bigr) = \bigl(v_{1\alpha}^*(t), v_{2\alpha}^*(t)\bigr).$$

Definition 2.1 For any positive solution $(x_1(t), x_2(t), y(t))$ of system (2.1), if there are positive constants m and M such that

$$m \le x_i(t) \le M, \qquad m \le y(t) \le M, \quad i = 1, 2, \text{as } t \to \infty,$$

then system (2.1) is said to be permanent.

Lemma 2.3 ([29]) *Assume that for* $y(t) > 0$, $t \ge 0$, *it holds that*

$$\dot{y}(t) \le y(t)\bigl(a - by(t-\tau)\bigr) \tag{2.5}$$

with initial conditions, $y(s) = \phi(s) \ge 0$ *for* $s \in [-\tau, 0]$, *where* a, b *are positive constants. Then*

$$\limsup_{t\to+\infty} y(t) \le \frac{ae^{a\tau}}{b}. \tag{2.6}$$

3 Main results

Theorem 3.1 *Suppose that assumptions* (H_1)–(H_3) *hold. If*

(H_4) $k_2c_2 \min_{t\in[0,T]} v_2^*(t) > r_3$,

then system (2.1) *is permanent.*

Proof We first prove the ultimate boundedness of all positive solutions of system (2.1). Let $(x_1(t), x_2(t), y(t))$ be any positive solution of system (2.1). Then we obtain

$$\begin{cases} \dot{x}_1(t) = r_1x_1(t)\ln\frac{1}{x_1(t)}, & \\ & t \ne nT, \\ \dot{x}_2(t) \le r_2x_2(t)\ln\frac{1}{x_2(t)}, & \\ \Delta x_1(t) = d_1[kx_2(t-\tau_0) - x_1(t)], & \\ & t = nT, \\ \Delta x_2(t) = d_2[\frac{1}{k}x_1(t-\tau_0) - x_2(t)], & \end{cases} \tag{3.1}$$

for all $t > \tau_0$. Consider the auxiliary system (2.2). From Lemma 2.1 and the comparison theorem of impulsive differential equations, we have that, for any constant $\varepsilon > 0$ small enough, there is $T_0 > 0$ such that

$$x_i(t) \le v_i(t) < v_i^*(t) + \varepsilon \le \max_{t\in[0,T]} v_i^*(t) + \varepsilon \triangleq M_i, \quad i = 1, 2, \tag{3.2}$$

for all $t \ge T_0$. Hence, from the third equation of (2.1) and (3.2), we have

$$\dot{y}(t) \le y(t)\bigl[-r_3 + k_2c_2M_2 - c_3y(t-\tau_2)\bigr], \quad t \ge T_0 + \tau.$$

By Lemma 2.3, we can obtain

$$y(t) \le \frac{-r_3 + k_2 c_2 M_2}{c_3} e^{(-r_3 + c_2 k_2 M_2)\tau_2} \triangleq M_3, \quad t \ge T_0 + \tau, \tag{3.3}$$

where $-r_3 + k_2 c_2 M_2 > 0$ can be easily obtained by (H_4). Take $M = \max\{M_1, M_2, M_3\}$, then $x_i(t) \le M$, $y(t) \le M$, $i = 1, 2$, $t \ge T_0 + \tau$.

The proof of the permanence of species x is simple. In fact, let $(x_1(t), x_2(t), y(t))$ be any positive solution of system (2.1), then from systems (2.1) and (3.3) we obtain

$$\begin{cases} \dot{x}_1(t) = r_1 x_1(t) \ln \frac{1}{x_1(t)}, & \\ \dot{x}_2(t) \ge x_2(t)[r_2 \ln \frac{1}{x_2(t)} - \alpha], & t \ne nT, \\ \Delta x_1(t) = d_1[k x_2(t - \tau_0) - x_1(t)], & \\ \Delta x_2(t) = d_2[\frac{1}{k} x_1(t - \tau_0) - x_2(t)], & t = nT, \end{cases} \tag{3.4}$$

where $\alpha = c_1 M_3$. Consider the auxiliary system (2.3). From Lemma 2.2 and the comparison theorem of impulsive differential equations, we obtain that, for above $\varepsilon > 0$, there exist $T_1 \ge T_0 + \tau$ such that

$$x_i(t) \ge v_{i\alpha}(t) > v_{i\alpha}^*(t) - \varepsilon \ge \min_{t \in [0,T]} v_{i\alpha}^*(t) - \varepsilon \triangleq m_i, \quad i = 1, 2. \tag{3.5}$$

This shows that species x_i $(i = 1, 2)$ are permanent in system (2.1).

