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Exponential stabilization for nonlinear switched stochastic systems with interval time-varying delay under asynchronous switching

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Abstract

The paper investigates mean-square exponential stabilization for a class of nonlinear switched stochastic systems with interval time-varying delay under asynchronous switching. Specifically, the delay occurs not only in the state equation, but also in the switching signal from the controller, which brings the difficulty of controller design to achieve mean-square exponential stabilization. Based on the Lyapunov stability theory, a new piecewise multi-Lyapunov–Krasovskii functional dependent on the size of time delay is constructed. By utilizing the matrix inequality technique and the average dwell time approach, delay-dependent sufficient conditions are given to guarantee mean-square exponential stabilization for nonlinear switched stochastic systems under asynchronous switching. In accordance with the method, we also design state feedback controllers of the switched stochastic systems under asynchronous switching through special operations of matrices and Schur complement. Finally, a numerical example and a practical example of river pollution control are provided to show the effectiveness of the approach proposed in this paper.

MSC: 93D20; 93E10; 93C10; 34D20

Keywords: Switched stochastic system; Asynchronous switching; Interval time-varying delay; Average dwell time; Lyapunov Krasovskii functional

1 Introduction

During the last two decades, hybrid systems have become increasingly important in contemporary society both in science and technology due mainly to the fact that hybrid systems have been extensively applied in many fields such as pattern recognition [1], network control [2], power systems [3], automotive systems [4], communication systems [5], neural networks [6], and so on. A switched system is one of the special dynamic hybrid systems that comprise a collection of subsystems equipped with a switching law orchestrating among these systems. Many of switched system models appear in the fields of industrial manufacturing, artificial intelligence, biochemical systems, actuator failures [7], and population dynamics [8–10].

The analysis and synthesis are important issues in the study of switched systems, and they have attracted extensive attention from domestic and foreign scientific research. So far, much progress has been made and many remarkable achievements for various types of switched systems have been studied. For example, behavior analysis [11, 12], property characterization [13], fault detection [14], and control synthesis [15–17]. These results show that stability is a crucial and fundamental problem for switched system with time delay. Essentially, time delay, perturbation, and stochastic term are common phenomena often encountered in real dynamical systems [18–20]. Moreover, the effect of time delays generally exists in system states and will be a source of control system instability, oscillation, and performance deterioration. As a result, the study of delay and stochastic term plays an important role in the stability analysis of switched system. With respect to those problems, we just mention here some representative work. In [21], a new method of uncertain matrix was proposed. Based on this approach, an exponential stabilization condition of nonlinear uncertain systems with time-varying delay was firstly established. By following this idea, [22] also studied the delay-dependent stability analysis and relevant control problems for nonlinear switched with interval time-varying delay based on Lyapunov–Krasovskii functional method. Robust guaranteed cost control for a class of uncertain neutral system with time-varying delays was investigated in [23], delay-dependent and delay-independent criteria were proposed for the stabilization of considered systems, state feedback control was considered to stabilize the uncertain neutral system, and upper bounds on the closed-loop cost function were also given. [24] obtained sufficient conditions with delay-dependent guaranteeing the exponential stability by a common Lyapunov functional (CLF). Recently analogous results have been found in [25], and [25] constructed a suitable Lyapunov–Krasovskii functional containing some novel triple integral terms with sufficient information about the actual sampling pattern. Based on the above discussion, the theory of time-delay systems can be divided into two classes: delay-independent control and delay-dependent control. To the best of our knowledge, delay-dependent stabilization condition gives less conservative result than the delay-independent one as it makes full use of information of the system. Specifically, systems with delay are of significant interest not only for their applicability in practice but also for their interesting theoretical properties. This is motivated by the need for systematic approach to investigate switched systems with delay.

On the other hand, the switching between the controller and the subsystem of switched systems is synchronous in the ideal case. In fact, the asynchronous phenomenon often occurs in practical industrial systems. For instance, when the system and the controller communicate via a communication channel and the current subsystem is switched to the next one, it is necessary to take some time to identify the active subsystem and then switch the controller from the current one to the corresponding subsystem, further causing asynchronous switching. With the great development of switched systems, the asynchronous control problem for switched systems, which is quite practical and energy efficient, has received increasing attention. In the past few years, it is noted that some valid results have appeared in studying nonlinear switched systems under asynchronous switching [26–29]. Specifically, [30] investigated the problem of output tracking control for switched systems with time-varying delay under asynchronous switching. Moreover, based on the dwell time approach, some sufficient conditions of exponential stabilization for a given switched system and a tracking error system were proposed in terms of linear matrix inequalities

(LMIs). Due to the switching instants of the controllers lagging behind those of the subsystems, [31] dealt with the problem of stabilization for a class of switched delay systems with polytopic type uncertainties under asynchronous switching, and the running time was divided into two parts: matched periods $[t_k + \tau_d, t_{k+1}), k = 1, 2, \ldots$ and mismatched periods $[t_k, t_k + \tau_d), k = 1, 2, \ldots$. In addition, by constructing the parameter-dependent Lyapunov–Krasovskii functional and the average dwell time approach, the exponential stabilization problem for a class of nonlinear switched systems with mixed delays under asynchronous switching was investigated in [32]. In these papers mentioned above, the time delay is simple and no stochastic items are considered. Searching for delay-dependent mean-square stability criteria for nonlinear switched stochastic systems with interval time-varying delays is obviously more preferable and challenging.

Furthermore, it is well known that few results have been devoted to the stability of nonlinear switched stochastic systems with interval time-varying delay under asynchronous switching based on the average dwell time approach. This paper considers interval time-varying delay. It is natural to look for an alternative view to derive a less conservative condition for exponential stabilization of nonlinear switched stochastic systems under asynchronous switching. Moreover, we can hardly use the existing methods to investigate a stochastic switched system due to the impact of stochastic factor. This has motivated our present study on the following questions.

- Is it possible to find a delay-dependent multiple Lyapunov–Krasovskii functional that studies the matched periods and the mismatched periods of the nonlinear switched stochastic systems, respectively?
- Based on the average dwell time approach and Jense's inequality, can we obtain a less conservative sufficient condition of mean-square exponential stabilization for nonlinear switched stochastic systems with interval time-varying delay under asynchronous switching?
- Can we design a mean-square exponentially stable feedback controller for switched nonlinear systems under asynchronous switching by the matrix deformation technique and Schur compensation?

The core of this paper is the further development of switched stochastic nonlinear systems with interval time-varying delay under asynchronous switching. Moreover, we have proposed a detailed study and solutions on the above issues. Compared with the existing results on switched systems, the main contributions of this paper can be summarized as follows. (i) We consider the actual situation. In fact, the system needs to take some time to identify the active subsystem, and then switch the controller from the current subsystem to the corresponding subsystem, further causing asynchronous switching. (ii) According to Lyapunov stability theory, the Lyapunov–Krasovskii functional constructed in this paper is time-delay-dependent and depends on the switching signal of the controller. At the same time, it is allowed to increase the running time of the active subsystem with mismatch controller. The established Lyapunov–Krasovskii functional facilitates the analysis of the proposed problem. (iii) By utilizing the matrix inequality technique and the average dwell time approach, delay-dependent sufficient conditions are given to guarantee mean-square exponential stabilization for nonlinear switched stochastic systems under asynchronous switching. Moreover, state feedback controllers and switching signal of the switched stochastic systems are designed simultaneously under asynchronous switching through special operations of matrices and Schur complement without resorting to additional constraints on the switching signal.

The remainder of this paper is organized as follows. In Sect. 2, the problem description and preliminaries and some necessary lemmas are presented. Section 3 is devoted to deriving the results on exponential stabilization for switching signals by the average dwell time approach and delay-dependent multi-Lyapunov–Krasovskii functional. Moreover, feedback controller of nonlinear switched stochastic systems with interval time-varying delay under asynchronous switching is designed, which is the main result of this paper. In Sect. 4, an example is given to illustrate the results. The paper is concluded in Sect. 5.

The notations used in this paper are fairly standard. R^n denotes the n-dimensional Euclidean space. A^T denotes the transpose of A. The symbol $*$ is used to denote the corresponding transposed block matrix. Diag $\{\cdots\}$ is a block-diagonal matrix. I represents the identity matrix in the block matrix, and 0 represents a zero matrix with appropriate dimensions. The notation $P > 0$ indicates that P is a real symmetric and positive definite matrix, and $\lambda_{\min}$ ($\lambda_{\max}$) is the minimum (maximum) eigenvalue of P.

