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Spreading dynamics of a preferential information model with hesitation psychology on scale-free networks

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Abstract

Considering the influence of the preferential degree, the hesitation psychology of customers and the heterogeneity of underlying networks in preferential information spreading, we propose a novel model called *AHFB* (adherent–hesitator–forwarder–beneficiary) model to illustrate the dynamic behaviors of preferential information spreading on scale-free networks. The mean-field theory is adopted to describe the formulas of *AHFB* model. To begin with, we analyze the spreading dynamics of preferential information. Then we determine the basic reproductive number and equilibria by the next generation matrix method. The relationship among the basic reproductive number, preferential degree, and hesitation parameter is also analyzed. In addition, the globally asymptotical stability of information-eliminate equilibrium and the permanence of the preferential information spreading are proved in detail. Furthermore, a preferential information competition model is proposed, and the corresponding dynamic behaviors are studied. We have found that under certain conditions, a greater competitive advantage can be achieved by improving a certain range of preferential strengths. Numerical simulations are also presented to verify and extend theoretical results.

Keywords: Attractivity parameter; Preferential information spread model; Heterogeneity; Scale-free networks; Hesitation psychology

1 Introduction

With the continuous prosperity of mobile Internet technology and the appearance of the 5G, e-commerce technology is rapidly developing and multifarious applications are emerging. As a kind of Internet networks, e-commerce networks not only play an important role in spreading information and promoting communication, but also penetrate people's daily life, such as convenient shopping, mobile payment [1–3]. In most instances, e-commerce networks provide businesses with a platform to release preferential information and display merchandises [4, 5]. In order to obtain discounts or small gifts from the store during the shopping process, customers need to forward the corresponding preferential information [6]. Therefore, the preferential information will be spread widely.

In the field of complex networks, researchers have made some achievements in information spreading [7–14] and network control [15–19]. Based on the characteristics of network structure, the effects of different network topologies, such as small world net-

works, random networks, scale-free networks, community structure networks, and multiplex networks, on information spreading models are studied in detail. The authors of [20] described information spreading by ODE (ordinary differential equation) dynamic system and built a linear algebra model in small world networks. They studied the relationship between the ODE function groups and the graph topology, and also found the rule of the information's distribution in networks. Lim et al. [21] studied the influence of clustering coefficients under the *SIR* model with heterogeneous contact rates and provided a novel iterative algorithm to estimate the conditions and sizes of global cascades. In order to study the influence of human subjective value and psychological status on information dissemination, Liu et al. [22] proposed a novel social network information dissemination with negative feedback *NFSIR* (negative feedback–susceptible–infected–removed) model on scale-free networks, which showed that the intensity of information feedback has a significant impact on the process of information dissemination. The authors of [23, 24] presented a privacy protection and emotional behavior information spreading model with a community structure on social networks. Zhang et al. [25, 26] proposed a node measurement augmented system model in multirate systems networks with dynamic quantization. Recently, scholars have begun to take serious consideration about the role of human behavior and the multiplex networks structure in information spreading [27–29]. The interplay between the epidemic spreading and the diffusion of awareness in multiplex networks was analyzed in [30, 31].

Another generalization of the initial simple determinacy is to focus on different models and spreading mechanisms. The typical information behavior in social networks is a kind of information sharing and interaction. Ally et al. [32] proposed two rewiring *SIR* (susceptible–infected–removed) models on information spreading in scale-free and small world networks. In order to study the effect of hesitation mechanism, the authors of [33, 34] proposed an *SEIR* (susceptible–exposed–infected–removed) model by introducing the exposed nodes between the ignorant nodes and the spread nodes in heterogeneity networks, they found that hesitant individuals have a very important influence on the spreading of information. To further investigate the influence of heterogeneity of the underlying complex networks and quarantine strategy, an *SIQRS* (susceptible–infected–quarantined–recovered–susceptible) epidemic model on the scale-free networks has been proposed [35, 36].

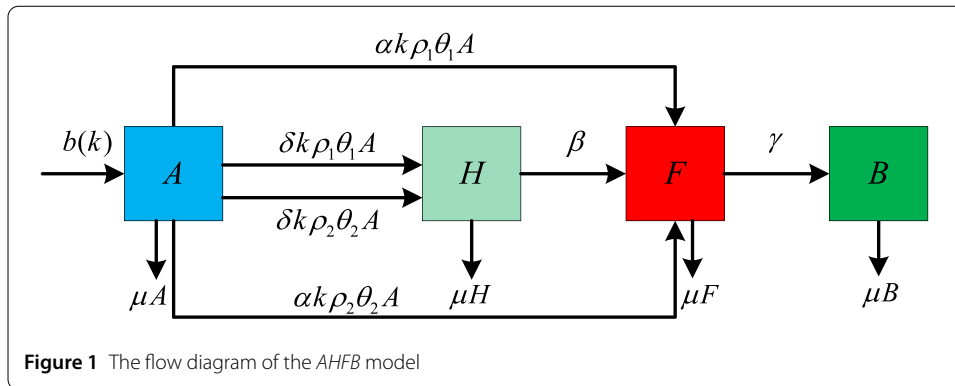
It has been found that most real networks are actually scale-free networks, so the scale-free property is a particularly important one in social networks [37, 38]. To further understand the information spreading dynamics in real world, the scale-free property of social networks has been taken into account by many information spreading models [39–41]. In networks, nodes represent individual and edges represent the relationship of people, as for preferential spreading dynamics, that relationship is contacted to forward the preferential information. With the rapid development of social networks and new media, increasingly many business groups choose social networks to promote their products by publishing preferential information. Hence, it is of great importance to study the spreading dynamics of preferential information on scale-free networks. Due to the influences of the merchandise's preferential degree, personal economic level, psychological and other factors, some customers will not immediately forward the preferential information after they know it in the real social life, and they will become hesitant at first. However, in the literature on preferential information spreading, some researchers ignore the hesitating mechanism,

which will affect people's attitudes and behaviors on preferential information and further influence its spreading. People in the hesitating state will read some comments or opinions about the products because of the attractiveness of preferential information, which may make one enter the forwarder status to forward the preferential information with a certain probability. An *FSFC* (follower–super-forwarder–client) preferential information model was proposed by Fu et al. [42] in online social networks, who found that different infection rates have a great impact on large- and small-degree node forwarders. Wan et al. [43] proposed an *SIB* (susceptible–infected–beneficial) model based on scale-free networks, and they further studied the influence of the preferential degree and the heterogeneity of underlying networks on the spread of preferential e-commerce information. However, the influence of the parameters in the model on the basic reproductive number has not been analyzed in detail, and the hesitation psychology of people is also not considered. Motivated by the above, we establish a novel *AHFB* preferential information spreading model with hesitation mechanism on scale-free networks.

The remainder of this paper is organized as follows. Section 2 presents an *AHFB* preferential information spreading model. In Sect. 3, the basic reproductive number and equilibriums are obtained. Section 4 analyzes the globally asymptotic stability of equilibriums and the permanence of preferential information spreading. In Sect. 5, the modified preferential information spreading model with competitive mechanism is introduced and the corresponding dynamical behaviors are studied. In Sect. 6, numerical simulations are presented to illustrate our main results. Finally, we give the discussions and conclusions in Sect. 7.

2 Model formulation

During preferential information spreading in a population, individuals are divided into four categories: adherents, hesitators, forwarders, and beneficiaries. Adherents are individuals who never know the preferential information. Hesitators are individuals who know the preferential information in hesitation state and do not forward preferential information. Forwarders are individuals who know and forward the preferential information. Beneficiaries are individuals who know the preferential information and obtain the preferential policies. Taking the heterogeneity induced by the presence of nodes with different connectivity into account, we let $A_k(t)$, $H_k(t)$, $F_k(t)$, and $B_k(t)$ be the relative densities of adherent, hesitator, forwarder and beneficiary nodes of degree k at time t , respectively. Figure 1 illustrates the spreading rules of the *AHFB* model. We assume that new immigration individuals are adherent at rate $b(k)$, the emigration rate of all individuals is μ . The reconnection of these new immigration nodes follows the below propagation rules. If an adherent is connected to a forwarder, then he or she will be infected to become a hesitator or a forwarder with a probability of $\delta\rho_1$ or $\alpha\rho_1$, respectively. If an adherent is connected to a beneficiary, then he or she will be infected to become a hesitator or a forwarder with a probability of $\delta\rho_2$ or $\alpha\rho_2$, respectively. Here, the parameter δ is used to describe the hesitation degree of adherents, the attractivity parameter α is used to describe the preferential degree of preferential information. A hesitator becomes a forwarder with a probability of β . Parameter $\rho_1(\rho_2)$ is the spread rate for an adherent after contacting with a forwarder (beneficiary). A forwarder becomes a beneficiary with a probability of γ when he or she forwards the preferential information a specific number of times.