Now, in system (2.1) we prove the permanence of species y. From assumption (H_4), we take a constant $\varepsilon_0 > 0$ small enough such that

$$\delta \triangleq k_2 c_2 \Big(\min_{t \in [0,T]} v_2^*(t) - \varepsilon_0 \Big) - c_3 \varepsilon_0 - r_3 > 0. \tag{3.6}$$

For any constant $\alpha > 0$, according to assumptions (H_1)–(H_3), we have that system (2.3) has a unique globally attractive positive T-periodic solution $(v_{1\alpha}^*(t), v_{2\alpha}^*(t))$. Since system (2.3) is periodic, we obtain that $(v_{1\alpha}^*(t), v_{2\alpha}^*(t))$ is globally uniformly attractive. Hence, for above ε_0 and M, there is a constant $T^* = T^*(\varepsilon_0, M) > 0$ such that, for any initial value $(t_0, v_{1\alpha}(t_0), v_{2\alpha}(t_0))$ with $t_0 \ge 0$ and $0 < v_{i\alpha}(t_0) \le M$ $(i = 1, 2)$, we have

$$\big| v_{i\alpha}(t) - v_{i\alpha}^*(t) \big| < \frac{\varepsilon_0}{2} \quad \text{for all } t \ge t_0 + T^*. \tag{3.7}$$

Therefore, we further have

$$v_{i\alpha}(t) > v_{i\alpha}^*(t) - \frac{\varepsilon_0}{2} \quad \text{for all } t \ge t_0 + T^*. \tag{3.8}$$

By the continuity of solutions with respect to parameters, there is $\alpha_0 \in (0, \varepsilon_0)$ such that

$$\big| v_{i\alpha_0}^*(t) - v_i^*(t) \big| < \frac{\varepsilon_0}{2} \quad \text{for all } t \in R. \tag{3.9}$$

We further have

$$v_{i\alpha_0}^*(t) \ge v_i^*(t) - \frac{\varepsilon_0}{2}, \quad t \ge 0. \tag{3.10}$$

Let $\varepsilon_1 = \min\{\frac{\alpha_0}{c_1}, \varepsilon_0\}$. There are three cases as follows for species $y(t)$.

Case 1. For all $t \geq T_2$, there is a constant $T_2 \geq T_1$ such that $y(t) \leq \varepsilon_1$.

Case 2. For all $t \geq T_2$, there is a constant $T_2 \geq T_1$ such that $y(t) \geq \varepsilon_1$.

Case 3. There is an interval sequence $\{[s_k, t_k]\}$ with $T_1 \leq s_1 < t_1 < s_2 < t_2 < \cdots < s_k < t_k < \cdots$ and $\lim_{k\to\infty} s_k = \infty$ such that $y(t) \leq \varepsilon_1$ for all $t \in \bigcup_{k=1}^{\infty}[s_k, t_k]$, $y(t) \geq \varepsilon_1$ for all $t \notin \bigcup_{k=1}^{\infty}(s_k, t_k)$, and $y(s_k) = y(t_k) = \varepsilon_1$.

For Case 1, from system (2.1), we have

$$\begin{cases} \dot{x}_1(t) = r_1 x_1(t) \ln \frac{1}{x_1(t)}, & t \neq nT, \\ \dot{x}_2(t) \geq x_2(t)[r_2 \ln \frac{1}{x_2(t)} - \alpha_0], & \\ \Delta x_1(t) = d_1[k x_2(t-\tau_0) - x_1(t)], & t = nT. \\ \Delta x_2(t) = d_2[\frac{1}{k} x_1(t-\tau_0) - x_2(t)], & \end{cases} \tag{3.11}$$

