Song et al. *Advances in Difference Equations* (2019) 2019:299
https://doi.org/10.1186/s13662-019-2203-4

Advances in Difference Equations
a SpringerOpen Journal

RESEARCH

Open Access

A non-autonomous Leslie–Gower model with Holling type IV functional response and harvesting complexity

Jie Song[1,2], Yonghui Xia[3], Yuzhen Bai[4*], Yaoxiong Cai[2] and D. O'Regan[5]

*Correspondence:
baiyu99@126.com
[4]School of Mathematical Sciences, Qufu Normal University, Qufu, China
Full list of author information is available at the end of the article

Abstract

This paper considers a non-autonomous modified Leslie–Gower model with Holling type IV functional response and nonlinear prey harvesting. The permanence of the model is obtained, and sufficient conditions for the existence of a periodic solution are presented. Two examples and their simulations show the validity of our results.

Keywords: Periodic solutions; Functional response; Permanence; Non-autonomous; Predator-prey model

1 Introduction

It is well known that predation activities are ubiquitous in nature [1]. Modeling of predator-prey interaction has become an important topic in mathematical biology. Song and Yuan [2] studied bifurcation analysis in a predator-prey system with time delay. Ruan and Xiao [3] provided a global analysis in a predator-prey system with a nonmonotonic functional response, and they proved the existence of two limit cycles. Huang and Xiao [4] considered a bifurcation analysis and stability for a predator-prey system with Holling-IV functional response. Xiao and Ruan [5] and Xue and Duan [6] considered time-delay effects to a predator-prey model with Holling-IV type functional response, where stability and bifurcation of periodic solutions were investigated. For the non-autonomous case, Chen [7] proved the existence of two periodic solutions for a model with Holling-IV functional response, and Xia et al. [8] obtained some sufficient conditions for the existence of two periodic solutions in a stage-structured predator-prey model. Li et al. [9] established the existence of multiple periodic solutions for a stage-structured model with harvesting terms. Wang et al. [10] studied the existence of multiple periodic solutions for an impulsive model with a Holling IV type functional response. A two-species model (the so-called LG model) was proposed by Leslie and Gower [11] in 1960. Korobeinikov [12] proved the existence of the limit cycle in such a model. For autonomous predator-prey models with Holling II or III type functional response, the existence of a limit cycle was proved and for the non-autonomous case, the existence of periodic solutions was established. Yu [13] reported some important research for a modified Leslie–Gower model. The Leslie–Gower type predator-prey model with Holling type IV functional response is described by

$$\begin{cases} \frac{du}{dt} = [r(1-\frac{u}{K}) - \frac{mv}{\frac{u^2}{i}+u+a}]u, \\ \frac{dv}{dt} = s(1-\frac{nv}{u})v, \end{cases} \tag{1.1}$$

where $u \equiv u(t)$ and $v \equiv v(t)$ are the prey and predator population density, respectively, r and s are intrinsic growth rates of the prey and predator, respectively. K is the carrying capacity of prey population; here m and i denote the maximum per capita predation rate and a measure of the predator's immunity from or tolerance of the prey, respectively, and a and n are the half saturation constant and the number of prey required to support one predator at equilibrium, respectively. Upadhyay et al. [14] studied that interaction between prey and predator with a Holling type IV functional response. We know that there are three main types of harvesting in the biomodel article: (1) constant rate of harvesting, (2) proportional harvesting $H(x) = qEx$, and (3) nonlinear harvesting $H(u) = \frac{qEu}{m_1E+m_2u}$, where m_1, m_2 are suitable constants, E is the effort applied to harvest individuals and q is the catchability coefficient. Zhang et al. [15] introduced the nonlinear harvesting $H(u) = \frac{qEu}{m_1E+m_2u}$ into model (1.1), and it can be described by

$$\begin{cases} \frac{du}{dt} = [r(1-\frac{u}{K}) - \frac{mv}{\frac{u^2}{i}+u+a} - \frac{qE}{m_1E+m_2u}]u, \\ \frac{dv}{dt} = s(1-\frac{nv}{u})v. \end{cases} \tag{1.2}$$

Taking

$$u = Kx, \qquad t = rT, \qquad h = \frac{qE}{rm_2K}, \qquad c = \frac{m_1E}{m_2K}, \qquad v = \frac{rKy}{m}, \qquad \alpha = \frac{i}{k},$$
$$\beta = \frac{nr}{m}, \qquad \gamma = \frac{a}{K}, \qquad \delta = \frac{s}{r},$$

then system (1.2) becomes

$$\begin{cases} \frac{dx}{dt} = x(1-x) - \frac{xy}{\frac{x^2}{\alpha}+x+\gamma} - \frac{hx}{x+c}, \\ \frac{dy}{dt} = \delta y(1-\beta\frac{y}{x}). \end{cases} \tag{1.3}$$

In spite of a lot of works focused on the global dynamics and bifurcation analysis of the ecological systems (e.g., [2–23]), in realistic environment, ecological systems are usually affected by the seasonable perturbations or other unpredictable disturbances (e.g., see [24–36]). Thus the time-varying parameters are more reasonable when we try to consider the periodic environment. In this paper, we consider the following non-autonomous model:

$$\begin{cases} \frac{dx}{dt} = x(1-x) - \frac{xy}{\frac{x^2}{\alpha(t)}+x+\gamma(t)} - \frac{h(t)x}{x+c(t)}, \\ \frac{dy}{dt} = \delta(t)y(1-\beta(t)\frac{y}{x}). \end{cases} \tag{1.4}$$

The rest of this paper is organized as follows. In Sect. 2, we discuss the permanence for the general nonautonomous case. Section 3 is to obtain some sufficient conditions for the existence of periodic solution of system (1.4). Finally, we use numerical simulation to fully demonstrate the existence of our periodic solution.