2 Preliminaries

Consider the following switched stochastic nonlinear systems with interval time-varying delay:

$$\begin{aligned} dx(t) &= \bigl[A_{1\sigma(t)}x(t) + A_{2\sigma(t)}x\bigl(t-h(t)\bigr) + B_{\sigma(t)}u(t) \\ &\quad + C_{\sigma(t)}f_{\sigma(t)}\bigl(t,x(t),x\bigl(t-h(t)\bigr)\bigr)\bigr]\,dt + D_{\sigma(t)}x(t)\,d\omega(t), \\ x(s) &= \phi(s), \quad s \in [-h_M, 0], \end{aligned} \tag{1}$$

where $x(t) \in R^n$ and $u(t) \in R^m$ are, respectively, the state vector and the control input of switched systems. $\phi(s) \in R^n$ is the initial condition and $f_{\sigma(t)}(\cdot)$ are nonlinear functions. $\sigma(t) : [0,\infty] \to M = \{1,2,\ldots,n\}$ is the switching signal. Specifically, denote Σ : $\{(t_0,\sigma(t_0)),\ldots,(t_k,\sigma(t_k)),\ldots,k=0,1,2,\ldots\}$, where $0 = t_0 < t_1 < t_2 < \cdots < t_k < \cdots$, in which t_0 is the initial switching instant, t_k denotes the kth switching instant. For $\sigma(t_k) = i$, $A_{1i}, A_{2i}, B_i, C_i, D_i$ are constant matrices with appropriate dimensions. In this article, we assume that the delay function $h(t)$ is interval time-varying and satisfies

$$h_m \le d(t) \le h_M, \qquad \dot{h}(t) \le h < 1. \tag{2}$$

$\omega(t)$ is a one-dimensional Brownian motion on a probability space $(\Omega,\mathcal{F},\mathcal{P})$ and it satisfies the following cases:

$$E\bigl\{d\omega(t)\bigr\} = 0, \qquad E\bigl\{d\omega^2(t)\bigr\} = dt. \tag{3}$$

$f_i(t,x(t),x(t-h(t)))$ are nonlinear perturbation functions, which satisfy the following condition:

$$\begin{aligned} &f_i^T\bigl(t,x(t),x\bigl(t-h(t)\bigr)\bigr)f_i\bigl(t,x(t),x\bigl(t-h(t)\bigr)\bigr) \\ &\quad \le x^T(t)V_i^TV_ix(t) + x^T\bigl(t-h(t)\bigr)\Lambda_i^T\Lambda_ix\bigl(t-h(t)\bigr), \end{aligned} \tag{4}$$

where V_i and Λ_i are known constant matrices. Note that the assumption on the nonlinear perturbations is widely applicable in practice and considered by many researchers. When the controller synchronizes with the switching subsystem, the state feedback controller is often designed as

$$u(t) = K_{\sigma(t)}x(t), \tag{5}$$

where $K_i, i \in M$, denotes the feedback gain matrix.

In practice, since it inevitably takes some time to identify the system modes and apply the matched controllers, the switching instants of the controllers lag behind those of the subsystems. At this point, we consider the state feedback given by

$$u(t) = K_{\sigma(t-\tau_d)}x(t), \tag{6}$$

where τ_d is a known constant.

Remark 1 In this paper, τ_d represents the period that the switching instants of the controller lag behind those of the system. Specifically, the running time of switched system is divided into two parts: matched periods $[t_k + \tau_d, t_{k+1}), k = 1, 2, \dots$, and mismatched periods $[t_k, t_k + \tau_d), k = 1, 2, \dots$. Correspondingly, we suppose that the jth subsystem is activated at the switching instant t_{k-1}, and the ith subsystem is activated at the switching instant t_k, then the corresponding switching controllers are activated at the switching $t_{k-1} + \tau_d$ and $t_k + \tau_d$, respectively.

The closed loop system of system (1) in the interval $[t_k, t_{k+1})$ can be represented as:

$$\begin{aligned} dx(t) &= \bigl[\bar{A}_{1ij}x(t)x(t) + A_{2i}x\bigl(t-h(t)\bigr) + C_i f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr)\bigr]\,dt \\ &\quad + D_i x(t)\,d\omega(t), \quad \forall t \in [t_k, t_k + \tau_d);\ \text{mismatched periods} \\ dx(t) &= \bigl[\bar{A}_{1i}x(t)x(t) + A_{2i}x\bigl(t-h(t)\bigr) + C_i f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr)\bigr]\,dt \\ &\quad + D_i x(t)\,d\omega(t), \quad \forall t \in [t_k + \tau_d, t_{k+1});\ \text{matched periods,} \end{aligned} \tag{7}$$

where $\bar{A}_{1ij} = A_{1i} + B_iK_j, \bar{A}_{1i} = A_{1i} + B_iK_i$.

Definition 1 ([28]) The equilibrium $x^* = 0$ of the closed-loop system (7) is said to be mean-square exponentially stable under switching signal $\sigma(t)$ if the solution $x(t)$ of system satisfies

$$E\bigl\{\bigl\|x(t)\bigr\|^2\bigr\} \le k \sup_{-h_M \le \theta \le 0} E\bigl\{\bigl\|x(t_0+\theta)\bigr\|^2\bigr\}e^{-\alpha(t-t_0)}, \quad \forall t \ge t_0 \tag{8}$$

for constants $k \ge 1, \alpha > 0$.

Definition 2 ([33]) For any $T_2 > T_1 \ge 0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$ on an interval (T_1, T_2). If

$$N_\sigma(T_1, T_2) \ge N_0 + (T_2 - T_1)/\tau_\alpha \tag{9}$$

holds for given $N_0 \geq 0, \tau_\alpha \geq 0$, then the constant τ_α is called the average dwell time and N_0 is the chatter bound. Without loss of generality, we choose $N_0 = 0$ in this paper.

Lemma 1 (Schur complement) *For a given matrix* $\left(\begin{smallmatrix} S_{11} & S_{12} \\ * & S_{22} \end{smallmatrix}\right)$ *with* $S_{11} = S_{11}^T, S_{22} = S_{22}^T$, *then the following conditions are equivalent*:

(1) $S < 0$,

(2) $S_{11} < 0, S_{22} - S_{12}^T S_{11}^{-1} S_{12} < 0$,

(3) $S_{22} < 0, S_{11} - S_{12} S_{22}^{-1} S_{12}^T < 0$.

Lemma 2 (Jensen's inequality) *For any symmetric and positive definite constant matrix* $G \in R^{l \times l}$, *scalars* α *and* β: $\beta < \alpha$, *vector function* $x : [\beta, \alpha] \to R^l$ *such that the integration concerned are well defined, then*

$$-\int_\beta^\alpha x^T(s) G x(s)\, ds \leq -\frac{1}{\alpha - \beta} \left(\int_\beta^\alpha x(s)\, ds \right)^T G \left(\int_\beta^\alpha x(s)\, ds \right).$$

3 Main results

In this section, based on the Lyapunov stability theory, a new piecewise multi-Lyapunov–Krasovskii functional dependent on the size of time delay is constructed. Moreover, we give sufficient conditions for the mean-square exponential stabilization of system (7) by the average dwell time approach and Jensen's inequality. In addition, the state feedback controllers of nonlinear switched systems are designed under asynchronous switching.

Theorem 1 *For given positive constants* α, β, h, *and* $\mu \geq 1$, *if there exist symmetric and positive definite matrices* $P_i, Q_{1i}, Q_{2i}, Q_{3i}, R_{1i}, R_{2i}$ *such that the following matrix inequalities hold*:

$$\begin{aligned} &P_i \leq \mu P_j, \qquad Q_{1i} \leq \mu Q_{1j}, \qquad Q_{2i} \leq \mu Q_{2j}, \\ &Q_{3i} \leq \mu Q_{3j}, \qquad R_{1i} \leq \mu R_{1j}, \qquad R_{2i} \leq \mu R_{2j}, \quad i, j \in M, i \neq j, \end{aligned} \tag{10}$$

$$\Xi_i = \begin{pmatrix} \phi_{11}^i & P_i A_{2i} & 0 & 0 & P_i C_i & 0 & 0 \\ * & \phi_{22}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & \phi_{33}^i & 0 & 0 & 0 & 0 \\ * & * & * & \phi_{44}^i & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & \phi_{66}^i & 0 \\ * & * & * & * & * & * & \phi_{77}^i \end{pmatrix} < 0, \tag{11}$$

$$\Omega_i = \begin{pmatrix} \varphi_{11}^i & P_i A_{2i} & 0 & 0 & P_i C_i & 0 & 0 \\ * & \varphi_{22}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & -Q_{2i} & 0 & 0 & 0 & 0 \\ * & * & * & -Q_{3i} & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 \\ * & * & * & * & * & -R_{1i} & 0 \\ * & * & * & * & * & * & -R_{2i} \end{pmatrix} < 0, \tag{12}$$