Based on the above assumptions and conditions of hypothesis, the AHFB preferential information spreading dynamics model can be described by

$$\begin{cases} \frac{dA_k(t)}{dt} = b(k) - \alpha k(\rho_1\theta_1(t) + \rho_2\theta_2(t))A_k(t) - \delta k(\rho_1\theta_1(t) + \rho_2\theta_2(t))A_k(t) - \mu A_k(t), \\ \frac{dH_k(t)}{dt} = \delta k(\rho_1\theta_1(t) + \rho_2\theta_2(t))A_k(t) - \beta H_k(t) - \mu H_k(t), \\ \frac{dF_k(t)}{dt} = \alpha k(\rho_1\theta_1(t) + \rho_2\theta_2(t))A_k(t) + \beta H_k(t) - \gamma F_k(t) - \mu F_k(t), \\ \frac{dB_k(t)}{dt} = \gamma F_k(t) - \mu B_k(t), \end{cases} \quad (2.1)$$

where $\theta_1(t), \theta_2(t)$ denote the probabilities of an adherent contacts with a forwarder or a beneficiary at time t , respectively, which satisfies the relations

$$\begin{cases} \theta_1(t) = \sum_{i=1}^n \frac{\varphi(i)}{i} P(i|k) \frac{F_i(t)}{N_i(t)} = \langle k \rangle^{-1} \sum_{i=1}^n \frac{\varphi(i)}{\eta_i} P(i) F_i(t), \\ \theta_2(t) = \sum_{i=1}^n \frac{\varphi(i)}{i} P(i|k) \frac{B_i(t)}{N_i(t)} = \langle k \rangle^{-1} \sum_{i=1}^n \frac{\varphi(i)}{\eta_i} P(i) B_i(t). \end{cases} \quad (2.2)$$

Here, the factor $1/i$ stands for the probability that one of the forwarder or beneficiary neighbors of a node with degree i will contact this node at the present time step; $P(i|k)$ denotes the conditional probability that a node with degree k is connected to a node with degree i . In this paper, $P(i|k) = iP(k)/\langle k \rangle$ and $\sum_{k=1}^n P(k) = 1$; $\langle k \rangle = \sum_{k=1}^n kP(k)$ denotes the average degree. The function $\varphi(t)$ is the forwarder node with degree k , and $\rho(t) = \rho_1\theta_1(t) + \rho_2\theta_2(t)$. And $N_k(t)$ stands for the total number of nodes with degree k at time t , which satisfies

$$N_k(t) = A_k(t) + H_k(t) + F_k(t) + B_k(t). \quad (2.3)$$

It is easy to see that $dN_k(t)/dt = b(k) - \mu N_k(t)$, so $N_k(t) = (N_0 - b(k)/\mu)e^{-\mu t} + b(k)/\mu$, where $N_0 = N_k(0)$, and then $\lim_{t \rightarrow \infty} N_k(t) = b(k)/\mu = \eta_k$. Thus, since system (2.1) monitors human population, it is reasonable to assume that all its state variables and parameters are nonnegative for all $t \geq 0$. It implies that the region

$$\begin{aligned} \Omega = \{A_k(t), H_k(t), F_k(t), B_k(t) \in \mathbb{R}_+^{4n} | A_k(t) + H_k(t) + F_k(t) + B_k(t) \leq \eta_k, \\ k = 1, 2, \dots, n\} \end{aligned} \quad (2.4)$$

is the positively invariant set for (2.1). Thus, each solution of system (2.1) with initial conditions and the limit sets are contained in Ω in this paper.

Since $A_k(t) = \eta_k - H_k(t) - F_k(t) - B_k(t)$ is at steady-state, it is sufficient to study the limiting system

$$\begin{cases} \frac{dH_k(t)}{dt} = \delta k(\rho_1\theta_1(t) + \rho_2\theta_2(t))(\eta_k - H_k(t) - F_k(t) - B_k(t)) \\ \quad - \beta H_k(t) - \mu H_k(t), \\ \frac{dF_k(t)}{dt} = \alpha k(\rho_1\theta_1(t) + \rho_2\theta_2(t))(\eta_k - H_k(t) - F_k(t) - B_k(t)) \\ \quad + \beta H_k(t) - \gamma F_k(t) - \mu F_k(t), \\ \frac{dB_k(t)}{dt} = \gamma F_k(t) - \mu B_k(t). \end{cases} \quad (2.5)$$

3 The basic reproductive number and existence of equilibriums

3.1 The basic reproductive number R_0

System (2.5) has an information-elimination equilibrium $E_0\{(0, 0, 0)\}_k$. Using the next generation method in [44], system (2.5) can be written as

$$\frac{dx}{dt} = M(x) - V(x),$$

where

$$\begin{aligned} x &= (H_k, F_k, B_k)^T, \\ M(x) &= \begin{pmatrix} \delta k \rho(t)(\eta_k - H_k - F_k - B_k) \\ \alpha k \rho(t)(\eta_k - H_k - F_k - B_k) \\ 0 \end{pmatrix}, \\ V(x) &= \begin{pmatrix} (\beta + \mu)H_k \\ -\beta H_k + (\gamma + \mu)F_k \\ \mu B_k - \gamma F_k \end{pmatrix}. \end{aligned}$$

The Jacobian matrices of $M(x)$ and $V(x)$ at the information-elimination equilibrium E_0 are as follows:

$$M(x) = DM(E_0) = \begin{pmatrix} 0 & M_{12} & M_{13} \\ 0 & M_{22} & M_{23} \\ 0 & 0 & 0 \end{pmatrix}, \quad V(x) = DV(E_0) = \begin{pmatrix} V_{11} & 0 & 0 \\ V_{21} & V_{22} & 0 \\ 0 & V_{32} & V_{33} \end{pmatrix}, \quad (3.1)$$

where

$$M_{12} = \frac{\delta \eta_k \rho_1}{\langle k \rangle} \begin{pmatrix} \frac{\varphi(1)}{\eta_1} P(1) & \frac{\varphi(2)}{\eta_1} P(2) & \dots & \frac{\varphi(n)}{\eta_1} P(n) \\ \frac{\varphi(1)}{\eta_2} 2P(1) & \frac{\varphi(2)}{\eta_2} 2P(2) & \dots & \frac{\varphi(n)}{\eta_2} 2P(n) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\varphi(1)}{\eta_n} nP(1) & \frac{\varphi(2)}{\eta_n} 2nP(2) & \dots & \frac{\varphi(n)}{\eta_n} nP(n) \end{pmatrix}, \quad (3.2)$$

$$M_{13} = \frac{\delta \eta_k \rho_2}{\langle k \rangle} \begin{pmatrix} \frac{\varphi(1)}{\eta_1} P(1) & \frac{\varphi(2)}{\eta_1} P(2) & \dots & \frac{\varphi(n)}{\eta_1} P(n) \\ \frac{\varphi(1)}{\eta_2} 2P(1) & \frac{\varphi(2)}{\eta_2} 2P(2) & \dots & \frac{\varphi(n)}{\eta_2} 2P(n) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\varphi(1)}{\eta_n} nP(1) & \frac{\varphi(2)}{\eta_n} 2nP(2) & \dots & \frac{\varphi(n)}{\eta_n} nP(n) \end{pmatrix}, \quad (3.3)$$

$$M_{22} = \frac{\alpha \eta_k \rho_1}{\langle k \rangle} \begin{pmatrix} \frac{\varphi(1)}{\eta_1} P(1) & \frac{\varphi(2)}{\eta_1} P(2) & \dots & \frac{\varphi(n)}{\eta_1} P(n) \\ \frac{\varphi(1)}{\eta_2} 2P(1) & \frac{\varphi(2)}{\eta_2} 2P(2) & \dots & \frac{\varphi(n)}{\eta_2} 2P(n) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\varphi(1)}{\eta_n} nP(1) & \frac{\varphi(2)}{\eta_n} 2nP(2) & \dots & \frac{\varphi(n)}{\eta_n} nP(n) \end{pmatrix}, \quad (3.4)$$

$$M_{23} = \frac{\alpha \eta_k \rho_2}{\langle k \rangle} \begin{pmatrix} \frac{\varphi(1)}{\eta_1} P(1) & \frac{\varphi(2)}{\eta_1} P(2) & \dots & \frac{\varphi(n)}{\eta_1} P(n) \\ \frac{\varphi(1)}{\eta_2} 2P(1) & \frac{\varphi(2)}{\eta_2} 2P(2) & \dots & \frac{\varphi(n)}{\eta_2} 2P(n) \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\varphi(1)}{\eta_n} nP(1) & \frac{\varphi(2)}{\eta_n} 2nP(2) & \dots & \frac{\varphi(n)}{\eta_n} nP(n) \end{pmatrix}, \quad (3.5)$$

here $V_{11} = (\beta + \mu)I$, $V_{21} = -\beta I$, $V_{22} = (\gamma + \mu)I$, $V_{32} = -\gamma I$, $V_{33} = \mu I$, I is the identity matrix, and $\mathbf{0}$ is the zero matrix. The basic reproductive number is denoted by

$$R_0 = \rho(MV^{-1}) = \frac{\langle \varphi(k)k \rangle (\mu \rho_1 + \gamma \rho_2) (\alpha(\mu + \beta) + \beta \delta)}{\langle k \rangle \mu (\gamma + \mu) (\beta + \mu)},$$

where $\langle \varphi(k)k \rangle = \sum_{i=1}^n k \varphi(k) P(k)$, $i = 1, 2, \dots, n$.