2 Permanence

In this section, we assume that $\alpha(t), \gamma(t), h(t), c(t), \delta(t)$, and $\beta(t)$ are all continuous and bounded above and below by positive constants. Let $\mathbb{R}^2_+ := \{(x,y) \in \mathbb{R}^2 \mid x \geq 0, y \geq 0\}$. For a continuous bounded function $f(t)$ on $\mathbb{R}$, denote

$$f^u := \sup_{t\in\mathbb{R}} f(t), \qquad f^l := \inf_{t\in\mathbb{R}} f(t).$$

From a biological viewpoint, we assume that the initial conditions satisfy

$$x(t_0) = x_0 > 0, \qquad y(t_0) = y_0 > 0.$$

Definition 2.1 If a positive solution $(x(t), y(t))$ of system (1.4) satisfies

$$\min\Big\{\lim_{t\to\infty}\inf x(t), \lim_{t\to\infty}\inf y(t)\Big\} = 0,$$

then system (1.4) is non-persistent.

Definition 2.2 If there exist two positive constants ϕ and $\varphi (0 < \phi < \varphi)$ with

$$\begin{aligned}
&\min\Big\{\lim_{t\to\infty}\inf x(t), \lim_{t\to\infty}\inf y(t)\Big\} \geq \phi,\\
&\max\Big\{\lim_{t\to\infty}\sup x(t), \lim_{t\to\infty}\sup y(t)\Big\} \leq \varphi,
\end{aligned}$$

then system (1.4) is permanent.

Define the collections:

$$\begin{aligned}
S_1 &= \big\{\big(c^u, c^l, h^u, \gamma^l, \alpha^u\big) \mid c^u > 1, c^l > h^u, 4\gamma^l < \alpha^u\big\};\\
S_2 &= \big\{\big(c^u, c^l, h^u, h^l, \gamma^l, \alpha^u, \beta^l\big) \mid c^u > 1, c^l > h^u, 4\gamma^l > \alpha^u,\\
&\quad \beta^l\big(c^u - 1\big)\big(c^l - h^u\big)\big(4\gamma^l - \alpha^u\big) > 4c^l\big(c^u - h^l\big)\big\};\\
S_3 &= \Bigg\{\big(c^u, c^l, h^u, h^l, \gamma^l, \alpha^u, \beta^l\big) \mid c^u > h^l, c^u > 1, c^l < h^u, 4\gamma^l < \alpha^u,\\
&\quad \frac{4c^l(c^u - h^l)}{(4\gamma^l - \alpha^u)} > \beta^l\big(c^u - 1\big)\big(c^l - h^u\big)\Bigg\};\\
S_4 &= \big\{\big(c^u, c^l, h^u, h^l, \gamma^l, \alpha^u, \beta^l\big) \mid c^u < h^l, c^u < 1, 4\gamma^l < \alpha^u,\\
&\quad 4c^l\big(c^u - h^l\big) > \beta^l\big(c^u - 1\big)\big(c^l - h^u\big)\big(4\gamma^l - \alpha^u\big)\big\}.
\end{aligned}$$

The set $\varGamma$ is defined by

$$\varGamma = \big\{(x,y) \in \mathbb{R}^2 \mid 0 < g_1 \leq x \leq G_1, 0 < g_2 \leq y \leq G_2\big\}, \tag{2.1}$$

where

$$G_1 = \frac{c^u - h^l}{c^u - 1}, \qquad g_1 = \frac{\beta^l (c^u - 1)(c^l - h^u)(4\gamma^l - \alpha^u) - 4c^l (c^u - h^l)}{\beta^l c^l (c^u - 1)(4\gamma^l - \alpha^u)}, \tag{2.2}$$

$$G_2 = \frac{c^u - h^l}{\beta^l (c^u - 1)}, \qquad g_2 = \frac{\beta^l (c^u - 1)(c^l - h^u)(4\gamma^l - \alpha^u) - 4c^l (c^u - h^l)}{\beta^u \beta^l c^l (c^u - 1)(4\gamma^l - \alpha^u)}. \tag{2.3}$$

Theorem 2.3 *If $S_1 \cup S_2 \cup S_3 \cup S_4 \neq \emptyset$, then the set Γ is a positively invariant and bounded region with respect to system* (1.4).

Proof Let $(x(t), y(t))$ be any solution of system (1.4) satisfying the initial values $(x(t_0), y(t_0)) = (x_0, y_0) \in \Gamma$. It suffices to show that all the solutions starting from the point in Γ keep inside Γ. From the first equation of system (1.4), we get

$$\begin{aligned}
\dot{x}(t) &\leq x(t)\left(1 - x(t) - \frac{h^l}{x(t) + c^u}\right) \\
&= \frac{x(t)}{x(t) + c^u}\left\{x(t) - x^2(t) + c^u - c^u x(t) - h^l\right\} \\
&\leq \frac{x(t)}{x(t) + c^u}\left\{c^u - h^l - \left(c^u - 1\right)x(t)\right\} \\
&\leq \frac{x(t)(c^u - 1)}{x(t) + c^u}\left\{\frac{c^u - h^l}{c^u - 1} - x(t)\right\} \\
&= \frac{x(t)(c^u - 1)}{x(t) + c^u}\left\{G_1 - x(t)\right\},
\end{aligned}$$