where

$$\phi_{11}^i = \bar{A}_{1i}^T P_i + P_i \bar{A}_{1i} + Q_{1i} + Q_{2i} + Q_{3i} + h_m^2 R_{1i} + h_M^2 R_{2i} + \alpha P_i + V_i^T V_i,$$

$$\phi_{22}^i = \Lambda_i^T \Lambda_i - (1-h)e^{-\alpha h_M} Q_{1i}, \qquad \phi_{33}^i = -e^{-\alpha h_m} Q_{2i},$$

$$\phi_{44}^i = -e^{-\alpha h_M} Q_{3i}, \qquad \phi_{66}^i = -e^{-\alpha h_m} R_{1i}, \qquad \phi_{77}^i = -e^{-\alpha h_M} R_{2i}.$$

$$\varphi_{11}^i = \bar{A}_{1ij}^T P_i + P_i \bar{A}_{1ij} + Q_{1i} + Q_{2i} + Q_{3i} + h_m^2 R_{1i} + h_M^2 R_{2i} - \beta P_i + V_i^T V_i,$$

$$\varphi_{22}^i = \Lambda_i^T \Lambda_i - (1-h) Q_{1i}.$$

If the average dwell time of the switching signal $\sigma(t)$ *satisfies*

$$\tau_a > \tau_a^* = \frac{\ln \mu + (\alpha + \beta)\tau_d}{\alpha}, \tag{13}$$

then the closed-loop system (7) *is mean-square exponentially stabilizable under arbitrary switching signal for the feedback control* (6).

Proof When $t \in [t_k + \tau_d, t_{k+1})$, $\sigma(t_k) = i \in M$, switched systems run in matched periods. The closed-loop system (7) is active within the ith subsystem, and the corresponding ith switching controller is also activated. We choose the Lyapunov–Krasovskii functional candidate as follows:

$$\begin{aligned} V_{1\sigma(t)}(t) &= x^T(t) P_{\sigma(t)} x(t) + \int_{t-h(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_{1\sigma(t)} x(s)\, ds \\ &\quad + \int_{t-h_m}^{t} e^{\alpha(s-t)} x^T(s) Q_{2\sigma(t)} x(s)\, ds \\ &\quad + \int_{t-h_M}^{t} e^{\alpha(s-t)} x^T(s) Q_{3\sigma(t)} x(s)\, ds \\ &\quad + h_m \int_{-h_m}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{1\sigma(t)} x(s)\, ds\, d\theta \\ &\quad + h_M \int_{-h_M}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{2\sigma(t)} x(s)\, ds\, d\theta. \end{aligned} \tag{14}$$

According to Itô's differential formula, the stochastic differential is

$$dV_{1i}(t) = \mathcal{L} V_{1i}\, dt + 2x^T(t) P_i D_i x(t)\, d\omega(t) \tag{15}$$

with the infinitesimal operator

$$\begin{aligned} \mathcal{L} V_{1i} &= 2x^T(t) P_i \bigl[\bar{A}_{1i} x(t) x(t) + A_{2i} x\bigl(t-h(t)\bigr) + C_i f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr)\bigr] \\ &\quad + x^T(t) Q_{1i} x(t) - \bigl(1 - \dot{h}(t)\bigr) e^{-\alpha h(t)} x^T\bigl(t-h(t)\bigr) Q_{1i} x\bigl(t-h(t)\bigr) \\ &\quad + x^T(t) D_i^T P_i D_i x(t) + x^T(t) Q_{2i} x(t) - e^{-\alpha h_m} x^T(t-h_m) Q_{2i} x(t-h_m) \\ &\quad + x^T(t) Q_{3i} x(t) - e^{-\alpha h_M} x^T(t-h_M) Q_{3i} x(t-h_M) + h_m^2 x^T(t) R_{1i} x(t) \\ &\quad - h_m \int_{-h_m}^{0} e^{\alpha\theta} x^T(t+\theta) R_{1i} x(t+\theta)\, d\theta + h_M^2 x^T(t) R_{2i} x(t) \end{aligned}$$

$$
\begin{aligned}
&\quad - \alpha \int_{t-h_m}^{t} e^{\alpha(s-t)} x^T(s) Q_{2i} x(s)\,ds - \alpha \int_{t-h_M}^{t} e^{\alpha(s-t)} x^T(s) Q_{3i} x(s)\,ds \\
&\quad - \alpha \int_{-h_m}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{1i} x(s)\,d\theta \\
&\quad - \alpha \int_{-h_M}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{2i} x(s)\,d\theta \\
&\quad - h_M \int_{-h_M}^{0} e^{\alpha\theta} x^T(t+\theta) R_{2i} x(t+\theta)\,d\theta - \alpha \int_{t-h(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_{1i} x(s)\,ds \\
&\leq x^T(t)\bigl[P_i \bar{A}_{1i} + \bar{A}_{1i}^T P_i + Q_{1i} + Q_{2i} + Q_{3i} + h_m^2 R_{1i} + h_M^2 R_{2i} + D_i^T P_i D_i\bigr] x(t) \\
&\quad - \alpha \int_{-h_m}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{1i} x(s)\,ds \\
&\quad - \alpha \int_{-h_M}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{2i} x(s)\,ds - \alpha \int_{t-h(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_{1i} x(s)\,ds \\
&\quad - h_m \int_{t-h_m}^{t} e^{-\alpha h_m} x^T(s) R_{1i} x(s)\,ds \\
&\quad - \alpha \int_{t-h_m}^{t} e^{\alpha(s-t)} x^T(s) Q_{2i} x(s)\,ds \\
&\quad - h_M \int_{t-h_M}^{t} e^{-\alpha h_M} x^T(s) R_{2i} x(s)\,ds \\
&\quad - \alpha \int_{t-h_M}^{t} e^{\alpha(s-t)} x^T(s) Q_{3i} x(s)\,ds - e^{-\alpha h_m} x^T(t-h_m) Q_{2i} x(t-h_m) \\
&\quad + x^T(t) P_i A_{2i} x\bigl(t-h(t)\bigr) + x^T\bigl(t-h(t)\bigr) A_{2i}^T P_i x(t) \\
&\quad + f_i^T\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) C_i^T P_i x(t) e^{-\alpha h_M} x^T(t-h_M) Q_{3i} x(t-h_M) \\
&\quad - (1-h) e^{-\alpha h_M} x^T\bigl(t-h(t)\bigr) Q_{1i} x\bigl(t-h(t)\bigr) \\
&\quad + x^T(t) P_i C_i f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr).
\end{aligned}
\tag{16}
$$

Inequality (4) can be written as follows:

$$
\begin{aligned}
&x^T(t) V_i^T V_i x(t) + x^T\bigl(t-h(t)\bigr) \Lambda_i^T \Lambda_i x\bigl(t-h(t)\bigr) \\
&\quad - f_i^T\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) \geq 0.
\end{aligned}
\tag{17}
$$

By using Lemma 2, we get

$$
\begin{aligned}
&-h_m \int_{t-h_m}^{t} e^{-\alpha h_m} x^T(s) R_{1i} x(s)\,ds \\
&\quad \leq -e^{-\alpha h_m} \left(\int_{t-h_m}^{t} x(s)\,ds \right)^T R_{1i} \left(\int_{t-h_m}^{t} x(s)\,ds \right), \\
&-h_M \int_{t-h_M}^{t} e^{-\alpha h_M} x^T(s) R_{2i} x(s)\,ds \\
&\quad \leq -e^{-\alpha h_M} \left(\int_{t-h_M}^{t} x(s)\,ds \right)^T R_{2i} \left(\int_{t-h_M}^{t} x(s)\,ds \right).
\end{aligned}
\tag{18}
$$

From (16), (17), and (18) we have

$$\begin{aligned}\mathcal{L}V_{1i} + \alpha V_{1i} \le{}& x^T(t)\big[P_i\bar{A}_{1i} + \bar{A}_{1i}^T P_i + Q_{1i} + Q_{2i} + Q_{3i}\\ &+ \alpha P_i + h_m^2 R_{1i} + h_M^2 R_{2i} + D_i^T P_i D_i + V_i^T V_i\big]x(t)\\ &+ x^T(t)P_iC_if_i\big(t,x(t),x\big(t-h(t)\big)\big)\\ &+ f_i^T\big(t,x(t),x\big(t-h(t)\big)\big)C_i^T P_i x(t)\\ &- f_i^T\big(t,x(t),x\big(t-h(t)\big)\big)f_i\big(t,x(t),x\big(t-h(t)\big)\big)\\ &+ x^T\big(t-h(t)\big)\big(\Lambda_i^T\Lambda_i - (1-h)e^{-\alpha h_M}Q_{1i}\big)x\big(t-h(t)\big)\\ &+ x^T(t)P_iA_{2i}x\big(t-h(t)\big) + x^T\big(t-h(t)\big)A_{2i}^T P_i x(t)\\ &- e^{-\alpha h_m}\bigg(\int_{t-h_m}^t x(s)\,ds\bigg)^T R_{1i}\bigg(\int_{t-h_m}^t x(s)\,ds\bigg)\\ &- e^{-\alpha h_m}x^T(t-h_m)Q_{2i}x(t-h_m)\\ &- e^{-\alpha h_M}\bigg(\int_{t-h_M}^t x(s)\,ds\bigg)^T R_{2i}\bigg(\int_{t-h_M}^t x(s)\,ds\bigg)\\ &- e^{-\alpha h_M}x^T(t-h_M)Q_{3i}x(t-h_M).\end{aligned} \tag{19}$$