3.2 Existence of equilibriums

Theorem 1 Consider system (2.1) and define $R_0 = \frac{\langle \varphi(k)k \rangle (\alpha(\mu + \beta) + \beta \delta) (\mu \rho_1 + \gamma \rho_2)}{\langle k \rangle \mu (\beta + \mu) (\mu + \gamma)}$. There always exists an information-elimination equilibrium $E_0(\eta_k, 0, 0, 0)$ when $R_0 < 1$. When $R_0 > 1$, the system has an information-prevailing equilibrium $E^*(A_k^*, H_k^*, F_k^*, B_k^*)$.

Proof To get the information-prevailing equilibrium solution $E^*(A_k^*, H_k^*, F_k^*, B_k^*)$, we need to make the right-hand side of the system equal to zero. It should satisfy

$$\begin{cases} b(k) - \alpha k A_k^*(t) (\rho_1 \theta_1^*(t) + \rho_2 \theta_2^*(t)) - \delta k A_k^*(t) (\rho_1 \theta_1^*(t) + \rho_2 \theta_2^*(t)) - \mu A_k^*(t) = 0, \\ \delta k A_k^*(t) (\rho_1 \theta_1^*(t) + \rho_2 \theta_2^*(t)) - \beta H_k^*(t) - \mu H_k^*(t) = 0, \\ \alpha k A_k^*(t) (\rho_1 \theta_1^*(t) + \rho_2 \theta_2^*(t)) + \beta H_k^*(t) - \gamma F_k^*(t) - \mu F_k^*(t) = 0, \\ \gamma F_k^*(t) - \mu B_k^*(t) = 0, \end{cases} \quad (3.6)$$

where $\theta_1^* = \langle k \rangle^{-1} \sum_{i=1}^n \frac{\varphi(k)}{\eta_k} P(k) F_k^*(t)$, $\theta_2^* = \langle k \rangle^{-1} \sum_{i=1}^n \frac{\varphi(k)}{\eta_k} P(k) B_k^*(t)$. We obtain from (3.6) that

$$\begin{aligned} A^* &= \frac{(\beta + \mu)(\gamma + \mu)}{k \rho^* (\alpha(\beta + \mu) + \beta \delta)} F^*, \\ H^* &= \frac{\delta(\gamma + \mu)}{\alpha(\beta + \mu) + \beta \delta} F^*, \\ B^* &= \frac{\gamma}{\mu} F^*, \end{aligned} \quad (3.7)$$

where $\rho = \rho_1\theta_1 + \rho_2\theta_2$. From (3.7) we get $\theta_2^* = \gamma\theta_1^*/\mu$. Considering the normalization condition, we obtain

$$F^* = \frac{\mu k \rho^* (\alpha(\beta + \mu) + \delta\beta)\eta_k}{(k\rho^*(\delta + \alpha) + \mu)(\gamma + \mu)(\beta + \mu)}. \quad (3.8)$$

Inserting it into (2.2), we obtain that

$$\theta_1^* = \frac{1}{\langle k \rangle} \sum_{i=1}^n \frac{\varphi(k)}{\eta_k} P(k) \frac{k(\mu\rho_1 + \gamma\rho_2)(\alpha(\beta + \mu) + \delta\beta)\eta_k}{(k\rho^*(\delta + \alpha) + \mu)(\beta + \mu)(\gamma + \mu)} \theta_1^* = f(\theta_1^*). \quad (3.9)$$

Clearly, $\theta_1^* = 0$ satisfies above equation, and if $\theta_1^* = 0$, then $\theta_2^* = 0$ and $H_k^* = F_k^* = B_k^* = 0$, which is an information-elimination equilibrium of (2.1). It is easy to verify that $f(\theta_1^*)$ is a convex and increasing function when $f(\theta_1^*)|_{\theta_1^*=1} < 1$. So if θ_1^* has another solution in $[0, 1]$, it must satisfy

$$\left. \frac{df(\theta_1^*)}{d\theta_1^*} \right|_{\theta_1^*=0} = \frac{\langle \varphi(k)k \rangle}{\langle k \rangle} \frac{(\alpha(\mu + \beta) + \delta\beta)(\mu\rho_1 + \gamma\rho_2)}{\mu(\beta + \mu)(\mu + \gamma)} = R_0 > 1, \quad (3.10)$$

and we can obtain the basic reproductive number

$$R_0 = \frac{\langle \varphi(k)k \rangle}{\langle k \rangle} \frac{(\alpha(\mu + \beta) + \delta\beta)(\mu\rho_1 + \gamma\rho_2)}{\mu(\beta + \mu)(\mu + \gamma)} > 1. \quad (3.11)$$

So, a nontrivial solution exists if and only if $R_0 > 1$. Inserting the nontrivial solution of (3.7) into Eq. (3.6), we obtain F_k^* . Then by (3.7) and (3.8), we can easily get $0 < A_k^*, H_k^*, F_k^*, B_k^* < \eta_k$ for $k = 1, 2, \dots, n$. Thus, the equilibrium $E^*(A_k^*, H_k^*, F_k^*, B_k^*)$ is well-defined. Hence, when $R_0 > 1$, only one positive equilibrium $E^*(A_k^*, H_k^*, F_k^*, B_k^*)$ of system (2.1) exists. The proof is completed. \square

Remark

- (1) The basic reproductive number R_0 depends on some model parameters and fluctuations of the degree distribution. It can be found that R_0 has no correlation with the degree-dependent new immigration individuals $b(k)$. It seems that the attraction parameters α and the infection rate ρ_1, ρ_2 have the same effects, because R_0 will increase when they increase, the effects will be explored by the detailed numerical calculation.
- (2) If $\varphi(k) = k$, $\beta = 0$, and $b(k) = \mu$ (i.e., new immigration individuals are balanced by emigration rate), then the model can be simplified to the network based *SIB* model with $R_0 = \frac{\langle k^2 \rangle}{\langle k \rangle} \frac{\nu(l\beta_1 + \varepsilon\beta_2)}{l(l + \varepsilon)}$, which is investigated in [43].

4 Global dynamics of the model

In this section, qualitative analysis of the model is presented. Firstly, we consider the local asymptotical stability of the information-elimination equilibrium E_0 .

Theorem 2 For system (2.5), the information-elimination equilibrium E_0 is locally asymptotically stable if $R_0 < 1$.

Proof For the purpose of using the comparison theorem to prove the global stability of the information-elimination equilibrium, we assume that $H_1 = z_1, H_2 = z_2, \dots, H_n = z_n, F_1 = z_{n+1}, F_2 = z_{n+2}, \dots, F_n = z_{2n}, B_1 = z_{2n+1}, B_2 = z_{2n+2}, \dots, B_n = z_{3n}$, and $z = (z_1, \dots, z_n, z_{n+1}, \dots, z_{2n}, z_{2n+1}, \dots, z_{3n})^T$, then system (2.5) can be written as

$$\frac{dz}{dt} = Az + N(z),$$

where

$$A = M - V, \quad N(z) = - \begin{pmatrix} \rho(z_1 + z_{n+1} + z_{2n+1}) \\ 2\rho(z_2 + z_{n+2} + z_{2n+2}) \\ \vdots \\ n\rho(z_n + z_{2n} + z_{3n}) \\ (n+1)\rho(z_n + z_{2n} + z_{3n}) \\ \vdots \\ 2n\rho(z_n + z_{2n} + z_{3n}) \\ 0 \\ \vdots \\ 0 \end{pmatrix}_{3n \times 1},$$

the matrices M and V satisfy (3.1); and, as one can see above, $N(z)$ is negative. So $\frac{dz}{dt} \leq Az$. We can find that the off-diagonal elements of matrix A are nonnegative, hence, the quasimonotone system $\frac{dz^*}{dt} \leq Az^*$ is obtained, which is a linear system and only has one equilibrium $E_0\{(0, 0, 0)\}$. Based on Lemma 1 in [45], we find that the eigenvalues of the matrix A all have negative real parts when $R_0 < 1$, so the linear system is stable, and then $H_k \rightarrow 0, F_k \rightarrow 0, B_k \rightarrow 0$ as $t \rightarrow \infty, k = 1, 2, 3, \dots, n$. Using the comparison theorem in [44], it follows that $H_k \rightarrow 0, F_k \rightarrow 0, B_k \rightarrow 0$ as $t \rightarrow \infty, k = 1, 2, 3, \dots, n$, for the nonlinear system (2.5) when $R_0 < 1$. So the information-elimination equilibrium is globally asymptotically stable in Ω^* when $R_0 < 1$. This completes the proof. \square

In this section, the global attractivity of the information-prevailing equilibrium is discussed.