which implies

$$0 \leq x(t_0) \leq G_1 \Rightarrow x(t) \leq G_1, \quad t \geq t_0.$$

From the second equation of system (1.4), we obtain

$$\begin{aligned}
\dot{y}(t) &\leq \delta^u y(t)\left[1 - \frac{\beta^l y(t)}{G_1}\right] \\
&= \frac{\delta^u \beta^l (c^u - 1) y(t)}{c^u - h^l}\left[\frac{c^u - h^l}{\beta^l (c^u - 1)} - y(t)\right] \\
&= \frac{\delta^u \beta^l (c^u - 1) y(t)}{c^u - h^l}\left[G_2 - y(t)\right],
\end{aligned}$$

which implies

$$0 \leq y(t_0) \leq G_2 \Rightarrow y(t) \leq G_2, \quad t \geq t_0.$$

Similarly, we have

$$\begin{aligned}
\dot{x}(t) &\geq x(t)\left[1 - x(t) - \frac{h^u}{c^l} - \frac{G_2 \alpha^u}{x(t)^2 + \alpha^u x(t) + \alpha^u \gamma^l}\right] \\
&\geq x(t)\left[1 - x(t) - \frac{h^u}{c^l} - \frac{4G_2 \alpha^u}{4\alpha^u \gamma^l - (\alpha^u)^2}\right]
\end{aligned}$$

$$= x(t)\left[\frac{\beta^l(c^u-1)(c^l-h^u)(4\gamma^l-\alpha^u)-4c^l(c^u-h^l)}{\beta^l c^l(c^u-1)(4\gamma^l-\alpha^u)} - x(t)\right]$$
$$= x(t)\left[g_1 - x(t)\right],$$

which leads to

$$x(t_0) \geq g_1 \Rightarrow x(t) \geq g_1, \quad t \geq t_0.$$

Moreover, it follows from the predator equation that

$$\begin{aligned}\dot{y}(t) &\geq \delta^l y(t)\left[1-\frac{\beta^u y}{g_1}\right]\\ &= \frac{\beta^u\delta^l y(t)}{g_1}\left[\frac{\beta^l(c^u-1)(c^l-h^u)(4\gamma^l-\alpha^u)-4c^l(c^u-h^l)}{\beta^u\beta^l c^l(c^u-1)(4\gamma^l-\alpha^u)} - y\right]\\ &= \frac{\beta^u\delta^l y(t)}{g_1}[g_2-y],\end{aligned}$$

and hence,

$$y(t_0) \geq g_2 \Rightarrow y(t) \geq g_1, \quad t \geq t_0.$$

This completes the proof of Theorem 2.3. □

Theorem 2.4 *Assume that the condition in Theorem* 2.3 *is satisfied. Then the set Γ is the ultimately bounded region of system* (1.4).

3 Periodic case

This section is to obtain some sufficient conditions for the existence of a periodic solution of system (1.4). When we study the non-autonomous periodic system, we focus on obtaining the existence of positive periodic solutions. Therefore, we assume that all the parameters of system (1.4) are periodic in t of period $\omega > 0$. It is easy to follow from Brouwer's fixed point theorem that

Theorem 3.1 *In addition to the conditions of Theorem* 2.3, *system* (1.4) *has at least one positive periodic solution of period ω, say $(x^*(t), y^*(t))$, which lies in Γ, i.e., $g_1 \leq x^*(t) \leq G_2, g_2 \leq y^*(t) \leq g_2(t)$, where $g_i, G_i, i = 1, 2$, are defined in* (2.2).

Alternatively, we can employ another method (coincidence degree theory) to investigate periodic solutions of system (1.4). We adopt the notations and lemmas from [24, 27, 37–39]. We denote $\bar{f} := \frac{1}{\omega}\int_0^\omega f(t)\,dt$ when $f(t)$ is a periodic and continuous function with period ω (see [31]). Let

$$H_1 := \ln\left\{1+\frac{\bar{h}}{\bar{c}}\right\} + 2\omega, \qquad H_2 := \ln\left\{\frac{\exp\{H_1\}}{\bar{\beta}}\right\} + 2\bar{\delta}\omega,$$
$$H_3 := \ln\left\{\frac{(\bar{c}-\bar{h})(4\gamma^l-\alpha^u)-4\exp\{H_2\}\bar{c}}{\bar{c}(4\gamma^l-\alpha^u)}\right\} - 2\omega, \qquad H_4 := \ln\left\{\frac{\exp\{H_3\}}{\bar{\beta}}\right\} - 2\bar{\delta}\omega,$$

and define the collections

$$\bar{S}_1 = \{(\bar{c},\bar{h},\gamma^l,\alpha^u) \mid 4\gamma^l < \alpha^u, \bar{c} > \bar{h}\};$$
$$\bar{S}_2 = \{(\bar{c},\bar{h},\gamma^l,\alpha^u,H_2) \mid 4\gamma^l > \alpha^u, \bar{c} > \bar{h}, (\bar{c}-\bar{h})(4\gamma^l-\alpha^u) > 4\exp\{H_2\}\bar{c}\};$$
$$\bar{S}_3 = \{(\bar{c},\bar{h},\gamma^l,\alpha^u,H_2) \mid 4\gamma^l < \alpha^u, \bar{c} < \bar{h}, (\bar{c}-\bar{h})(4\gamma^l-\alpha^u) < 4\exp\{H_2\}\bar{c}\}.$$

Theorem 3.2 *If $(S_1 \cup S_2 \cup S_3 \cup S_4) \cap (\bar{S}_1 \cup \bar{S}_2 \cup \bar{S}_3) \neq \emptyset$, then system (1.4) has at least one positive ω periodic solution, namely $(x^*(t), y^*(t))$.*