Let

$$\begin{aligned}\xi(t) = \bigg(&x^T(t) \quad x^T\big(t-h(t)\big) \quad x^T(t-h_m) \quad x^T(t-h_M)\\ &f_i^T\big(t,x(t),x\big(t-h(t)\big)\big) \quad \bigg(\int_{t-h_m}^t x(s)\,ds\bigg)^T \quad \bigg(\int_{t-h_M}^t x(s)\,ds\bigg)^T\bigg)^T.\end{aligned}$$

According to (19), we can obtain

$$\mathcal{L}V_{1i} + \alpha V_{1i} \le \xi^T(t)\Xi_i\xi(t). \tag{20}$$

We can get

$$\mathcal{L}V_{1i} \le -\alpha V_{1i}. \tag{21}$$

Then, using (9) and (13), we have

$$\begin{aligned}d\big(e^{\alpha t}V_{1i}\big) &= e^{\alpha t}\big[\alpha V_{1i}\,dt + \mathcal{L}V_{1i}\,dt + 2x^T(t)P_iD_ix(t)\,d\omega(t)\big]\\ &\le e^{\alpha t}\big[\alpha V_{1i}\,dt - \alpha V_{1i}\,dt + 2x^T(t)P_iD_ix(t)\,d\omega(t)\big]\\ &= 2e^{\alpha t}x^T(t)P_iD_ix(t)\,d\omega(t).\end{aligned} \tag{22}$$

When $t \in [t_k, t_k + \tau_d)$, switched systems run in mismatched periods. The closed-loop system (7) is active within the ith subsystem and the corresponding jth switching controller is also activated. We choose the Lyapunov–Krasovskii functional candidate as follows:

$$V_{2\sigma(t)}(t) = x^T(t)P_{\sigma(t)}x(t) + \int_{t-h(t)}^t e^{\beta(t-s)}x^T(s)Q_{1\sigma(t)}x(s)\,ds$$

$$
\begin{aligned}
&+ \int_{t-h_m}^{t} e^{\beta(t-s)} x^T(s) Q_{2\sigma(t)} x(s)\,ds \\
&+ \int_{t-h_M}^{t} e^{\beta(t-s)} x^T(s) Q_{3\sigma(t)} x(s)\,ds \\
&+ h_m \int_{-h_m}^{0} \int_{t+\theta}^{t} e^{\beta(t-s)} x^T(s) R_{1\sigma(t)} x(s)\,ds\,d\theta \\
&+ h_M \int_{-h_M}^{0} \int_{t+\theta}^{t} e^{\beta(t-s)} x^T(s) R_{2\sigma(t)} x(s)\,ds\,d\theta.
\end{aligned}
\tag{23}
$$

According to Itô's differential formula, we have

$$
\begin{aligned}
\mathcal{L}V_{2i} ={}& 2x^T(t)P_i\bigl[\bar{A}_{1ij}x(t)x(t) + A_{2i}x\bigl(t-h(t)\bigr) + C_i f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr)\bigr] \\
&+ x^T(t)Q_{1i}x(t) + x^T(t)D_i^T P_i D_i x(t) - + x^T(t)Q_{2i}x(t) \\
&- \bigl(1-\dot{h}(t)\bigr)e^{\beta h(t)} x^T\bigl(t-h(t)\bigr)Q_{1i}x\bigl(t-h(t)\bigr) \\
&- e^{\beta h_m} x^T(t-h_m)Q_{2i}x(t-h_m) + h_m^2 x^T(t)R_{1i}x(t) \\
&+ x^T(t)Q_{3i}x(t) - e^{\beta h_M} x^T(t-h_M)Q_{3i}x(t-h_M) \\
&- h_m \int_{-h_m}^{0} e^{-\beta\theta} x^T(t+\theta)R_{1i}x(t+\theta)\,d\theta + h_M^2 x^T(t)R_{2i}x(t) \\
&- h_M \int_{-h_M}^{0} e^{-\beta\theta} x^T(t+\theta)R_{2i}x(t+\theta)\,d\theta \\
&+ \beta \int_{t-h(t)}^{t} e^{\beta(t-s)} x^T(s)Q_{1i}x(s)\,ds \\
&+ \beta \int_{t-h_m}^{t} e^{\beta(t-s)} x^T(s)Q_{2i}x(s)\,ds \\
&+ \beta \int_{t-h_M}^{t} e^{\beta(t-s)} x^T(s)Q_{3i}x(s)\,ds \\
&+ \beta \int_{-h_m}^{0} \int_{t+\theta}^{t} e^{\beta(t-s)} x^T(s)R_{1i}x(s)\,ds\,d\theta \\
&+ \beta \int_{-h_M}^{0} \int_{t+\theta}^{t} e^{\beta(t-s)} x^T(s)R_{2i}x(s)\,ds\,d\theta \\
\le{}& x^T(t)\bigl[P_i\bar{A}_{1ij} + \bar{A}_{1ij}^T P_i + Q_{1i} + Q_{2i} + Q_{3i} + h_m^2 R_{1i} + h_M^2 R_{2i} \\
&+ D_i^T P_i D_i\bigr]x(t) - (1-h)x^T\bigl(t-h(t)\bigr)Q_{1i}x\bigl(t-h(t)\bigr) \\
&+ x^T(t)P_i C_i f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) + f_i^T\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr)C_i^T P_i x(t) \\
&- x^T(t-h_M)Q_{3i}x(t-h_M) + \beta \int_{-h_m}^{0} \int_{t+\theta}^{t} e^{\beta(t-s)} x^T(s)R_{1i}x(s)\,ds\,d\theta \\
&+ x^T\bigl(t-h(t)\bigr)A_{2i}^T P_i x(t) + \beta \int_{t-h(t)}^{t} e^{\beta(t-s)} x^T(s)Q_{1i}x(s)\,ds \\
&- h_m \int_{t-h_m}^{t} x^T(s)R_{1i}x(s)\,ds + \beta \int_{t-h_m}^{t} e^{\alpha(t-s)} x^T(s)Q_{2i}x(s)\,ds
\end{aligned}
$$

$$
\begin{aligned}
&- h_M \int_{t-h_M}^{t} x^T(s) R_{2i} x(s)\,ds + \beta \int_{t-h_M}^{t} e^{\beta(t-s)} x^T(s) Q_{3i} x(s)\,ds \\
&- x^T(t-h_m) Q_{2i} x(t-h_m) + \beta \int_{-h_M}^{0} \int_{t+\theta}^{t} e^{\beta(t-s)} x^T(s) R_{2i} x(s)\,ds\,d\theta \\
&+ x^T(t) P_i A_{2i} x\bigl(t-h(t)\bigr).
\end{aligned}
\tag{24}
$$

Following the similar way, we have

$$
\begin{aligned}
\mathcal{L}V_{2i} - \beta V_{2i} \le{}& x^T(t)\bigl[P_i \bar{A}_{1ij} + \bar{A}_{1ij}^T P_i + Q_{1i} + Q_{2i} + Q_{3i} - \beta P_i + h_m^2 R_{1i} \\
&+ h_M^2 R_{2i} + D_i^T P_i D_i + V_i^T V_i\bigr] x(t) + x^T\bigl(t-h(t)\bigr) A_{2i}^T P_i x(t) \\
&+ x^T(t) P_i C_i f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) + x^T(t) P_i A_{2i} x\bigl(t-h(t)\bigr) \\
&+ f_i^T\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) C_i^T P_i x(t) \\
&- f_i^T\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) f_i\bigl(t, x(t), x\bigl(t-h(t)\bigr)\bigr) \\
&+ x^T\bigl(t-h(t)\bigr)\bigl(\Lambda_i^T \Lambda_i - (1-h) Q_{1i}\bigr) x\bigl(t-h(t)\bigr) \\
&- x^T(t-h_m) Q_{2i} x(t-h_m) - x^T(t-h_M) Q_{3i} x(t-h_M) \\
&- \left(\int_{t-h_m}^{t} x(s)\,ds\right)^T R_{1i} \left(\int_{t-h_m}^{t} x(s)\,ds\right) \\
&- \left(\int_{t-h_M}^{t} x(s)\,ds\right)^T R_{2i} \left(\int_{t-h_M}^{t} x(s)\,ds\right).
\end{aligned}
\tag{25}
$$