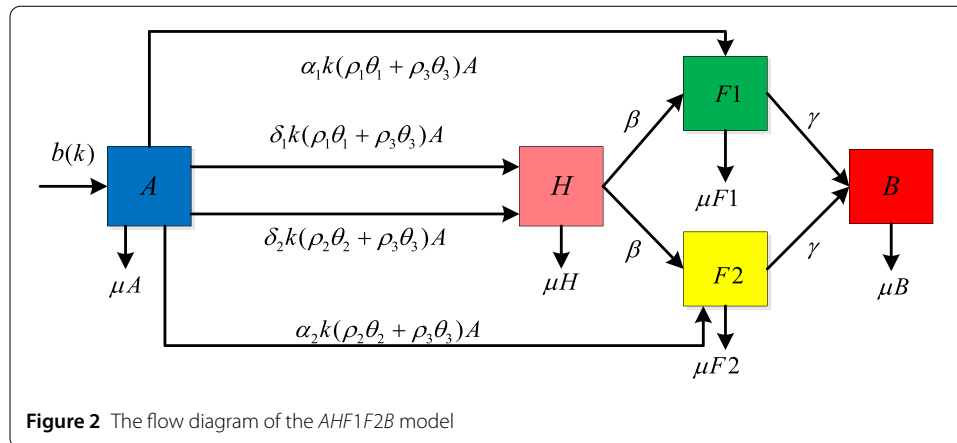
Theorem 3 Suppose that $(A_k(t), H_k(t), F_k(t), B_k(t))$ is a solution of (2.1), satisfying

$$A_k(0) > 0, \quad H_k(0) > 0, \quad F_k(0) > 0 \quad \text{and} \quad B_k(0) > 0.$$

If $R_0 > 1$, then $\lim_{t \rightarrow \infty} \inf\{A_k(t), H_k(t), F_k(t), B_k(t)\} = \{A_k^*, H_k^*, F_k^*, B_k^*\}$, where $(A_k^*, H_k^*, F_k^*, B_k^*)$ is the unique information-prevailing equilibrium of (2.1) for $k = 1, 2, \dots, n$.

Proof Let us consider the Lyapunov function:

$$\begin{aligned} W(t) &= [(A_k - A_k^*) - (H_k - H_k^*) - (F_k - F_k^*) - (B_k - B_k^*)]^2, \\ W'(t) &= 2[(A_k - A_k^*) - (H_k - H_k^*) - (F_k - F_k^*) - (B_k - B_k^*)][A_k' + H_k' + F_k' + B_k'] \\ &= 2[(A_k - A_k^*) - (H_k - H_k^*) - (F_k - F_k^*) - (B_k - B_k^*)] \end{aligned}$$



$$\times [b(k) - \mu A_k - \mu H_k - \mu F_k - \mu B_k].$$

Because $E^*(A_k^*, H_k^*, F_k^*, B_k^*)$ satisfies (2.2), it follows that

$$b(k) - \mu A_k^* - \mu H_k^* - \mu F_k^* - \mu B_k^* = 0.$$

In other words, $b(k) = \mu A_k^* - \mu H_k^* - \mu F_k^* - \mu B_k^*$. Then, we can get

$$\begin{aligned} W'(t) &= 2[(A_k - A_k^*) - (H_k - H_k^*) - (F_k - F_k^*) - (B_k - B_k^*)] \\ &\quad \times [\mu A_k^* + \mu H_k^* + \mu F_k^* + \mu B_k^* - \mu A_k - \mu H_k - \mu F_k - \mu B_k] \\ &= 2[(A_k - A_k^*) - (H_k - H_k^*) - (F_k - F_k^*) - (B_k - B_k^*)] \\ &\quad \times [\mu(A_k^* - A_k) - \mu(H_k^* - H_k) - \mu(F_k^* - F_k) - \mu(B_k^* - B_k)] \\ &= -2\mu[(A_k - A_k^*) - (H_k - H_k^*) - (F_k - F_k^*) - (B_k - B_k^*)]^2 \leq 0. \end{aligned}$$

Thus, the information-prevailing equilibrium $E^*(A_k^*, H_k^*, F_k^*, B_k^*)$ is globally asymptotically stable by LaSalle Invariance Principle [46]. This completes the proof. \square

5 A preferential information spreading model with competitive mechanism

Competition is ubiquitous in human society [47, 48]. Bo Zhao et al. studied the competing spreading processes of epidemic and awareness in multiplex networks, they found that strong capacities of awareness diffusion and self-protection of individuals could lead to a much higher spreading threshold and a smaller outbreak size [49]. Comparing to e-commerce networks, a product may have multiple merchants on sale, and different merchants may also offer different degrees of preferential for this product, which will trigger competition. Therefore, a preferential information spreading model with competitive mechanism is proposed. In the process of competition, we consider the difference of the preferential degree of the product and people's hesitation psychology. The model flow diagram is shown in Fig. 2.

Here $F1_k(t), F2_k(t)$ represent the individuals who forwarded preferential information from shop 1 and shop 2, respectively. The parameter β represents the probability that a hesitator becomes a forwarder. A forwarder becomes a beneficiary with a probability of γ .

The attractivity parameters α_1 and α_2 are used to describe the preferential degree of the product in shops 1 and 2, respectively. The hesitation parameters δ_1 and δ_2 are used to describe the hesitation degree of adherents to the product preferential information in shops 1 and 2, respectively. Parameter $\rho_1(\rho_2)$ is the spread rate for an adherent after contacting with a forwarder in shop 1 (shop 2), ρ_3 is the spread rate for an adherent after contacting with a beneficiary. In this model, we assume $b(k) = \mu$. Based on these assumptions above, the model can be described by the following system of ordinary differential equations:

$$\begin{cases} \frac{dA_k(t)}{dt} = b(k) - (\alpha_1 + \delta_1)k(\rho_1\theta_1(t) + \rho_3\theta_3(t))A_k(t) \\ \quad - (\alpha_2 + \delta_2)k(\rho_2\theta_2(t) + \rho_3\theta_3(t))A_k(t) - \mu A_k(t), \\ \frac{dH_k(t)}{dt} = \delta_1k(\rho_1\theta_1(t) + \rho_3\theta_3(t))A_k(t) \\ \quad + \delta_2k(\rho_2\theta_2(t) + \rho_3\theta_3(t))A_k(t) - 2\beta H_k(t) - \mu H_k(t), \\ \frac{dF1_k(t)}{dt} = \alpha_1k(\rho_1\theta_1(t) + \rho_3\theta_3(t))A_k(t) + \beta H_k(t) - \gamma F1_k(t) - \mu F1_k(t), \\ \frac{dF2_k(t)}{dt} = \alpha_2k(\rho_2\theta_2(t) + \rho_3\theta_3(t))A_k(t) + \beta H_k(t) - \gamma F2_k(t) - \mu F2_k(t), \\ \frac{dB_k(t)}{dt} = \gamma F1_k(t) + \gamma F2_k(t) - \mu B_k(t). \end{cases} \quad (5.1)$$

Here $\theta_1(\theta_2)$ denotes the probability that an adherent contacts with a forwarder in shop 1 (shop 2) at time t , θ_3 denotes the probability that an adherent contacts with a beneficiary at time t , which satisfies the relation

$$\begin{cases} \theta_1(t) = \sum_{i=1}^n \frac{\varphi(i)}{i} P(i|k) \frac{F1_i(t)}{N_i(t)} = \langle k \rangle^{-1} \sum_{i=1}^n \frac{\varphi(i)}{\eta_i} P(i) F1_i(t), \\ \theta_2(t) = \sum_{i=1}^n \frac{\varphi(i)}{i} P(i|k) \frac{F2_i(t)}{N_i(t)} = \langle k \rangle^{-1} \sum_{i=1}^n \frac{\varphi(i)}{\eta_i} P(i) F2_i(t), \\ \theta_3(t) = \sum_{i=1}^n \frac{\varphi(i)}{i} P(i|k) \frac{B_i(t)}{N_i(t)} = \langle k \rangle^{-1} \sum_{i=1}^n \frac{\varphi(i)}{\eta_i} P(i) B_i(t). \end{cases} \quad (5.2)$$

Theorem 4 Consider model (5.1) and define

$$R_0^* = \frac{\langle \varphi(k)k \rangle}{\langle k \rangle} \times \frac{[(\alpha_1(2\beta + \mu) + \beta\delta_1)\mu\rho_1 + (\alpha_2(2\beta + \mu) + \beta\delta_2)\mu\rho_2 + ((\alpha_1 + \alpha_2)(2\beta + \mu) + 2\beta(\delta_1 + \delta_2))\gamma\rho_3]}{\mu(2\beta + \mu)(\mu + \gamma)},$$

then the following statements hold:

- (1) There always exists an information-elimination equilibrium $E_0^* = \{(1, 0, 0, 0, 0)\}_k$ when $R_0^* < 1$.
- (2) There is an information-prevailing equilibrium $E_1^* = \{(A_k^*, H_k^*, F1_k^*, F2_k^*, B_k^*)\}_k$ when $R_0^* > 1$.