Proof We make the change of variables:

$$x(t) = \exp\{\tilde{x}(t)\}, \qquad y(t) = \exp\{\tilde{y}(t)\}.$$

Then system (1.4) becomes

$$\begin{cases} \tilde{x}'(t) = 1 - \exp\{\tilde{x}(t)\} - \frac{\alpha(t)\exp\{\tilde{y}(t)\}}{(\exp\{\tilde{x}(t)\})^2 + \alpha(t)\exp\{\tilde{x}(t)\} + \alpha(t)\gamma(t)} - \frac{h(t)}{\exp\{\tilde{x}(t)\} + c(t)}, \\ \tilde{y}'(t) = \delta(t)(1 - \beta(t)\frac{\exp\{\tilde{y}(t)\}}{\exp\{\tilde{x}(t)\}}). \end{cases} \tag{3.1}$$

We denote

$$\mathcal{X} = \mathcal{Y} = \{(\tilde{x},\tilde{y}) \in C(\mathbb{R},\mathbb{R}^2) | \tilde{x}(t+\omega) = \tilde{x}, \tilde{y}(t+\omega) = \tilde{y}\},$$
$$\|(\tilde{x},\tilde{y})\| = \max_{t\in[0,\omega]}(|\tilde{x}(t)| + |\tilde{y}(t)|), \quad (\tilde{x},\tilde{y}) \in \mathcal{X} \text{ (or } \mathcal{Y}).$$

Clearly, $\mathcal{X}$ and $\mathcal{Y}$ are Banach spaces. Let

$$N\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix} 1 - \exp\{\tilde{x}(t)\} - \frac{\alpha(t)\exp\{\tilde{y}(t)\}}{(\exp\{\tilde{x}(t)\})^2 + \alpha(t)\exp\{\tilde{x}(t)\} + \alpha(t)\gamma(t)} - \frac{h(t)}{\exp\{\tilde{x}(t)\} + c(t)} \\ \delta(t)(1 - \beta(t)\frac{\exp\{\tilde{y}(t)\}}{\exp\{\tilde{x}(t)\}}) \end{bmatrix},$$
$$L\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix}\tilde{x}'\\ \tilde{y}'\end{bmatrix}, \qquad P\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = Q\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix}\frac{1}{\omega}\int_0^\omega \tilde{x}(t)\,dt \\ \frac{1}{\omega}\int_0^\omega \tilde{y}(t)\,dt\end{bmatrix}.$$

We easily see that the inverse $Kp : \operatorname{Im} L \to \operatorname{Dom} L \cap \ker P$ exists, and a simple computation leads to

$$QN\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix}\frac{1}{\omega}\int_0^\omega [1 - \exp\{\tilde{x}(t)\} - \frac{\alpha(t)\exp\{\tilde{y}(t)\}}{(\exp\{\tilde{x}(t)\})^2 + \alpha(t)\exp\{\tilde{x}(t)\} + \alpha(t)\gamma(t)} - \frac{h(t)}{\exp\{\tilde{x}(t)\} + c(t)}]\,dt \\ \frac{1}{\omega}\int_0^\omega [\delta(t)(1 - \beta(t)\frac{\exp\{\tilde{y}(t)\}}{\exp\{\tilde{x}(t)\}})]\,dt\end{bmatrix}$$

and

$$Kp(I-Q)N\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix}\int_0^t N_1(s)\,ds - \frac{1}{\omega}\int_0^\omega \int_0^t N_1(s)\,ds\,dt - (\frac{t}{\omega} - \frac{1}{2})\int_0^\omega N_1(s)\,ds \\ \int_0^t N_2(s)\,ds - \frac{1}{\omega}\int_0^\omega \int_0^t N_2(s)\,ds\,dt - (\frac{t}{\omega} - \frac{1}{2})\int_0^\omega N_2(s)\,ds\end{bmatrix}.$$

Also, it is easy to prove that N is L-compact on $\overline{\Omega}$ with any open bounded set $\Omega \subset X$. Now we find an appropriate open bounded subset Ω for the application of the continuation theorem of [24, 37]. According to the equation $Lx = \lambda Nx, \lambda \in (0,1)$, we get

$$\begin{cases} \tilde{x}'(t) = \lambda[1 - \exp\{\tilde{x}(t)\} - \frac{\alpha(t)\exp\{\tilde{y}(t)\}}{(\exp\{\tilde{x}(t)\})^2+\alpha(t)\exp\{\tilde{x}(t)\}+\alpha(t)\gamma(t)} - \frac{h(t)}{\exp\{\tilde{x}(t)\}+c(t)}], \\ \tilde{y}'(t) = \lambda[\delta(t)(1-\beta(t)\frac{\exp\{\tilde{y}(t)\}}{\exp\{\tilde{x}(t)\}})]. \end{cases} \tag{3.2}$$

Assume that $(\tilde{x}(t),\tilde{y}(t))$ is an arbitrary solution of system (3.1) with certain $\lambda \in (0,1)$. Integration on both sides of system (3.2) over the interval $[0,\omega]$ leads to

$$\begin{cases} \omega = \int_0^\omega [\exp\{\tilde{x}(t)\} + \frac{\alpha(t)\exp\{\tilde{y}(t)\}}{(\exp\{\tilde{x}(t)\})^2+\alpha(t)\exp\{\tilde{x}(t)\}+\alpha(t)\gamma(t)} + \frac{h(t)}{\exp\{\tilde{x}(t)\}+c(t)}]\,dt, \\ \bar{\delta}\omega = \int_0^\omega [\delta(t)\beta(t)\frac{\exp\{\tilde{y}(t)\}}{\exp\{\tilde{x}(t)\}}]\,dt. \end{cases} \tag{3.3}$$