According to (12), we have

$$
\mathcal{L}V_{2i} - \beta V_{2i} \le \xi^T(t) \Omega_i \xi(t). \tag{26}
$$

Then

$$
\mathcal{L}V_{2i} \le \beta V_{2i}, \tag{27}
$$

we can get

$$
\begin{aligned}
d\bigl(e^{-\beta t} V_{2i}\bigr) &= e^{-\beta t}\bigl[-\beta V_{2i}\,dt + \mathcal{L}V_{2i}\,dt + 2x^T(t) P_i D_i x(t)\,d\omega(t)\bigr] \\
&\le e^{-\beta t}\bigl[-\beta V_{2i}\,dt + \beta V_{2i}\,dt + 2x^T(t) P_i D_i x(t)\,d\omega(t)\bigr] \\
&= 2e^{-\beta t} x^T(t) P_i D_i x(t)\,d\omega(t).
\end{aligned}
\tag{28}
$$

By recalling (2), we have

$$
\begin{aligned}
&\int_{t-h(t)}^{t} e^{\alpha(s-t)} x^T(s) Q_{1i} x(s)\,ds + \int_{t-h_m}^{t} e^{\alpha(s-t)} x^T(s) Q_{2i} x(s)\,ds \\
&\quad + \int_{t-h_M}^{t} e^{\alpha(s-t)} x^T(s) Q_{3i} x(s)\,ds + \int_{-h_m}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{1i} x(s)\,ds\,d\theta \\
&\quad + \int_{-h_M}^{0} \int_{t+\theta}^{t} e^{\alpha(s-t)} x^T(s) R_{2i} x(s)\,ds\,d\theta
\end{aligned}
$$

$$\begin{aligned}
&\leq \int_{t-h(t)}^{t} x^T(s)Q_i x(s)\,ds + \int_{t-h_m}^{t} x^T(s)Q_{2i}x(s)\,ds \\
&\quad + \int_{t-h_M}^{t} x^t(s)Q_{3i}x(s)\,ds + \int_{-h_m}^{0}\int_{t+\theta}^{t} x^T(s)R_{1i}x(s)\,ds\,d\theta \\
&\quad + \int_{-h_M}^{0}\int_{t+\theta}^{T} x^T(s)R_{2i}x(s)\,ds\,d\theta \\
&\leq \int_{t-h(t)}^{t} e^{\beta(t-s)}x^T(s)Q_{1i}x(s)\,ds + \int_{t-h_m}^{t} e^{\beta(t-s)}x^T(s)Q_{2i}x(s)\,ds \\
&\quad + \int_{t-h_M}^{t} e^{\beta(t-s)}x^T(s)Q_{3i}x(s)\,ds + \int_{-h_m}^{0}\int_{t+\theta}^{t} e^{\beta(t-s)}x^T(s)R_{1i}x(s)\,ds\,d\theta \\
&\quad + \int_{-h_M}^{0}\int_{t+\theta}^{t} e^{\beta(t-s)}x^T(s)R_{2i}x(s)\,ds\,d\theta. \qquad (29)
\end{aligned}$$

Therefore,

$$V_{1i}(t) \leq V_{2i}(t). \qquad (30)$$

Considering the whole interval $[t_0, t)$, the Lyapunov–Krasovskii functional $V(t)$ is expressed as

$$\begin{aligned}
&V_{1\sigma(t)}(t) \quad \forall t \in [t_k + \tau_d, t_{k+1}), k = 0, 1, 2, \ldots; \\
&V_{2\sigma(t)}(t) \quad \forall t \in [t_k, t_k + \tau_d), k = 0, 1, 2, \ldots.
\end{aligned} \qquad (31)$$

When $t \in [t_k + \tau_d, t_{k+1})$, integrating both sides of (22) from $t_k + \tau_d$ to t and taking expectation, we have

$$\begin{aligned}
E\{V(t)\} = E\{V_{1i}(t)\} &\leq e^{-\alpha(t-(t_k+\tau_d))}E\{V_{1i}((t_k+\tau_d)^+)\} \\
&\leq e^{-\alpha(t-(t_k+\tau_d))}E\{V_{2i}((t_k+\tau_d)^-)\} \\
&\leq e^{-\alpha(t-(t_k+\tau_d))}e^{\beta\tau_d}E\{V_{2i}((t_k)^+)\} \\
&\leq \mu e^{-\alpha(t-(t_k+\tau_d))}e^{\beta\tau_d}E\{V_{1i}((t_k)^+)\} \\
&\leq \cdots \\
&\leq \mu^k e^{(k+1)\beta\tau_d}e^{-\alpha(t-t_0-(k+1)\tau_d)}E\{V(t_0)\} \\
&\leq e^{(\alpha+\beta)\tau_d}e^{\{[\ln\mu+(\alpha+\beta)\tau_d]/\tau_a-\alpha\}(t-t_0)}E\{V(t_0)\}. \qquad (32)
\end{aligned}$$

When $t \in [t_k, t_k + \tau_d)$, integrating both sides of (28) and taking expectation, we obtain

$$\begin{aligned}
E\{V(t)\} = E\{V_{2i}(t)\} &\leq e^{\beta(t-(t_k))}E\{V_{2i}((t_k)^+)\} \\
&\leq \mu e^{\beta(t-t_k)}E\{V_{1i}((t_k)^-)\} \\
&\leq \cdots \\
&\leq \mu^k e^{(k+1)\beta\tau_d}e^{-\alpha(t-t_0-(k+1)\tau_d)}E\{V(t_0)\} \\
&\leq e^{(\alpha+\beta)\tau_d}e^{\{[\ln\mu+(\alpha+\beta)\tau_d]/\tau_a-\alpha\}(t-t_0)}E\{V(t_0)\}. \qquad (33)
\end{aligned}$$

Notice from (14) and (23) that

$$E\{V(t)\} \geq aE\{\|x(t)\|^2\}, \qquad E\{V(t_0)\} \leq b \sup_{-h_M \leq \theta \leq 0} E\{\|x(t_0+\theta)\|^2\}, \tag{34}$$

where

$$\begin{aligned} a &= \min_{i\in M} \lambda_{\min}(P_i), \\ b &= \max_{i\in M} \lambda_{\max}(P_i) + h_M \max_{i\in M} \lambda_{\max}(Q_{1i}) + h_m \max_{i\in M} \lambda_{\max}(Q_{2i}) \\ &\quad + h_M \max_{i\in M} \lambda_{\max}(Q_{3i}) + \frac{h_m^3}{2} \max_{i\in M} \lambda_{\max}(R_{1i}) + \frac{h_M^3}{2} \max_{i\in M} \lambda_{\max}(R_{2i}). \end{aligned}$$

Finally, we can get

$$\begin{aligned} E\{\|x(t)\|^2\} &\leq e^{(\alpha+\beta)\tau_d} \frac{b}{a} \sup_{-h_M \leq \theta \leq 0} E\{\|x(t_0+\theta)\|^2\} \\ &\quad \times e^{\{\alpha - [\ln\mu + (\alpha+\beta)\tau_d]/\tau_a\}(t-t_0)}. \end{aligned} \tag{35}$$

By Definition 1, we know that the closed-loop system (7) is mean-square exponentially stabilizable. This completes the proof. □

Theorem 2 *For given positive constants* α, β, h, *and* $\mu \geq 1$, *if there exist symmetric and positive definite matrices* $X_i, S_{1i}, S_{2i}, S_{3i}, T_{1i}, T_{2i}$ *and any* Y_i *such that the following matrix inequalities hold*:

$$\begin{aligned} &X_i \leq \mu X_j, \qquad S_{1i} \leq \mu S_{1j}, \qquad S_{2i} \leq \mu S_{2j}, \\ &S_{3i} \leq \mu S_{3j}, \qquad T_{1i} \leq \mu T_{1j}, \qquad T_{2i} \leq \mu T_{2j}, \quad i,j \in M, i \neq j, \end{aligned} \tag{36}$$

$$\begin{pmatrix} \Phi_{11}^i & \Phi_{12}^i \\ * & \Phi_{22}^i \end{pmatrix} < 0, \tag{37}$$

$$\begin{pmatrix} \Psi_{11}^i & \Phi_{12}^i \\ * & \Phi_{22}^i \end{pmatrix} < 0, \tag{38}$$

where

$$\Phi_{11}^i = \begin{pmatrix} \hat{\phi}_{11}^i & \hat{\phi}_{12}^i & 0 & 0 & C_i & 0 & 0 & \hat{\phi}_{18}^i & 0 \\ * & \hat{\phi}_{22}^i & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\phi}_{29}^i \\ * & * & \hat{\phi}_{33}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\phi}_{44}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \hat{\phi}_{66}^i & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\phi}_{77}^i & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -I \end{pmatrix},$$