Proof According to the formulas of system (5.1) and due to $b(k) = \mu$, one can easily find that $E_0^*(1, 0, 0, 0, 0)$ is always an equilibrium of system (5.1), which is called information-elimination equilibrium. To get the information-prevailing equilibrium $E_1^* = (A^*, H^*, F1^*, F2^*, B^*)$, we need to make the right-hand side of system (5.1) equal to zero,

so it should satisfy

$$\begin{cases} b - (\alpha_1 + \delta_1)k(\rho_1\theta_1^*(t) + \rho_3\theta_3^*(t))A_k^*(t) \\ \quad - (\alpha_2 + \delta_2)k(\rho_2\theta_2^*(t) + \rho_3\theta_3^*(t))A_k^*(t) - \mu A_k^*(t) = 0, \\ \delta_1 k(\rho_1\theta_1^*(t) + \rho_3\theta_3^*(t))A_k^*(t) + \delta_2 k(\rho_2\theta_2^*(t) + \rho_3\theta_3^*(t))A_k^*(t) \\ \quad - 2\beta H_k^*(t) - \mu H_k^*(t) = 0, \\ \alpha_1 k(\rho_1\theta_1^*(t) + \rho_3\theta_3^*(t))A_k^*(t) + \beta H_k^*(t) - \gamma F1_k^*(t) - \mu F1_k^*(t) = 0, \\ \alpha_2 k(\rho_2\theta_2^*(t) + \rho_3\theta_3^*(t))A_k^*(t) + \beta H_k^*(t) - \gamma F2_k^*(t) - \mu F2_k^*(t) = 0, \\ \gamma F1_k^*(t) + \gamma F2_k^*(t) - \mu B_k^*(t) = 0, \end{cases} \quad (5.3)$$

and a direct calculation yields

$$\begin{aligned} A^* &= \frac{\mu}{(\mu + k)[(\delta_1 + \alpha_1)(\rho_1\theta_1^* + \rho_3\theta_3^*) + (\delta_2 + \alpha_2)(\rho_2\theta_2^* + \rho_3\theta_3^*)]}, \\ H^* &= \frac{\mu k(\delta_1(\rho_1\theta_1^* + \rho_3\theta_3^*) + \delta_2(\rho_2\theta_2^* + \rho_3\theta_3^*))}{(2\beta + \mu)(\mu + k[(\delta_1 + \alpha_1)(\rho_1\theta_1^* + \rho_3\theta_3^*) + (\delta_2 + \alpha_2)(\rho_2\theta_2^* + \rho_3\theta_3^*)])}, \\ F1^* &= \frac{\mu k((2\beta + \mu)\alpha_1(\rho_2\theta_2^* + \rho_3\theta_3^*) + \beta[\delta_1(\rho_1\theta_1^* + \rho_3\theta_3^*) + \delta_2(\rho_2\theta_2^* + \rho_3\theta_3^*)])}{(2\beta + \mu)(\mu + k[(\delta_1 + \alpha_1)(\rho_1\theta_1^* + \rho_3\theta_3^*) + (\delta_2 + \alpha_2)(\rho_2\theta_2^* + \rho_3\theta_3^*)])}, \\ F2^* &= \frac{\mu k((2\beta + \mu)\alpha_2(\rho_2\theta_2^* + \rho_3\theta_3^*) + \beta[\delta_1(\rho_1\theta_1^* + \rho_3\theta_3^*) + \delta_2(\rho_2\theta_2^* + \rho_3\theta_3^*)])}{(2\beta + \mu)(\mu + k[(\delta_1 + \alpha_1)(\rho_1\theta_1^* + \rho_3\theta_3^*) + (\delta_2 + \alpha_2)(\rho_2\theta_2^* + \rho_3\theta_3^*)])}, \\ B^* &= \frac{\gamma \mu k((2\beta + \mu)[\alpha_1(\rho_1\theta_1^* + \rho_3\theta_3^*) + \alpha_2(\rho_2\theta_2^* + \rho_3\theta_3^*)] + 2\beta[\delta_1(\rho_1\theta_1^* + \rho_3\theta_3^*) + \delta_2(\rho_2\theta_2^* + \rho_3\theta_3^*)])}{\mu(\gamma + \mu)(2\beta + \mu)(\mu + k[(\delta_1 + \alpha_1)(\rho_1\theta_1^* + \rho_3\theta_3^*) + (\delta_2 + \alpha_2)(\rho_2\theta_2^* + \rho_3\theta_3^*)])}, \end{aligned} \quad (5.4)$$

as well as $\rho(t) = \sum_i P(i)(\rho_1 F1_i + \rho_2 F2_i + \rho_3 B_i) \leq \rho_1 + \rho_2 + \rho_3$.

Obviously, $\rho = 0$ satisfies (5.3). Hence, $A_k = 1$ and $H_k = F1_k = F2_k = B_k = 0$ is an equilibrium of (5.1), which is called the information-elimination equilibrium.

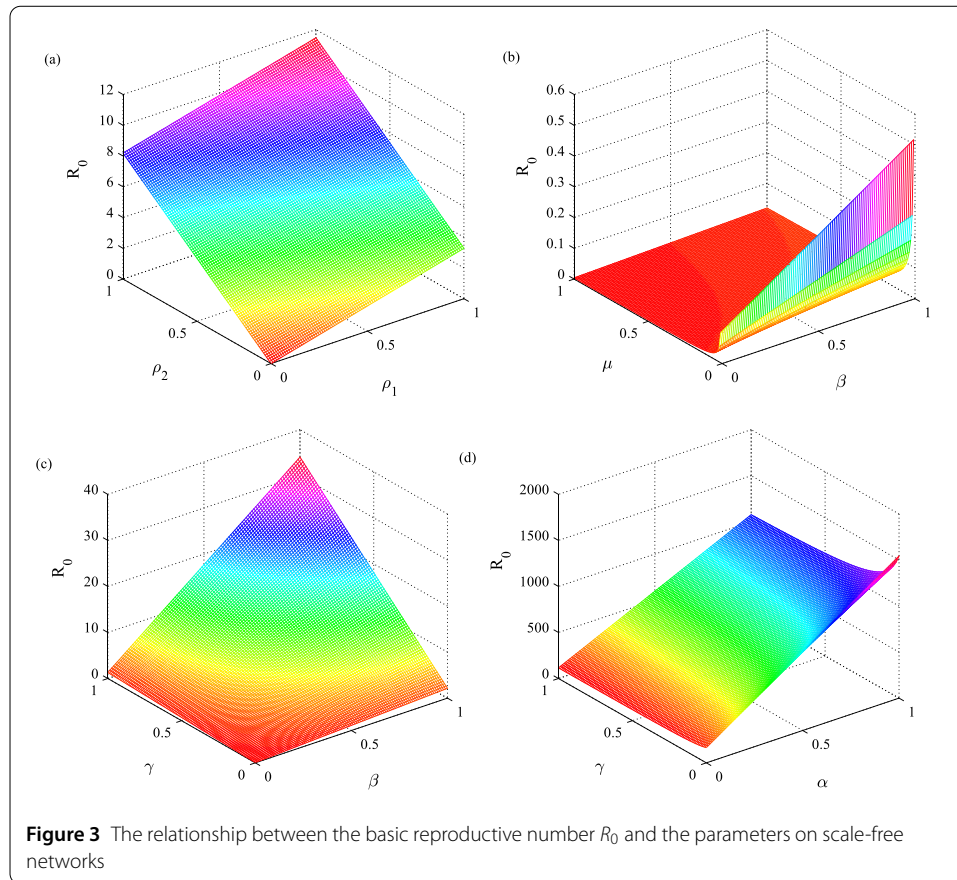
Substituting $F1_k$, $F2_k$, and B_k of (5.1) into ρ , an equation of the form $\rho f(\rho) = 0$ is obtained, where

$$\begin{aligned} f(\rho) &= 1 - \frac{(\alpha_1(2\beta + \mu) + \beta\delta_1)\mu\rho_1 + (\alpha_2(2\beta + \mu) + \beta\delta_2)\mu\rho_2 + ((2\beta + \mu)(\alpha_1 + \alpha_2) + 2\beta(\delta_1 + \delta_2))\gamma\rho_3}{\langle k \rangle \mu(\gamma + \mu)(2\beta + \mu)} \\ &\quad \times \sum_i \varphi(k)kP(i). \end{aligned}$$