According to system (3.2) and (3.3), we get

$$\begin{cases} \int_0^\omega |\tilde{x}'(t)|\,dt \\ \quad \le \lambda[\int_0^\omega 1\,dt + \int_0^\omega \exp\{\tilde{x}(t)\}\,dt + \int_0^\omega \frac{\alpha(t)\exp\{\tilde{y}(t)\}}{(\exp\{\tilde{x}(t)\})^2+\alpha(t)\exp\{\tilde{x}(t)\}+\alpha(t)\gamma(t)}\,dt \\ \qquad + \int_0^\omega \frac{h(t)}{\exp\{\tilde{x}(t)\}+c(t)}\,dt] \\ \quad < 2\omega, \\ \int_0^\omega |\tilde{y}'(t)|\,dt \\ \quad \le \lambda[\int_0^\omega \delta(t)\,dt + \int_0^\omega \delta(t)\beta(t)\frac{\exp\{\tilde{y}(t)\}}{\exp\{\tilde{x}(t)\}}\,dt] \\ \quad < 2\bar{\delta}\omega. \end{cases} \tag{3.4}$$

Since $(\tilde{x}(t),\tilde{y}(t)) \in \mathcal{X}$, we know that there exist ξ_i and $\eta_i \in [0,\omega], i = 1,2$, such that

$$\begin{aligned} &\tilde{x}(\xi_1) = \min_{t\in[0,\omega]} \tilde{x}(t), \qquad \tilde{x}(\eta_1) = \max_{t\in[0,\omega]} \tilde{x}(t), \\ &\tilde{y}(\xi_2) = \min_{t\in[0,\omega]} \tilde{y}(t), \qquad \tilde{y}(\eta_2) = \max_{t\in[0,\omega]} \tilde{y}(t). \end{aligned} \tag{3.5}$$

According to the first equation of system (3.3), we have

$$\begin{aligned} &\omega \ge \int_0^\omega \exp\{\tilde{x}(\xi_1)\}\,dt - \int_0^\omega \frac{h(t)}{c(t)}\,dt = \exp\{\tilde{x}(\xi_1)\}\omega - \frac{\bar{h}}{\bar{c}}\omega, \\ &\tilde{x}(\xi_1) \le \ln\left\{1 + \frac{\bar{h}}{\bar{c}}\right\}. \end{aligned}$$

From systems (3.4) and (3.5), we obtain

$$\tilde{x}(t) \le \tilde{x}(\xi_1) + \int_0^\omega \left|\tilde{x}'(t)\right|dt < \ln\left\{1 + \frac{\bar{h}}{\bar{c}}\right\} + 2\omega := H_1. \tag{3.6}$$

According to system (3.5) and the second equation of system (3.3), we have

$$\begin{aligned} &\bar{\delta}\omega \ge \int_0^\omega \beta(t)\delta(t)\frac{\exp\{\tilde{y}(\xi_2)\}}{\exp\{H_1\}}\,dt = \bar{\beta}\bar{\delta}\frac{\exp\{\tilde{y}(\xi_2)\}}{\exp\{H_1\}}\omega, \\ &\tilde{y}(\xi_2) \le \ln\left\{\frac{\exp\{H_1\}}{\bar{\beta}}\right\}, \end{aligned}$$

and hence,

$$\tilde{y}(t) \le \tilde{y}(\xi_2) + \int_0^\omega \left|\tilde{y}'(t)\right| dt < \ln\left\{\frac{\exp\{H_1\}}{\bar{\beta}}\right\} + 2\bar{\delta}\omega := H_2. \tag{3.7}$$

From the first equation of system (3.3), we also obtain

$$\begin{aligned}\omega &\le \int_0^\omega \left[\exp\left\{\tilde{x}(\eta_1)\right\} + \frac{4\alpha(t)\exp\{H_2\}}{4\alpha(t)\gamma(t) - \{\alpha(t)\}^2} + \frac{h(t)}{c(t)}\right] dt \\ &= \exp\left\{\tilde{x}(\eta_1)\right\}\omega + \frac{4\exp\{H_2\}}{4\gamma^l - \alpha^u}\omega + \frac{\bar{h}}{\bar{c}}\omega,\end{aligned}$$

and therefore,

$$\begin{aligned}\exp\left\{\tilde{x}(\eta_1)\right\} &\ge 1 - \frac{4\exp\{H_2\}}{4\gamma^l - \alpha^u} - \frac{\bar{h}}{\bar{c}} \\ &= \frac{4\gamma^l\bar{c} - \alpha^u\bar{c} - 4\exp\{H_2\}\bar{c} - \bar{h}(4\gamma^l - \alpha^u)}{\bar{c}(4\gamma^l - \alpha^u)} \\ &= \frac{(\bar{c} - \bar{h})(4\gamma^l - \alpha^u) - 4\exp\{H_2\}\bar{c}}{\bar{c}(4\gamma^l - \alpha^u)},\end{aligned}$$

which implies

$$\tilde{x}(\eta_1) \ge \ln\left\{\frac{(\bar{c} - \bar{h})(4\gamma^l - \alpha^u) - 4\exp\{H_2\}\bar{c}}{\bar{c}(4\gamma^l - \alpha^u)}\right\}.$$

Thus,

$$\tilde{x}(t) \ge \tilde{x}(\eta_1) - \int_0^\omega \left|\tilde{x}'(t)\right| dt > \ln\left\{\frac{(\bar{c} - \bar{h})(4\gamma^l - \alpha^u) - 4\exp\{H_2\}\bar{c}}{\bar{c}(4\gamma^l - \alpha^u)}\right\} - 2\omega := H_3. \tag{3.8}$$