$$\Phi_{12}^i = \begin{pmatrix} X_i D_i^T & X_i & X_i & X_i & h_m X_i & h_M X_i \\ \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} \end{pmatrix},$$

$$\Phi_{22}^i = \operatorname{diag}\{-X_i, -S_{1i}, -S_{2i}, -S_{3i}, -T_{1i}, -T_{2i}\},$$

$$\Psi_{11}^i = \begin{pmatrix} \hat{\varphi}_{11}^i & \hat{\phi}_{12}^i & 0 & 0 & C_i & 0 & 0 & \hat{\varphi}_{18}^i & 0 \\ * & \hat{\varphi}_{22}^i & 0 & 0 & 0 & 0 & 0 & 0 & \hat{\varphi}_{29}^i \\ * & * & \hat{\varphi}_{33}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \hat{\varphi}_{44}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \hat{\varphi}_{66}^i & 0 & 0 & 0 \\ * & * & * & * & * & * & \hat{\varphi}_{77}^i & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -I \end{pmatrix}$$

$$\hat{\phi}_{11}^i = A_{1i}X_i + B_iK_iX_i + (A_{1i}X_i + B_iK_iX_i)^T + \alpha X_i,$$

$$\hat{\phi}_{22}^i = (1-h)e^{-\alpha h_M}(S_{1i} - 2X_i), \qquad \hat{\phi}_{12}^i = A_{2i}X_i, \qquad \hat{\phi}_{18}^i = X_iV_i^T,$$

$$\hat{\phi}_{29}^i = X_i\Lambda_i^T, \qquad \hat{\phi}_{33}^i = e^{-\alpha h_m}(S_{2i} - 2X_i), \qquad \hat{\phi}_{44}^i = e^{-\alpha h_M}(S_{3i} - 2X_i),$$

$$\hat{\phi}_{66}^i = e^{-\alpha h_m}(R_{1i} - 2X_i), \qquad \hat{\phi}_{77}^i = e^{-\alpha h_M}(R_{2i} - 2X_i),$$

$$\hat{\varphi}_{11}^i = A_{1i}X_i + B_iK_jX_i + (A_{1i}X_i + B_iK_jX_i)^T - \beta X_i,$$

$$\hat{\varphi}_{22}^i = (1-h)(S_{1i} - 2X_i), \qquad \hat{\varphi}_{33}^i = S_{2i} - 2X_i, \qquad \hat{\varphi}_{44}^i = S_{3i} - 2X_i,$$

$$\hat{\phi}_{66}^i = T_{1i} - 2X_i, \qquad \hat{\phi}_{77}^i = T_{2i} - 2X_i,$$

then system (1) *is mean-square exponentially stabilizable for arbitrary switching signal with the average dwell time satisfying* (13). *In addition, the feedback controller can be designed by the following formula*:

$$K_i = Y_iX_i^{-1}, \quad i \in M. \tag{39}$$

Proof According to $S_i > 0$, $T_i > 0$, we have

$$(S_{pi} - X_i)^T {S_{pi}}^{-1}(S_{pi} - X_i) \geq 0 \quad (p = 1, 2, 3),$$

$$(T_{qi} - X_i)^T {T_{qi}}^{-1}(T_{qi} - X_i) \geq 0 \quad (q = 1, 2).$$

Then the following inequality can be obtained:

$$S_{pi} - 2X_i \geq -X_i{S_{pi}}^{-1}X_i, \qquad T_{qi} - 2X_i \geq -X_i{T_{qi}}^{-1}X_i. \tag{40}$$

Substituting (40) into (37) and multiplying both sides of (37) by $\operatorname{diag}\{X_i^{-1}, X_i^{-1}, X_i^{-1}, X_i^{-1}, I, X_i^{-1}, X_i^{-1}, I, I, I, I, I, I, I, I\}$, we can get the following inequality:

$$\begin{pmatrix} \bar{\Phi}_{11}^i & \bar{\Phi}_{12}^i \\ * & \Phi_{22}^i \end{pmatrix} < 0, \tag{41}$$

where

$$\bar{\Phi}_{11}^i = \begin{pmatrix} \tilde{\phi}_{11}^i & \tilde{\phi}_{12}^i & 0 & 0 & X_i^{-1}C_i & 0 & 0 & V_i^T & 0 \\ * & \tilde{\phi}_{22}^i & 0 & 0 & 0 & 0 & 0 & 0 & \Lambda_i^T \\ * & * & \tilde{\phi}_{33}^i & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & \tilde{\phi}_{44}^i & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & \tilde{\phi}_{66}^i & 0 & 0 & 0 \\ * & * & * & * & * & * & \tilde{\phi}_{77}^i & 0 & 0 \\ * & * & * & * & * & * & * & -I & 0 \\ * & * & * & * & * & * & * & * & -I \end{pmatrix}$$

$$\bar{\Phi}_{12}^i = \begin{pmatrix} D_i^T & I & I & I & h_m & h_M \\ \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} & \mathbf{0}_{8\times 1} \end{pmatrix},$$

$$\tilde{\phi}_{11}^i = X_i^{-1}A_{1i} + X_i^{-1}B_iY_iX_i^{-1} + \left(X_i^{-1}A_{1i} + X_i^{-1}B_iY_iX_i^{-1}\right)^T + \alpha X_i^{-1},$$

$$\tilde{\phi}_{12}^i = X_i^{-1}A_{2i}, \qquad \tilde{\phi}_{22}^i = -(1-h)e^{-\alpha h_M}S_{1i}^{-1}, \qquad \tilde{\phi}_{33}^i = -e^{-\alpha h_m}S_{2i}^{-1},$$

$$\hat{\phi}_{29}^i = X_i\Lambda_i^T, \qquad \hat{\phi}_{33}^i = e^{-\alpha h_m}(S_{2i} - 2X_i), \qquad \hat{\phi}_{44}^i = e^{-\alpha h_M}(S_{3i} - 2X_i),$$

$$\tilde{\phi}_{44}^i = -e^{-\alpha h_M}S_{3i}^{-1}, \qquad \tilde{\phi}_{66}^i = -e^{-\alpha h_m}R_{1i}^{-1}, \qquad \tilde{\phi}_{77}^i = -e^{-\alpha h_M}R_{2i}^{-1}.$$

Then set

$$\begin{aligned} &Y_i = K_iX_i, \qquad X_i^{-1} = P_i, \qquad S_{1i}^{-1} = Q_{1i}, \qquad S_{2i}^{-1} = Q_{2i}, \\ &S_{3i}^{-1} = Q_{3i}, \qquad T_{1i}^{-1} = R_{1i}, \qquad T_{2i}^{-1} = R_{2i}. \end{aligned} \tag{42}$$

Using Schur complement in (41), it can be concluded that (12) holds. By the same method, (38) implies (13). Correspondingly, controller gains are given by (39). The proof is completed. □

Remark 2 It is noticed that the Lyapunov–Krasovskii functional is delay-dependent in this paper. On the one hand, the important information of h_m and h_M is taken into full consideration, which may overcome the conservatism of quadratic mean-square exponential stability conditions for nonlinear switched stochastic systems with interval time-varying delay under asynchronous switching. On the other hand, the delay-dependent Lyapunov–Krasovskii functional is allowed to rise at both switching instants. However, the delay-dependent Lyapunov–Krasovskii functional is decreasing on the entire interval and the mean-square exponential stabilization for nonlinear switched stochastic systems is also guaranteed.

Remark 3 [34] investigated the stabilization problem for a class of positive switched nonlinear systems under asynchronous switching. The author mainly focused on the study of positive switched systems in [34]. [35] addressed the problem of robust control for uncertain switched nonlinear systems with time delay under asynchronous switching. However, stochastic disturbance was not considered in [34] and [35]. The problem of robust reliable control for a class of uncertain stochastic switched nonlinear systems under asynchronous switching was investigated in [36], but the interval time-varying delay was not considered in [36] under asynchronous switching. In this paper, we consider stochastic disturbance and interval time-varying delay. Compared with [34–36], it is obvious that we have considered more external factors, and our switched systems model is more in line with engineering practice from the application level.

Remark 4 [18] obtained sufficient conditions with delay-dependent guaranteeing the exponential stability by a common Lyapunov functional (CLF). In fact, we deeply realize that common Lyapunov functional (CLF) may not satisfy all subsystems and become conservative for switched systems. The multi-Lyapunov–Krasovskii functional (MLKF) and delay-dependent method are better choices. They provide a powerful framework for analyzing the stability of switched nonlinear systems with interval time-varying delay.

Remark 5 We now summarize the controller design procedures as follows.