Since $f'(\rho) > 0$ and $f(\rho_1 + \rho_2 + \rho_3) > 0$, the equation $f(\rho) = 0$ has a unique nontrivial solution if and only if $f(0) < 0$, i.e., $[(\alpha_1(2\beta + \mu) + \beta\delta_1)\mu\rho_1 + (\alpha_2(2\beta + \mu) + \beta\delta_2)\mu\rho_2 + \gamma((2\beta + \mu)(\alpha_1 + \alpha_2) + 2\beta(\delta_1 + \delta_2))\rho_3] / \langle \varphi(k)k \rangle / \langle k \rangle (\gamma + \mu)(2\beta + \mu) > 1$. So, we can get the basic reproductive number

$$\begin{aligned} R_0^* &= \frac{\langle \varphi(k)k \rangle}{\langle k \rangle} \\ &\quad \times \frac{[(\alpha_1(2\beta + \mu) + \beta\delta_1)\mu\rho_1 + (\alpha_2(2\beta + \mu) + \beta\delta_2)\mu\rho_2 + ((\alpha_1 + \alpha_2)(2\beta + \mu) + 2\beta(\delta_1 + \delta_2))\gamma\rho_3]}{\mu(2\beta + \mu)(\mu + \gamma)}, \end{aligned}$$

hence, a nontrivial solution exists if and only if $R_0^* > 1$, completing the proof. \square

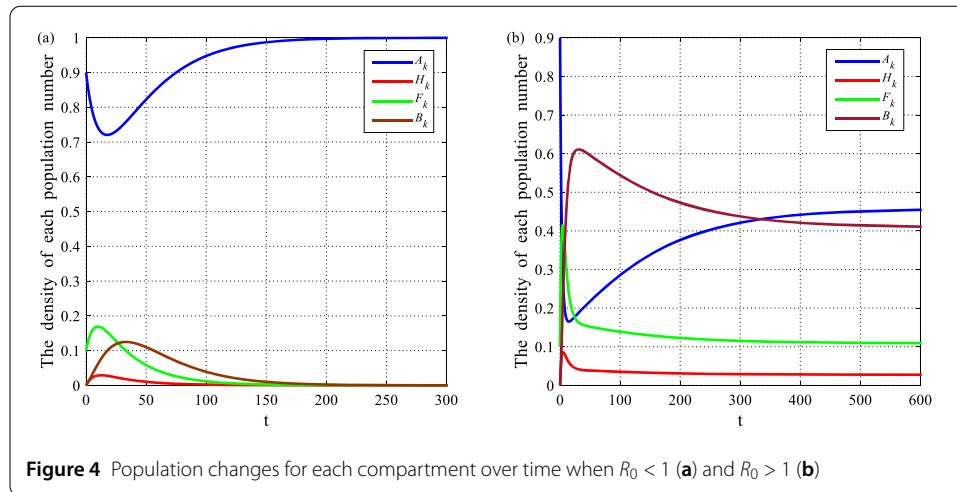


6 Numerical simulation and discussion

First, we perform some sensitivity analysis of the basic reproductive number R_0 in terms of model (2.1) parameters on scale-free networks. Obviously,

$$\begin{aligned}\frac{\partial R_0}{\partial \rho_1} &= \frac{\langle \varphi(k)k \rangle (\alpha(\beta + \mu) + \delta\beta)}{\langle k \rangle (\beta + \mu)(\gamma + \mu)}, & \frac{\partial R_0}{\partial \rho_2} &= \frac{\langle \varphi(k)k \rangle \gamma (\alpha(\beta + \mu) + \delta\beta)}{\langle k \rangle \mu (\beta + \mu)(\gamma + \mu)}, \\ \frac{\partial R_0}{\partial \beta} &= \frac{\langle \varphi(k)k \rangle \delta (\mu\rho_1 + \gamma\rho_2)}{\langle k \rangle (\gamma + \mu)(\beta + \mu)^2}, & \frac{\partial R_0}{\partial \gamma} &= \frac{\langle \varphi(k)k \rangle (\rho_2 - \rho_1)(\alpha(\beta + \mu) + \delta\beta)}{\langle k \rangle (\beta + \mu)(\gamma + \mu)^2}, \\ \frac{\partial R_0}{\partial \alpha} &= \frac{\langle \varphi(k)k \rangle (\mu\rho_1 + \gamma\rho_2)}{\langle k \rangle \mu (\gamma + \mu)}, & \frac{\partial R_0}{\partial \delta} &= \frac{\langle \varphi(k)k \rangle \beta (\mu\rho_1 + \gamma\rho_2)}{\langle k \rangle \mu (\gamma + \mu)(\beta + \mu)}.\end{aligned}$$

Some interesting results can be found as follows. In Fig. 3(a), the parameters are chosen as follows: $\beta = 0.15, \gamma = 0.25, \alpha = 0.3, \delta = 0.1, \mu = 0.1$; we can see that larger ρ_1 or ρ_2 can lead to larger R_0 . In Fig. 3(b), the parameters are chosen as follows: $\rho_1 = 0.3, \rho_2 = 0.25, \alpha = 0.35, \delta = 0.4, \gamma = 0.4$; it shows that larger β can lead to larger R_0 , but R_0 increase as μ decreases. In Fig. 3(c), the parameters are chosen as follows: $\rho_1 = 0.25, \rho_2 = 0.3, \beta = 0.15$, which shows that larger γ or δ can lead to larger R_0 when $\rho_1 < \rho_2$. Figure 3(d) shows that larger α can lead to larger R_0 and larger γ can lead to a decrease in R_0 when $\rho_1 > \rho_2$. From Fig. 3(a), (c), and (d), we can see that the simultaneous effect of the two parameters has a greater impact on the basic reproductive number. At the same time, variance of degree distribution $\varphi(k)\lambda(k)$ manifests in the diversity in contact patterns. Particularly, the ratio



$\varphi(k)\lambda(k)/\langle k \rangle$ is the parameter defining the level of heterogeneity of the network, which shows the impact of network topology.

In this section, we conduct simulations to investigate the dynamics of the model parameters and network topology structures to support and explain our theoretical results. Here the degree distribution is $P(k) = ck^{-l}$ ($2 < l \leq 3$) in which $l = 3$ and c satisfies $\sum_{k=1}^n P(k) = 1$ [50], $\sum_{k=1}^n P(k) = 1$, $n = 4000$, $b(k) = b/n$; if we choose $\varphi(k) = k^m$ with $m = 1$, then some interesting phenomena can be observed in our simulations.

Figure 4 shows the time series for $A(t)$, $H(t)$, $F(t)$, and $B(t)$, which are the globally average densities of the four states. The initial values of Figs. 4–8 are $A(t) = 0.9$, $H(t) = 0$, $F(t) = 0.1$ and $B(t) = 0$. In Fig. 4(a), if we choose $\rho_1 = 0.02$, $\rho_2 = 0.01$, $\delta = 0.1$, $\mu = 0.04$, $\beta = 0.06$, $\gamma = 0.05$, $\alpha = 0.4$, then the basic reproductive number $R_0 = 0.758 < 1$. It shows that, when $R_0 < 1$, the number of forwarders will diminish finally over time, even for a large fraction of the forwarder nodes at the beginning, the preferential information spreading will die out eventually, and all nodes will become adherents, i.e., $A(t) = 1$, $H(t) = 0$, $F(t) = 0$, and $B(t) = 0$ as $t \rightarrow \infty$ when $R_0 < 1$. It suggests that the information-eliminate equilibrium is locally stable when $R_0 < 1$, in agreement with Theorem 2. The parameters in Fig. 4(b) are chosen as $\rho_1 = 0.25$, $\rho_2 = 0.1$, $\alpha = 0.6$, $\beta = 0.3$, $\gamma = 0.2$, $\delta = 4$, $\mu = 0.1$, $\alpha = 0.6$, and then the basic reproductive number $R_0 = 3.07 > 1$. It is shown that when $R_0 > 1$, even for a small fraction of the forwarder nodes at the beginning, the density of forwarder is maintaining at a positive constant, which means that the preferential information spreading will be persistent on the network, in accordance with Theorems 3 and 4.

Figure 5 shows the dynamics behaviors of forwarders with different degree when the basic reproductive number $R_0 < 1$ and $R_0 > 1$. We find that the larger degree leads to larger value of the preferential information spreading level.

Figure 6 shows the effects of attractivity parameter α in H_{200} and F_{200} . We choose the parameters as follows: $\rho_1 = 0.2$, $\rho_2 = 0.15$, $\gamma = 0.2$, $\delta = 0.15$, $\beta = 0.1$, with $\alpha = 0.1, 0.4, 0.7, 0.9$ from bottom to top. It can be observed that the higher the attractivity rate α , the fewer the hesitators. That is, the greater the degree of preferential, the less hesitant people will be to forward the preferential information.