The second equation of system (3.3) also produces

$$\begin{aligned}&\bar{\delta}\omega \le \int_0^\omega \beta(t)\delta(t)\frac{\exp\{\tilde{y}(\eta_2)\}}{\exp\{H_3\}}\, dt = \beta\delta\frac{\exp\{\tilde{y}(\xi_2)\}}{\exp\{H_3\}}\omega, \\ &\tilde{y}(\eta_2) \ge \ln\left\{\frac{\exp\{H_3\}}{\bar{\beta}}\right\};\end{aligned}$$

and consequently,

$$\tilde{y}(t) \ge \tilde{y}(\eta_2) - \int_0^\omega \left|\tilde{y}'(t)\right| dt > \ln\left\{\frac{\exp\{H_3\}}{\bar{\beta}}\right\} - 2\bar{\delta}\omega := H_4. \tag{3.9}$$

It follows from (3.6)–(3.9) that

$$\begin{cases}\max_{t\in[0,\omega]} |\tilde{x}(t)| \le \max\{|H_1|, |H_3|\} := C_1, \\ \max_{t\in[0,\omega]} |\tilde{y}(t)| \le \max\{|H_2|, |H_4|\} := C_2.\end{cases} \tag{3.10}$$

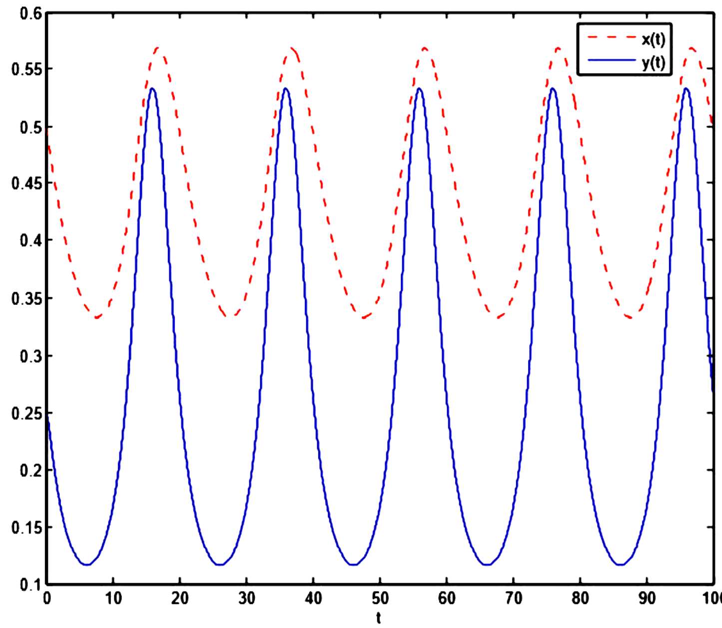

Figure 1 20-periodic solution

We choose $C > 0$ such that $C > C_1 + C_2$. Let $\Omega = \{(\tilde{x}, \tilde{y}) \in X \mid \|(\tilde{x}, \tilde{y})\| < C\}$. Then it is easy to verify that requirement (1) in the continuation theorem of [24, 37] is satisfied. Also,

$$QN\begin{bmatrix}\tilde{x}\\ \tilde{y}\end{bmatrix} = \begin{bmatrix} 1 - \frac{1}{\omega}\int_0^\omega \exp\{\tilde{x}(t)\}\,dt - \frac{1}{\omega}\int_0^\omega \frac{\alpha(t)\exp\{\tilde{y}(t)\}}{(\exp\{\tilde{x}(t)\})^2 + \alpha(t)\exp\{\tilde{x}(t)\} + \alpha(t)\gamma(t)}\,dt \\ -\frac{1}{\omega}\int_0^\omega \frac{h(t)}{\exp\{\tilde{x}(t)\}+c(t)}\,dt, \\ \frac{1}{\omega}\int_0^\omega \delta(t)\,dt - \frac{1}{\omega}\int_0^\omega \delta(t)\beta(t)\frac{\exp\{\tilde{y}(t)\}}{\exp\{\tilde{x}(t)\}}\,dt \end{bmatrix} \neq \begin{bmatrix}0\\0\end{bmatrix}.$$

In addition, we have $\deg\{JQN, \Omega \cap \operatorname{Ker} L, 0\} \neq 0$. Thus all the conditions in the continuation theorem are satisfied (see, e.g., [24, 37]). Hence, system (3.1) has at least one ω periodic solution $(\tilde{x}^*(t), \tilde{y}^*(t))$. It is easy to see that $x^*(t) = \exp\{\tilde{x}^*(t)\}, y^*(t) = \exp\{\tilde{y}^*(t)\}$, and then $(x^*(t), y^*(t))$ is an ω periodic solution of system (1.4). The proof of Theorem 3.2 is complete. □

4 Numerical simulations

To support the previous theoretical analysis, in this section, we present two numerical simulation results for the different coefficients of system (1.4).

Example 1 Consider the following model:

$$\begin{cases} \frac{dx}{dt} = x(1-x) - \frac{xy}{\frac{x^2}{[2+\sin(0.1\pi t)]} + x + [101+\sin(0.1\pi t)]} - \frac{[2+\sin(0.1\pi t)]x}{x+[3+\sin(0.1\pi t)]}, \\ \frac{dy}{dt} = [3+\sin(0.1\pi t)]y(1-[2+\sin(0.1\pi t)]\frac{y}{x}). \end{cases} \tag{4.1}$$

It is easy to verify that the coefficients of system (1.4) satisfy the conditions in Theorem 3.2. Thus, system (1.4) has a 20-*periodic solution*. Figure 1 shows the validity of our results.