Consider the auxiliary system (2.3). From Lemma 2.2, (3.8), (3.10), and the comparison theorem of impulsive differential equations, we have that

$$\begin{aligned} x_i(t) &\geq v_{i\alpha_0}(t) > v_{i\alpha_0}^*(t) - \frac{\varepsilon_0}{2} \\ &\geq v_i^*(t) - \varepsilon_0 \geq \min_{t\in[0,T]} v_i^*(t) - \varepsilon_0, \quad i = 1, 2, t \geq T_1 + T^*. \end{aligned} \tag{3.12}$$

Consider the third equation of system (2.1), we further obtain

$$\dot{y}(t) \geq y(t)\Big[-r_3 + k_2 c_2\Big(\min_{t\in[0,T]} v_2^*(t) - \varepsilon_0\Big) - c_3\varepsilon_0\Big], \quad t \geq T_1 + T^* + \tau. \tag{3.13}$$

For any $t = T_2 + n_1 T$, we choose an integer $n_1 \geq 0$, where $T_2 = T_1 + T^* + \tau$, and integrate (3.13) from T_2 to t, then from (3.6) we have

$$\begin{aligned} y(t) &\geq y(T_2) \exp\Big\{\Big[-r_3 + k_2 c_2\Big(\min_{t\in[0,T]} v_2^*(t) - \varepsilon_0\Big) - c_3\varepsilon_0\Big](t - T_2)\Big\} \\ &= y(T_2) e^{n_1 T\delta}. \end{aligned} \tag{3.14}$$

We have $y(t) \to \infty$ as $n_1 \to \infty$, which leads to a contradiction.

We now consider Case 3. For any $t \geq T_1$, when $t \in \bigcup_{k=1}^{\infty}[s_k, t_k]$, then $t \in [s_k, t_k]$ for some k. Assume $t_k - s_k \leq T^*$. Since for any $t \in [s_k, t_k]$

$$\dot{y}(t) \geq y(t)(-r_3 - c_3\varepsilon_0), \tag{3.15}$$

then we obtain

$$\begin{aligned} y(t) &\geq y(s_k) \exp\{-(r_3 + c_3\varepsilon_0)T^*\} \\ &= \varepsilon_1 \exp\{-(r_3 + c_3\varepsilon_0)T^*\} \\ &\triangleq m^*. \end{aligned} \tag{3.16}$$

Assume $t_k - s_k \geq T^*$. For any $t \in [s_k, t_k]$, if $t \leq s_k + T^*$, then according to the above discussion on the case of $t_k - s_k \leq T^*$, we obtain inequality (3.16). Particularly, we have

$y(s_k + T^*) \geq m^*$. Since $y(t) \leq \varepsilon_1$ for all $t \in [s_k, t_k]$, then according to the discussion on Case 1, we have inequality (3.13). For any $t \in [s_k + T^*, t_k]$, we choose an integer $n_2 \geq 0$ such that $t \in [s_k + T^* + n_2 T, s_k + T^* + (n_2 + 1)T)$. Then integrating (3.13) from $s_k + T^*$ to t, we obtain

$$\begin{aligned} y(t) &\geq y(s_k + T^*) \exp\left\{\int_{s_k+T^*}^{t} \left[-r_3 + k_2 c_2\left(\min_{t\in[0,T]} v_2^*(t) - \varepsilon_0\right) - c_3\varepsilon_0\right] dt\right\} \\ &\geq m^* \exp\left\{\int_{s_k+T^*}^{s_k+T^*+n_2T} \left[-r_3 + k_2 c_2\left(\min_{t\in[0,T]} v_2^*(t) - \varepsilon_0\right) - c_3\varepsilon_0\right] dt \right. \\ &\quad \left. + \int_{s_k+T^*+n_2T}^{t} \left[-r_3 + k_2 c_2\left(\min_{t\in[0,T]} v_2^*(t) - \varepsilon_0\right) - c_3\varepsilon_0\right] dt\right\} \\ &\geq m^* \exp\left\{\int_{s_k+T^*+n_2T}^{t} \left[-r_3 + k_2 c_2\left(\min_{t\in[0,T]} v_2^*(t) - \varepsilon_0\right) - c_3\varepsilon_0\right] dt\right\} \\ &\geq m^* \exp\{-(r_3 + c_3\varepsilon_0)T\} \\ &= \varepsilon_1 \exp\{-(r_3 + c_3\varepsilon_0)(T + T^*)\} \\ &\triangleq m_3. \end{aligned} \tag{3.17}$$