Figure 7 shows the effects of parameter δ in H_{200} and F_{200} . We choose $\rho_1 = 0.25$, $\rho_2 = 0.1$, $\gamma = 0.15$, $\alpha = 0.5$, $\beta = 0.1$, with $\delta = 0.05, 0.1, 0.3, 0.5$ from bottom to top. One can see that the higher the hesitation of the customer, the fewer individuals will be able to forward

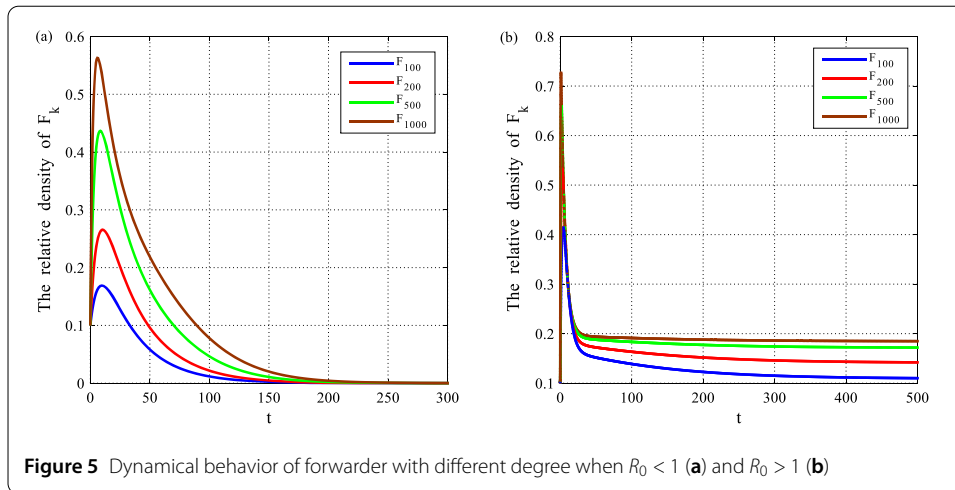


Figure 5 Dynamical behavior of forwarder with different degree when $R_0 < 1$ (a) and $R_0 > 1$ (b)

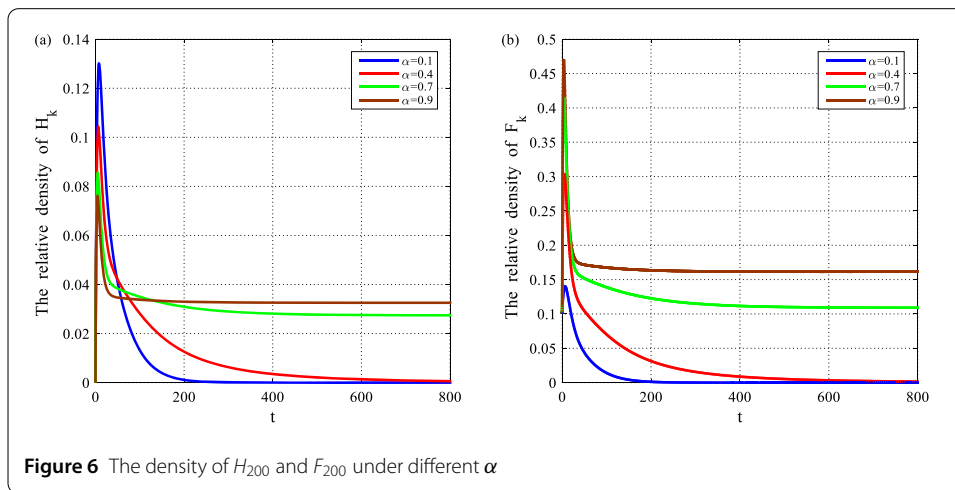


Figure 6 The density of H_{200} and F_{200} under different α

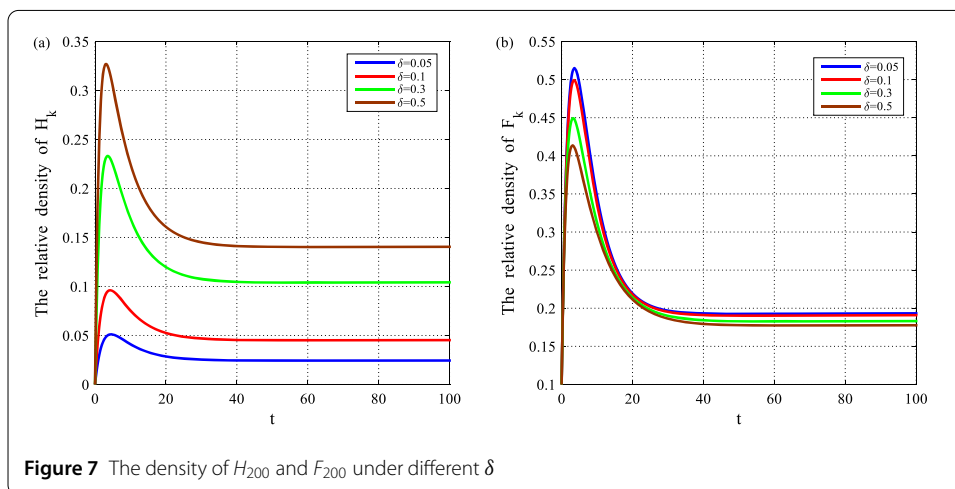


Figure 7 The density of H_{200} and F_{200} under different δ

the preferential information. Therefore, reducing the degree of hesitation of the customer, that is, improving the promotion strategy and effective customer service guidance, can promote the spreading of the preferential information.

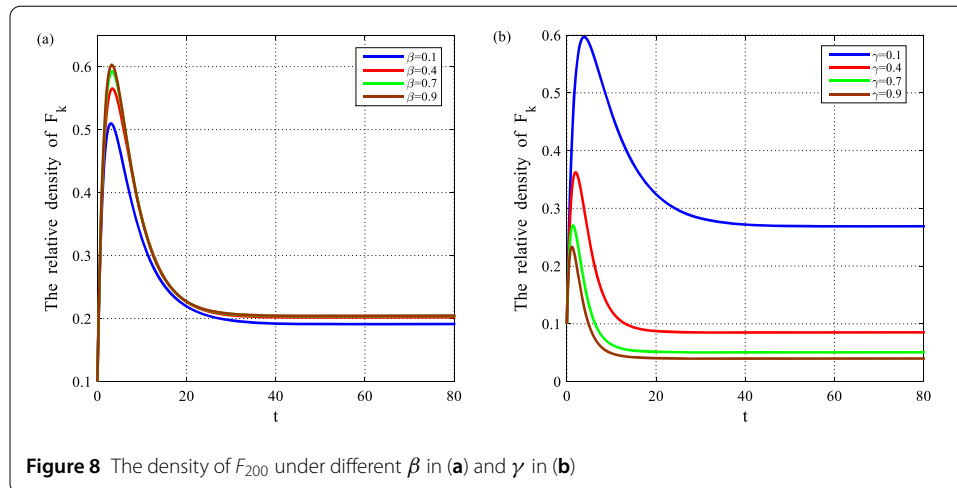


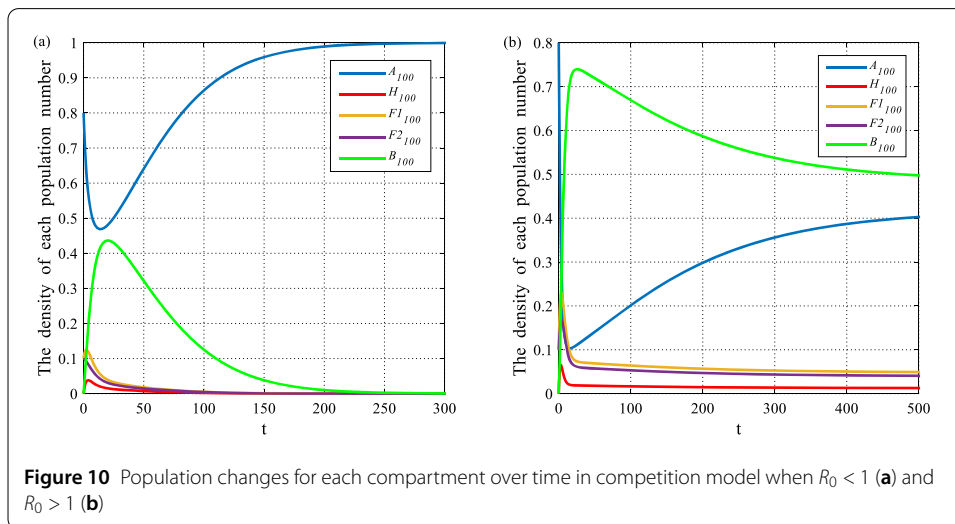
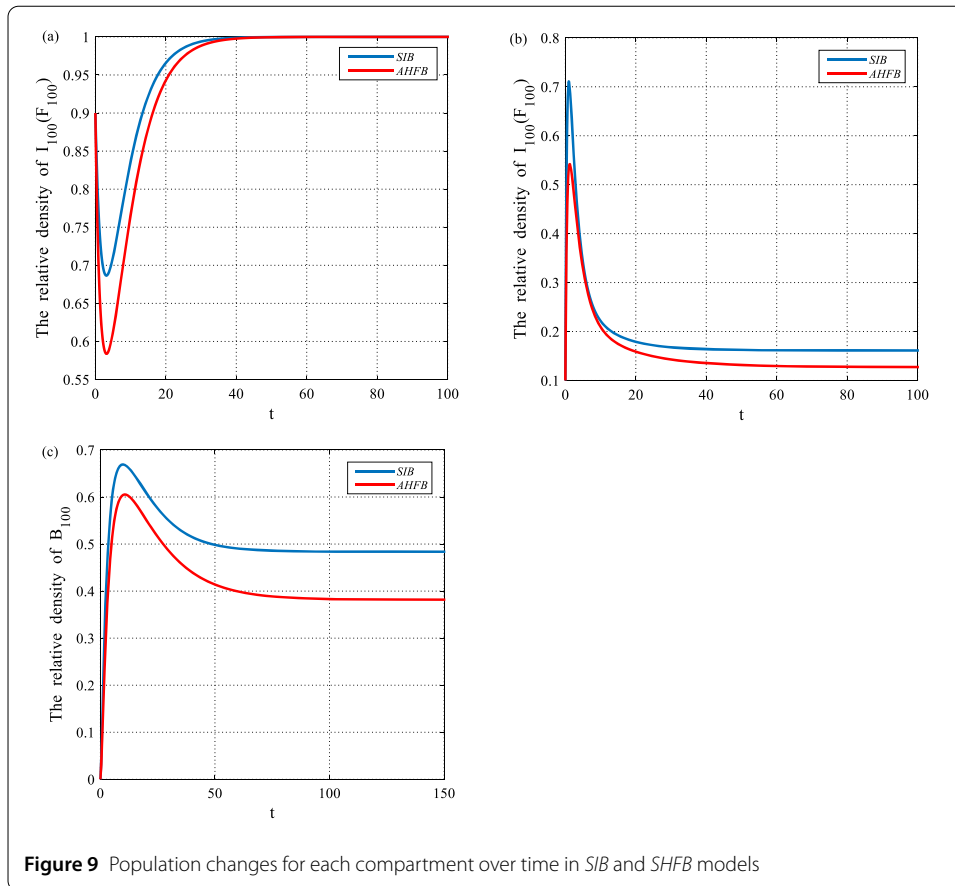
Figure 8 The density of F_{200} under different β in (a) and γ in (b)