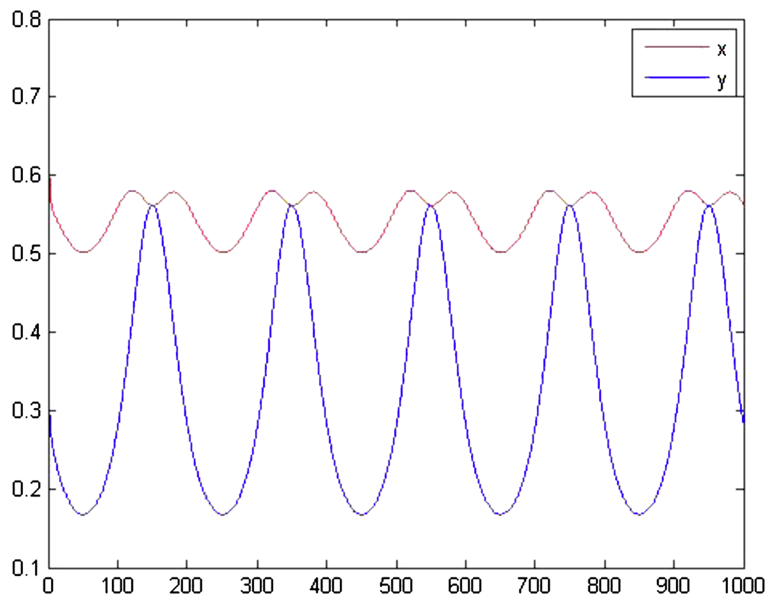

Figure 2 200-periodic solution

Example 2 Consider the following model:

$$\begin{cases} \frac{dx}{dt} = x(1-x) - \frac{xy}{\frac{x^2}{[100+\sin(0.01\pi t)]}+x+[3+\sin(0.01\pi t)]} - \frac{[2+\sin(0.01\pi t)]x}{x+[5+\sin(0.01\pi t)]}, \\ \frac{dy}{dt} = [3+\sin(0.01\pi t)]y(1-[2+\sin(0.01\pi t)]\frac{y}{x}). \end{cases} \tag{4.2}$$

It is easy to verify that the coefficients of system (1.4) satisfy the conditions in Theorem 3.2. Thus, system (1.4) has a 200-*periodic solution*. Figure 2 shows the validity of our results.

5 Conclusions

This paper considers a non-autonomous modified Leslie–Gower model with Holling type IV functional response and nonlinear prey harvesting. We study the permanence of the model. Sufficient conditions are obtained for the existence of a periodic solution by Brouwer fixed point theorem and coincidence degree theory, respectively. Also, we give examples and simulations to verify our theoretical analysis.

Funding

This work was supported by the National Natural Science Foundation of China (No. 11671176), the Natural Science Foundation of Fujian Province (No. 2018J01001), the Start-up Fund of Huaqiao University (Z16J0039), China Postdoctoral Science Foundation (No. 2014M551873), and Subsidized Project for Postgraduates' Innovative Fund in Scientific Research of Huaqiao University (18011020008).

Availability of data and materials

All data are fully available without restriction.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors have the same contributions. All authors read and approved the final manuscript.

Author details
[1]School of Economics and Finance, Huaqiao University, Quanzhou, China. [2]School of Mathematical Sciences, Huaqiao University, Quanzhou, China. [3]Department of Mathematics, Zhejiang Normal University, Jinhua, China. [4]School of Mathematical Sciences, Qufu Normal University, Qufu, China. [5]School of Mathematics, Statistics and Applied Mathematics, National University of Ireland, Galway, Ireland.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 14 February 2019 Accepted: 14 June 2019 Published online: 24 July 2019