From the above discussion, we obtain

$$y(t) \geq m_3 \quad \text{for all } t \in \bigcup_{k=1}^{\infty} [s_k, t_k]. \tag{3.18}$$

For any $t \notin \bigcup_{k=1}^{\infty} (s_k, t_k)$, we obviously have

$$y(t) \geq \varepsilon_1 > m_3 \quad \text{for all } t \geq T_1. \tag{3.19}$$

Hence, for Case 3 we finally have

$$y(t) \geq m_3 \quad \text{for all } t \geq T_1. \tag{3.20}$$

Lastly, we consider Case 2. Since $y(t) \geq \varepsilon_1$ for any $t \geq T_2$, we obtain

$$y(t) \geq m_3 \quad \text{for all } t \geq T_2. \tag{3.21}$$

Therefore, we finally have

$$y(t) \geq m_3 \quad \text{for all } t \geq T_2. \tag{3.22}$$

Take $m = \min\{m_1, m_2, m_3\}$, then $x_i(t) \geq m$ $(i = 1, 2)$, $y(t) \geq m$ hold as $t \to +\infty$. This completes the proof. □

For system (2.1), if we let $y(t) \equiv 0$, then system (2.1) degenerates into system (2.2). From Lemma 2.1 we know that system (2.2) has a unique globally attractive positive T-periodic solution $(v_1^*(t), v_2^*(t))$. Therefore, system (2.1) has a nonnegative T-periodic solution $(v_1^*(t), v_2^*(t), 0)$.

Next, we present conditions to ensure the global attractivity of a nonnegative T-periodic solution $(v_1^*(t), v_2^*(t), 0)$ of system (2.1).

Theorem 3.2 *Suppose that assumptions* (H_1)–(H_3) *hold. If*

(H_5) $k_2c_2\max_{t\in[0,T]} v_2^*(t) \le r_3$,

then system (2.1) *admits a predator-extinction periodic solution, which is globally attractive.*

Proof From Theorem 3.1, for any $\varepsilon > 0$ small enough, we have

$$x_i(t) \le v_i(t) < v_i^*(t) + \varepsilon \le \max_{t\in[0,T]} v_i^*(t) + \varepsilon, \quad i = 1, 2, t \ge T_0. \tag{3.23}$$

According to assumption (H_5), for any $\eta_1 > 0$, there is $\eta_0 \in (\varepsilon, \eta_1)$ such that

$$\sigma \triangleq k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3\eta_1 - r_3 < 0. \tag{3.24}$$

From the third equation of system (2.1) and (3.23), we have

$$\dot{y}(t) \le y(t)\Big[-r_3 + k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3y(t-\tau_2)\Big], \quad t \ge T_0 + \tau. \tag{3.25}$$

Assume $y(t) \ge \eta_1$ for all $t > T_0$. From (3.25) we obtain

$$\dot{y}(t) \le y(t)\Big[-r_3 + k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3\eta_1\Big], \quad t \ge T_0 + \tau. \tag{3.26}$$

For any $t \ge T_0 + \tau$, we choose an integer $n_3 \ge 0$ such that $t \in [n_3T + T_0 + \tau, (n_3 + 1)T + T_0 + \tau)$. Then integrating (3.26) from $T_0 + \tau$ to t, we have

$$\begin{aligned} y(t) &\le y(T_0+\tau)\exp\left\{\int_{T_0+\tau}^{t}\Big[-r_3 + k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3\eta_1)\Big]\,dt\right\} \\ &\le y(T_0+\tau)\exp\left\{\int_{T_0+\tau}^{n_3T+T_0+\tau}\Big[-r_3 + k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3\eta_1)\Big]\,dt\right. \\ &\quad \left. + \int_{n_3T+T_0+\tau}^{(n_3+1)T+T_0+\tau}\Big[-r_3 + k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3\eta_1)\Big]\,dt\right\} \\ &\le y(T_0+\tau)\exp\{n_3T\sigma + \lambda T\}, \end{aligned} \tag{3.27}$$