Step 1: The desired convergence rates α and β are given. Choose a positive and appropriate parameter μ.

Step 2: Define the variables $X_i, S_{1i}, S_{2i}, S_{3i}, T_{1i}, T_{2i}, Y_i, i \in \{1,2\}$ to be solved.

Step 3: Describe the block form to give a linear matrix inequalities (LMIs).
- $X_i > 0, S_{1i} > 0, S_{2i} > 0, S_{3i} > 0, T_{1i} > 0, T_{2i} > 0$;
- (36), (37), and (38) are established

Step 4: Complete LMIs model description.

Step 5: Solve LMIs problems.

Step 6: Solve (42).

Then the obtained feedback controller will make the desired performance indices be satisfied.

4 Numerical example

In this section, a numerical example is presented to confirm the effectiveness of the proposed approach.

Example 1 Consider system (1) composed of two subsystems with the following parameters:

$$
\begin{aligned}
&A_{11} = \begin{bmatrix} -0.85 & 0.1 \\ 0 & -0.9 \end{bmatrix}, \qquad A_{21} = \begin{bmatrix} -0.75 & 0 \\ 0.1 & -0.8 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0.4 \\ 0.5 \end{bmatrix}, \\
&A_{12} = \begin{bmatrix} -0.75 & 0 \\ 0.2 & -0.9 \end{bmatrix}, \qquad A_{22} = \begin{bmatrix} -0.7 & 0.15 \\ 0 & -0.85 \end{bmatrix}, \qquad B_2 = \begin{bmatrix} 0.6 \\ 0.5 \end{bmatrix}, \\
&D_1 = \begin{bmatrix} -0.3 & 0 \\ 0.1 & -0.4 \end{bmatrix}, \qquad D_2 = \begin{bmatrix} -0.3 & 0 \\ 0 & -0.2 \end{bmatrix}, \qquad C_1 = \begin{bmatrix} 0.3 & 0 \\ 0 & 0.4 \end{bmatrix}, \\
&V_1 = \begin{bmatrix} -0.65 & 0 \\ 0.05 & -0.7 \end{bmatrix}, \qquad V_2 = \begin{bmatrix} -0.4 & 0.1 \\ 0 & -0.6 \end{bmatrix}, \qquad C_2 = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.3 \end{bmatrix}, \\
&\Lambda_1 = \begin{bmatrix} -0.8 & 0.1 \\ 0 & -0.7 \end{bmatrix}, \qquad \Lambda_2 = \begin{bmatrix} -0.5 & 0.1 \\ 0 & -0.4 \end{bmatrix}.
\end{aligned}
$$

Table 1 Calculated upper delay bound h_M

α	0.55	0.6	0.65	0.7	0.75
h_M ($\beta = 0.7$)	0.677	0.733	0.827	0.889	0.935
h_M ($\beta = 0.75$)	0.746	0.762	0.841	0.865	1.016
h_M ($\beta = 0.8$)	0.804	0.852	0.906	1.119	1.352

Let $\alpha = 0.45, \beta = 0.65, \mu = 1.25, h_m = 0.1, h_M = 0.6, h = 0.5, h(t) = 0.5\sin(t) + 0.1, \tau_d = 0.2$. According to (13), we get the average dwell time

$$\tau_a > \tau_a^* = \frac{\ln\mu + (\alpha + \beta)\tau_d}{\alpha} = 0.9848.$$

Choose

$$f_1\bigl(t, x(t), x\bigl(t - h(t)\bigr)\bigr) = \begin{pmatrix} 0.2e^{-4t} \\ 0.1\sin(x_2(t)) \end{pmatrix},$$

$$f_2\bigl(t, x(t), x\bigl(t - d(t)\bigr)\bigr) = \begin{pmatrix} 0.1\sin(x_1(t - h(t))) \\ 0.1e^{-3t} \end{pmatrix}.$$

By solving (36), (37), and (38), we can get

$$X_1 = \begin{bmatrix} 0.5817 & -0.2809 \\ -0.2809 & 0.5537 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 0.5539 & 0.2561 \\ 0.2561 & 0.5298 \end{bmatrix},$$

$$S_{11} = \begin{bmatrix} 1.4886 & -0.3040 \\ -0.3040 & 1.0582 \end{bmatrix}, \qquad S_{12} = \begin{bmatrix} 1.6720 & 0.0467 \\ 0.0467 & 1.3186 \end{bmatrix},$$

$$S_{21} = \begin{bmatrix} 1.7736 & -0.0015 \\ -0.0015 & 1.3371 \end{bmatrix}, \qquad S_{22} = \begin{bmatrix} 1.4761 & 0.1202 \\ 0.1202 & 1.2674 \end{bmatrix},$$

$$S_{31} = \begin{bmatrix} 1.2352 & -0.0103 \\ -0.0103 & 0.9825 \end{bmatrix}, \qquad S_{32} = \begin{bmatrix} 1.1326 & -0.0831 \\ -0.0831 & 1.1141 \end{bmatrix},$$

$$T_{11} = \begin{bmatrix} 1.2828 & -0.1263 \\ -0.1263 & 1.1729 \end{bmatrix}, \qquad T_{12} = \begin{bmatrix} 1.2508 & 0.0585 \\ 0.0585 & 1.1796 \end{bmatrix},$$

$$T_{21} = \begin{bmatrix} 1.7573 & 0.0240 \\ 0.0240 & 1.3964 \end{bmatrix}, \qquad T_{22} = \begin{bmatrix} 1.5633 & 0.1690 \\ 0.1690 & 1.3723 \end{bmatrix},$$

$$Y_1 = \begin{bmatrix} 0.8214 & 0.4447 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 0.6154 & 0.7919 \end{bmatrix}.$$

Then the controller gains constructed by (39) are

$$K_1 = \begin{bmatrix} 2.3840 & 2.0128 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 0.5408 & 1.2334 \end{bmatrix}.$$

By Theorem 2, the maximum value of interval time-varying delay h_M for the switched systems (1) is provided in Table 1 for different values of α and β.

In order to show the effectiveness of the proposed method, the responses of state trajectories of the open-loop and closed-loop systems and switching signals (system signals

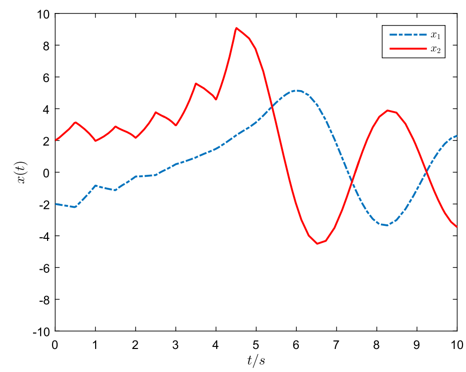


Figure 1 Responses of state trajectories of an open-loop system

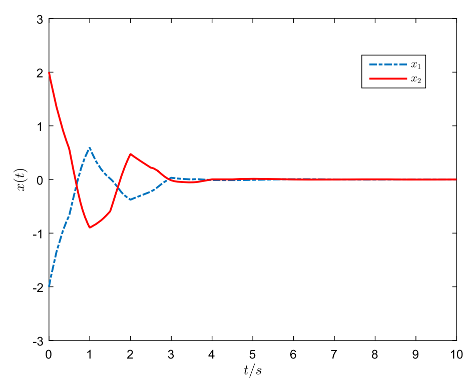


Figure 2 Responses of state trajectories of a closed-loop system

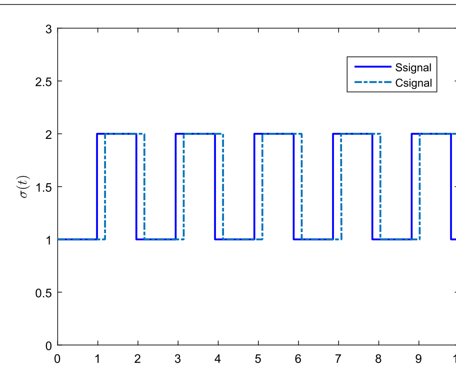


Figure 3 The switching law

and controller signals) are given in Figs. 1, 2, and 3, respectively. It is clear that the open-loop system with initial state $x(0) = (-2, 2)^T$ is not stable form in Fig. 1. The closed-loop system with initial state $x(0) = (-2, 2)^T$ is mean-square exponentially stabilizable under the designed asynchronous switching and controllers form in Fig. 2 and 3. Therefore, the effectiveness of the designed asynchronous switching and controllers is fully illustrated.

Example 2 The problem of water pollution is an important issue facing every country, and its development is of great significance to social development. In this section, an example of applying this system to water pollution control systems will be demonstrated.