References

1. Taylor, R.: Predation. Chapman and Hall, New York (1984)
2. Song, Y., Yuan, S.: Bifurcation analysis in a predator-prey system with time delay. Nonlinear Anal., Real World Appl. **7**(2), 265–284 (2006)
3. Ruan, S., Xiao, D.: Global analysis in a predator-prey system with nonmonotonic functional response. SIAM J. Appl. Math. **61**, 1445–1472 (2001)
4. Huang, J., Xiao, D.: Analysis of bifurcations and stability in a predator-prey system with Holling type-IV functional response. Acta Math. Appl. Sin. Engl. Ser. **20**, 167–178 (2004)
5. Xiao, D., Ruan, S.: Multiple bifurcations in a delayed predator-prey system with nonmonotonic functional response. J. Differ. Equ. **176**, 494–510 (2001)
6. Xue, Y., Duan, X.: The dynamic complexity of a Holling type-IV predator-prey system with stage structure and double delays. Discrete Dyn. Nat. Soc. **2011**, Article ID 509871 (2011)
7. Chen, Y.: Multiple periodic solution of delayed predator-prey systems with type IV functional responses. Nonlinear Anal., Real World Appl. **5**(1), 45–53 (2004)
8. Xia, Y., Cao, J., Cheng, S.: Multiple periodic solutions of a delayed stage-structured predator-prey model with non-monotone functional responses. Appl. Math. Model. **31**(9), 1947–1959 (2007)
9. Li, Z., Zhao, K., Li, Y.: Multiple positive periodic solutions for a non-autonomous stage-structured predatory-prey system with harvesting terms. Commun. Nonlinear Sci. Numer. Simul. **15**(8), 2140–2148 (2010)
10. Wang, Q., Dai, B., Chen, Y.: Multiple periodic solutions of an impulsive predator-prey model with Holling-type IV functional response. Math. Comput. Model. **49**(9–10), 1829–1836 (2009)
11. Leslie, P., Gower, J.: The properties of a stochastic model for the predator-prey type of interaction between two species. Biometrika **47**, 219–234 (1960)
12. Korobeinikov, A.: A Lyapunov function for Leslie–Gower predator-prey models. Appl. Math. Lett. **14**(6), 697–699 (2001)
13. Yu, S.: Global stability of a modified Leslie-Gower model with Beddington–DeAngelis functional response. Adv. Differ. Equ. **2014**, 84 (2014)
14. Upadhyay, R., Iyengar, S.: Introduction to Mathematical Modelling and Chaotic Dynamics. CRC Press, Taylor and Francis Group, London (2013)
15. Zhang, Z., Upadhyay, R., Datta, J.: Bifurcation analysis of a modified Leslie–Gower model with Holling type IV functional response and nonlinear prey harvesting. Adv. Differ. Equ. **2018**, 127 (2018)
16. Huang, C., Zhao, X., Wang, X., Wang, Z., Xiao, M., Cao, J.: Disparate delays-induced bifurcations in a fractional-order neural network. J. Franklin Inst. **356**(5), 2825–2846 (2019)
17. Zhang, B., Zhuang, J., Liu, H., Cao, J., Xia, Y.: Master-slave synchronization of a class of fractional-order Takagi–Sugeno fuzzy neural networks. Adv. Differ. Equ. **2018**, 473 (2018)
18. Huang, C., Cao, J.: Impact of leakage delay on bifurcation in high-order fractional BAM neural networks. Neural Netw. **98**, 223–235 (2018)
19. Huang, C., Li, T., Cai, L., Cao, J.: Novel design for bifurcation control in a delayed fractional dual congestion model. Phys. Lett. A **383**(5), 440–445 (2019)
20. Yu, X., Yuan, S., Zhang, T.: Survival and ergodicity of a stochastic phytoplankton-zooplankton model with toxin-producing phytoplankton in an impulsive polluted environment. Appl. Math. Comput. **347**(15), 249–264 (2019)
21. Xu, C., Yuan, S., Zhang, T.: Average break-even concentration in a simple chemostat model with telegraph noise. Nonlinear Anal. Hybrid Syst. **29**, 373–382 (2018)
22. Zhang, T., Liu, X., Meng, X., Zhang, T.: Spatio-temporal dynamics near the steady state of a planktonic system. Comput. Math. Appl. **75**(12), 4490–4504 (2018)
23. Huang, C., Li, H., Cao, J.: A novel strategy of bifurcation control for a delayed fractional predator-prey model. Appl. Math. Comput. **347**, 808–838 (2019)
24. Fan, M., Wang, Q., Zou, X.: Dynamics of a non-autonomous ratio-dependent predator-prey system. Proc. R. Soc. Edinb. A **133**, 97–118 (2003)
25. Liu, B., Teng, Z., Chen, L.: Analysis of a predator-prey model with Holling II functional response concerning impulsive control strategy. J. Comput. Appl. Math. **193**(1), 347–362 (2006)
26. Meng, X., Zhao, S., Feng, T., Zhang, T.: Dynamics of a novel nonlinear stochastic SIS epidemic model with double epidemic hypothesis. J. Math. Anal. Appl. **433**(1), 227–242 (2016)
27. Xia, Y.: Periodic solution of certain nonlinear differential equations: via topological degree theory and matrix spectral theory. Int. J. Bifurc. Chaos **22**(8), 940287 (2012)
28. Nie, L., Teng, Z., Hu, L., Peng, J.: The dynamics of a Lotka–Volterra predator-prey model with state dependent impulsive harvest for predator. Biosystems **98**, 67–72 (2009)
29. Song, J., Hu, M., Bai, Y., Xia, Y.: Dynamic analysis of a non-autonomous ratio-dependent predator-prey model with additional food. J. Appl. Anal. Comput. **8**(6), 1893–1909 (2018)
30. Song, Y., Tang, X.: Stability, steady-state bifurcations, and turing patterns in a predator-prey model with herd behavior and prey-taxis. Stud. Appl. Math. **139**(3), 371–404 (2017)

31. Wei, F.: Existence of multiple positive periodic solutions to a periodic predator-prey system with harvesting terms and Holling III type functional response. Commun. Nonlinear Sci. Numer. Simul. **16**(4), 2130–2138 (2011)
32. Wei, F., Fu, Q.: Hopf bifurcation and stability for predator-prey systems with Beddington–DeAngelis type functional response and stage structure for prey incorporating refuge. Appl. Math. Model. **40**(1), 126–134 (2016)
33. Yuan, S., Xu, C., Zhang, T.: Spatial dynamics in a predator-prey model with herd behavior. Chaos **23**, 033102 (2013)
34. Zhang, T., Zhang, T., Meng, X.: Stability analysis of a chemostat model with maintenance energy. Appl. Math. Lett. **68**, 1–7 (2017)
35. Bai, Y., Li, Y.: Stability and Hopf bifurcation for a stage-structured predator-prey model incorporating refuge for prey and additional food for predator. Adv. Differ. Equ. **2019**, 42 (2019)
36. Zhang, Z., Hou, Z.: Existence of four positive periodic solutions for a ratio-dependent predator-prey system with multiple exploited (or harvesting) terms. Nonlinear Anal., Real World Appl. **11**(3), 1560–1571 (2010)
37. Gaines, R., Mawhin, J.: Coincidence Degree and Nonlinear Differential Equations. Springer, Berlin (1977)
38. Zhang, T., Liu, J., Teng, Z.: Existence of positive periodic solutions of an SEIR model with periodic coefficients. Appl. Math. **57**(6), 601–616 (2012)
39. Zheng, H., Guo, L., Bai, Y., Xia, Y.: Periodic solutions of a non-autonomous predator-prey system with migrating prey and disease infection: via Mawhin's coincidence degree theory. J. Fixed Point Theory Appl. **21**, 37 (2019)