where $\lambda = k_2c_2(\max_{t\in[0,T]} v_2^*(t) + \eta_0)$. Since $n_3 \to \infty$ and $\sigma < 0$, then $y(t) \to 0$ as $t \to \infty$. This leads to a contradiction. Hence, there is $t_1 > T_0$ such that $y(t) \le \eta_1$. Since $y(t)$ is continuous for all $t \ge 0$, if further exists $t_3 > t_1$ such that $y(t_3) > \eta_1 e^{\lambda T}$, then there is $t_2 \in (t_1, t_3)$ such that $y(t_2) = \eta_1$ and $y(t) > \eta_1$ for any $t \in (t_2, t_3]$. When $t \in [t_2, t_3]$, we can easy find that inequality (3.26) holds. Further, we choose an integer $n_4 \ge 0$ such that $t_3 \in [t_2 + n_4T, t_2 + (n_4 + 1)T)$. Integrating (3.26) from t_2 to t_3, we obtain

$$\begin{aligned} y(t) &\le y(t_2)\exp\left\{\int_{t_2}^{t_3}\Big[-r_3 + k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3\eta_1)\Big]\,dt\right\} \\ &= y(t_2)\exp\left\{\int_{t_2}^{t_2+n_4T}\Big[-r_3 + k_2c_2\Big(\max_{t\in[0,T]} v_2^*(t) + \eta_0\Big) - c_3\eta_1)\Big]\,dt\right. \end{aligned}$$

$$+\int_{t_2+n_4T}^{t_3}\Big[-r_3+k_2c_2\Big(\max_{t\in[0,T]}v_2^*(t)+\eta_0\Big)-c_3\eta_1\Big)\Big]\,dt\Big\}$$
$$\le \eta_1 e^{\lambda T}, \tag{3.28}$$

which is a contradiction. So, we finally have

$$y(t)\le \eta_1 e^{\lambda T}\quad \text{for any } t>T_0. \tag{3.29}$$

Since η_1 is arbitrary and λ is a constant, from (3.29) we have

$$\lim_{t\to\infty} y(t)=0. \tag{3.30}$$

Therefore, for any $\varepsilon_2\ge 0$ small enough, there is $T_3>T_0$ such that $0<y(t)<\varepsilon_2$, $t>T_3$.

For the second equation of system (2.1), we have

$$\begin{cases}\dot{x}_1(t)=r_1x_1(t)\ln\frac{1}{x_1(t)}, & \\ \dot{x}_2(t)\ge x_2(t)[r_2\ln\frac{1}{x_2(t)}-\alpha_1], & t\ne nT,\\ \Delta x_1(t)=d_1[kx_2(t-\tau_0)-x_1(t)], & \\ \Delta x_2(t)=d_2[\frac{1}{k}x_1(t-\tau_0)-x_2(t)], & t=nT,\end{cases} \tag{3.31}$$

where $\alpha_1=c_1\varepsilon_2$. Consider the auxiliary system (2.3). From Lemma 2.2 and the comparison theorem of impulsive differential equations, we obtain that, for above ε, there is $T_4>0$ such that

$$x_i(t)\ge v_{i\alpha_1}(t)>v_{i\alpha_1}^*(t)-\frac{\varepsilon_0}{2}\ge v_i^*(t)-\varepsilon_0,\quad i=1,2,t\ge T_4. \tag{3.32}$$

Combining (3.23), (3.30), and (3.32), we have

$$x_i(t)\to v_i^*(t),\quad y(t)\to 0,\quad i=1,2,t\to\infty. \tag{3.33}$$

That is, system (2.1) admits a predator-extinction periodic solution, which is globally attractive. The proof of Theorem 3.2 is completed. □

Remark 3.1 In this paper, we have proposed a predator-prey model with prey impulsive diffusion and dispersal delay. By using the comparison theorem of impulsive differential equation and other analysis methods, we have established a set of easily verifiable sufficient conditions on the global attractivity of the predator-extinction periodic solution and the permanence of species. The highlight of this paper is that we considered the prey with impulsive diffusion and dispersal delay. However, we only discussed the case of the predator-prey model with prey impulsive diffusion in two patches. For this model with prey impulsive diffusion in multiple patches, the results that can be obtained are still important and interesting open problems.

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