To facilitate the creation of models for water pollution control systems, we record $p(t)$ and $q(t)$ as the concentrations per unit volume of biochemical oxygen demand and dissolved oxygen, respectively. Simultaneously, let p^* and q^* indicate the desired steady values of $p(t)$ and $q(t)$ in a reach of a polluted river, respectively. Moreover, we take p^* and q^* as corresponding to some measure of water quality standards, given by the following definition:

$$x_1(t) = p(t) - p^*, \qquad x_2(t) = q(t) - q^*, \qquad x(t) = \left[x_1^T(t)x_2^T(t)\right]^T.$$

As a result, the dynamic equation for $x(t)$ can be expressed as

$$dx(t) = \left[Ax(t) + \bar{A}x\bigl(t - h(t)\bigr) + Bu(t)\right]dt + x(t)\,d\omega(t), \tag{43}$$

where

$$A = \begin{bmatrix} -m_1 - \varepsilon_1 - \varepsilon_2 & 0 \\ -m_3 & -m_2 - \varepsilon_1 - \varepsilon_2 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} \varepsilon_2 & 0 \\ 0 & \varepsilon_2 \end{bmatrix}, \qquad B = \begin{bmatrix} \varepsilon_1 \\ 1 \end{bmatrix}, \tag{44}$$

$m_i (i = 1, 2, 3)$, ε_1 and ε_2 are known constants, and $\omega(t)$ is a one-dimensional Brownian motion that satisfies condition (3). Moreover, $u(t) = [u_1^T(t)u_2^T(t)]^T$ is the control variable of river pollution system. We can learn the engineering significance of these parameters from [1]. This paper assumes that system actuators have good performance or failure, and according to the actual situation, we know that at least one actuator can ensure the normal operation of the river pollution system. In addition, for simulation of our purposes, we do not consider the nonlinear perturbation term, and the nonlinear perturbation term is not also considered in [26]. As a consequence, the river pollution system (43) can be modeled as a switched system consisting of two subsystems:

$$\begin{aligned} dx(t) &= \left[A_{11}x(t) + A_{21}x\bigl(t - h(t)\bigr) + B_1u(t)\right]dt + x(t)\,d\omega(t), && \text{no failures occur} \\ dx(t) &= \left[A_{12}x(t) + A_{22}x\bigl(t - h(t)\bigr) + B_2u(t)\right]dt + x(t)\,d\omega(t), && \text{failures occur.} \end{aligned} \tag{45}$$

Next, we choose $m_1 = 1.1, m_2 = 0.6, m_3 = 1.3, \varepsilon_1 = 0.5, \varepsilon_2 = 0.4$ and get that

$$A = \begin{bmatrix} -2 & 0 \\ -1.3 & -1.5 \end{bmatrix}, \qquad \bar{A} = \begin{bmatrix} 0.4 & 0 \\ 0 & 0.4 \end{bmatrix}, \qquad B = \begin{bmatrix} 0.5 \\ 1 \end{bmatrix}.$$

Let $\alpha = 0.5, \beta = 0.6, \mu = 1.15, h_m = 0.15, h_M = 0.65, h = 0.3, h(t) = 0.3\sin(t) + 0.2, \tau_d = 0.1$.
By (36), (37), and (38), we have

$$X_1 = \begin{bmatrix} 0.3725 & 0.1019 \\ 0.1019 & 0.6832 \end{bmatrix}, \qquad X_2 = \begin{bmatrix} 0.4827 & 0.1024 \\ 0.1024 & 0.6728 \end{bmatrix},$$

$$S_{11} = \begin{bmatrix} 1.2871 & 0.2028 \\ 0.2028 & 0.9811 \end{bmatrix}, \qquad S_{12} = \begin{bmatrix} 1.1728 & 0.1422 \\ 0.1422 & 1.1107 \end{bmatrix},$$

$$S_{21} = \begin{bmatrix} 1.2821 & 0.0102 \\ 0.0102 & 1.1371 \end{bmatrix}, \qquad S_{22} = \begin{bmatrix} 1.0963 & 0.3294 \\ 0.3294 & 1.1623 \end{bmatrix},$$

$$S_{31} = \begin{bmatrix} 1.4372 & 0.2031 \\ 0.2031 & 1.0921 \end{bmatrix}, \qquad S_{32} = \begin{bmatrix} 1.2355 & 0.2832 \\ 0.2832 & 1.2362 \end{bmatrix},$$

Figure 4 State response of subsystem 1

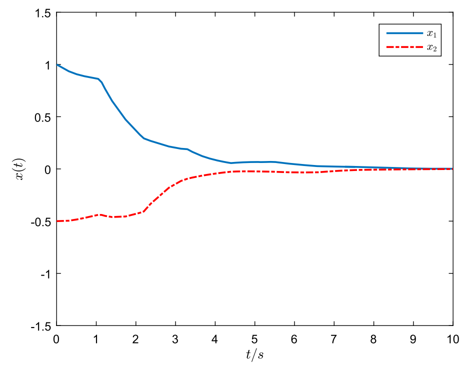
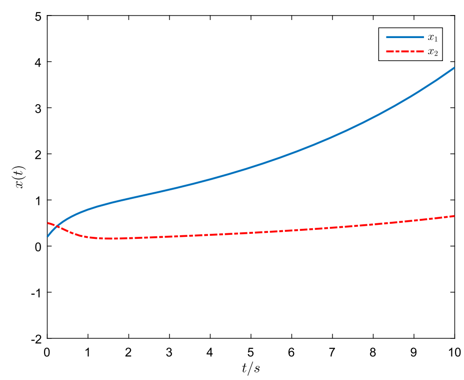


Figure 5 State response of subsystem 2



$$T_{11} = \begin{bmatrix} 1.0812 & 0.2733 \\ 0.2733 & 1.0128 \end{bmatrix}, \qquad T_{12} = \begin{bmatrix} 1.3562 & 0.0738 \\ 0.0738 & 1.3792 \end{bmatrix},$$
$$T_{21} = \begin{bmatrix} 1.3783 & 0.0367 \\ 0.0367 & 1.0917 \end{bmatrix}, \qquad T_{22} = \begin{bmatrix} 1.4923 & 0.2623 \\ 0.2623 & 1.8392 \end{bmatrix},$$
$$Y_1 = \begin{bmatrix} 0.9972 & 0.5377 \end{bmatrix}, \qquad Y_2 = \begin{bmatrix} 0.8362 & 0.5849 \end{bmatrix}.$$

Then the controller gains constructed by (39) are

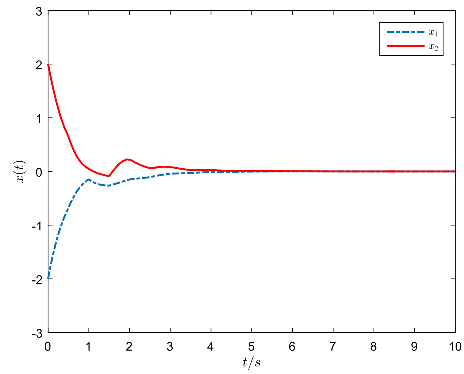
$$K_1 = \begin{bmatrix} 2.5665 & 0.4042 \end{bmatrix}, \qquad K_2 = \begin{bmatrix} 1.5996 & 0.6259 \end{bmatrix}.$$

Figure 4 describes state response of subsystem 1 with the initial condition $x(0) = (1, -0.5)^T$. Figure 5 describes state response of subsystem 2 with the initial condition $x(0) = (0.2, 0.5)^T$. Through the designed switching signal and our approach, we can get that system (43) with the initial condition $x(0) = (2, -2)^T$ is mean-square exponentially stabilizable for any switching signal under the feedback control form Fig. 6. As a consequence, this verifies the effectiveness of our results in the control of river pollution process.

5 Conclusions

The switching signal of the switched controller involves delay, which results in the asynchronous switching between the candidate controllers and subsystems. In the paper, we

Figure 6 State response of system (43)



have investigated the problem of asynchronous switching for nonlinear switched stochastic systems with interval time-varying delay based on time-dependent switching signal for the matched and mismatched sections, respectively. By constructing a new multi-Lyapunov–Krasovskii functional, which is related to the size of the time delay, using the matrix inequality technique and the average dwell time approach, the mean-square exponential stabilization criteria for nonlinear switched stochastic systems with interval time-varying delay are obtained under asynchronous switching. Then, the proposed approach is extended to design state feedback controller for switched stochastic systems by special operations of matrices. Finally, the numerical example illustrates the effectiveness of the theoretical results. Compared with the existing results, the new condition is less conservative. In order to better study the asynchronous switching issue, our future work will focus on extending the proposed method to a delay-dependent robust dissipative problem for a class of nonlinear switched systems with mixed delays and the stabilization of stochastic switched nonlinear systems with Markov jumps.

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Ethics approval and consent to participate
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Authors' contributions
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