Figure 8 shows the effects of parameter β and γ in F_{200} . The parameters in Fig. 8(a) are chosen as follows: $\rho_1 = 0.2, \rho_2 = 0.15, \gamma = 0.15, \alpha = 0.6, \delta = 0.2$, β is chosen as 0.1, 0.4, 0.7, 0.9. This implies that the product's attractiveness parameter (the degree of preferential) plays a strong role in contributing the spreading of preferential information. In Fig. 8(b), $\beta = 0.2$, the parameter γ is chosen as 0.1, 0.4, 0.7, 0.9. It can be seen that if it is easier to get preferential, the number of beneficiaries will increase.

Figure 9 compares the density of susceptible (adherent), sharer, beneficiary in the *SIB* and *AHFB* models. In Fig. 9(a), we choose parameters $\mu = 0.2, \beta = 0.2, \alpha = 0.4, \rho_1 = 0.1, \rho_2 = 0.01, R_0 = 0.5008$ in *AHFB* model. In Fig. 9(b) and (c), we choose $\mu = 0.1, \beta = 0.3, \alpha = 0.7, \rho_1 = 0.5, \rho_2 = 0.01, R_0 = 4.223$. It can be easily seen that the peak value of each state's density of *SIB* model is higher than that of *AHFB* model. It illustrates that the hesitation mechanism can reduce the level of preferential information spreading, but it can reflect people's identification of some preferential information and people's rational shopping during shopping.

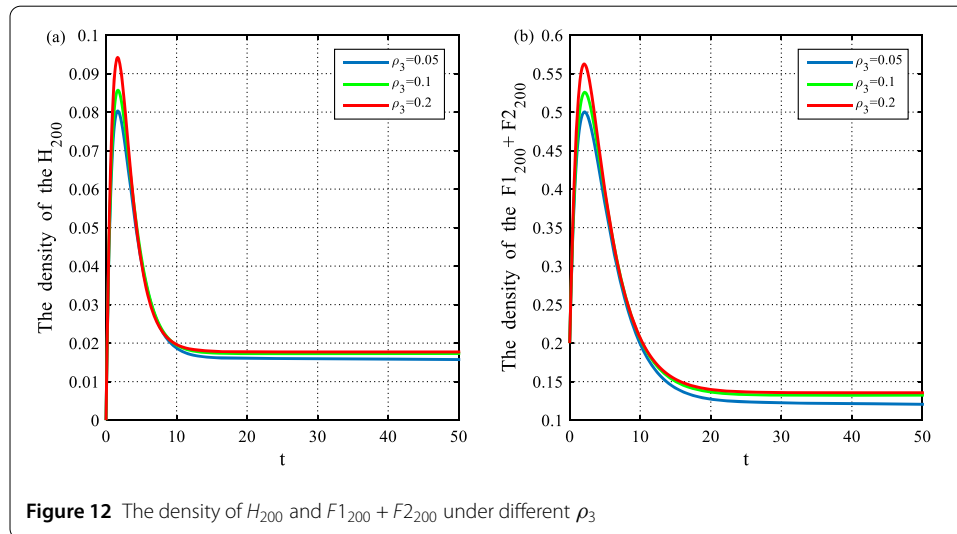
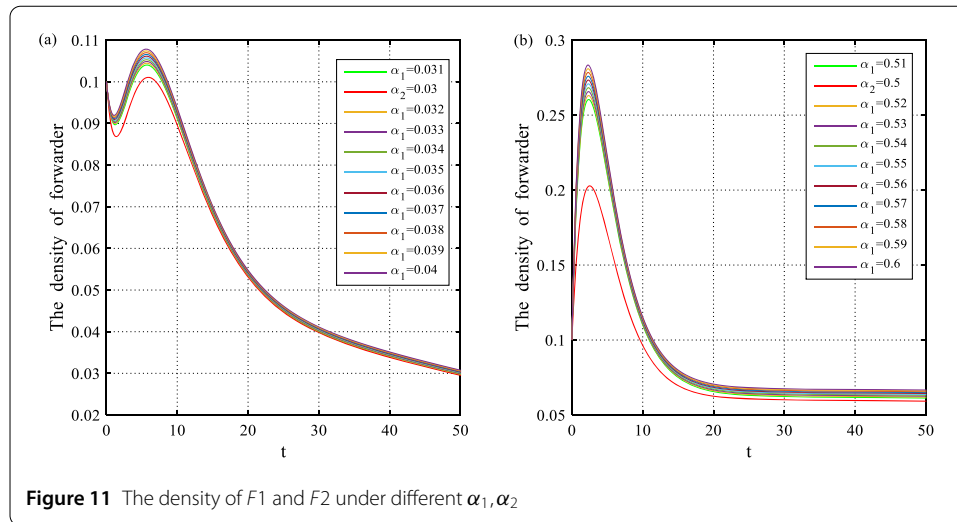
Figure 10 shows the time series for $A(t), H(t), F_1(t), F_2(t)$, and $B(t)$ in preferential information competition model, which are the globally average densities of the five states. The initial value of Fig. 10 is $A(t) = 0.8, H(t) = 0, F_1(t) = 0.1, F_2(t) = 0.1$, and $B(t) = 0$. In Fig. 10(a), we choose the parameters as $\mu = 0.04, \rho_1 = 0.05, \rho_2 = 0.05, \rho_3 = 0.01, \delta_1 = 0.15, \delta_2 = 0.1, \beta = 0.15, \gamma = 0.25, \alpha_1 = 0.4, \alpha_2 = 0.3$, forwarders 1 and 2 will disappear finally over time. And in Fig. 10(b), the parameters are chosen as $\rho_1 = 0.08, \rho_2 = 0.08, \rho_3 = 0.05, \delta_1 = 0.15, \delta_2 = 0.2, \beta = 0.24, \gamma = 0.22, \alpha_1 = 0.5, \alpha_2 = 0.4$. We can see that the population of forwarders will be maintained at a positive constant, which means that the information spreading will be persistent.

Figure 11 further studies the impact of preferential levels on competition mechanisms. In Figs. 11(a), if the merchant's preferential strength is less ($0 < \alpha_1, \alpha_2 \leq 0.1$), we choose $\alpha_2 = 0.03, \alpha_1 = 0.031, \dots, 0.039, 0.04$. It can be seen that in the process of competition, when the degree of merchandise preference is 0.001 more than that of competitors, a larger number of forwarders can be obtained, when the preference is increased again, the degree of increase is smaller. In Fig. 11(b), if the merchant's preferential strength is large ($0.1 < \alpha_1, \alpha_2 < 1$), we choose $\alpha_2 = 0.5, \alpha_1 = 0.51, \dots, 0.59, 0.6$. We can see that in the competition process, when the degree of merchandise preference is 0.01 higher than that of the competitor, the number of forwarders will increase significantly, and when the preference



is increased again, the degree of increase is not obvious. From this we can find the best preferential policies in the competition process.

Figure 12 shows the impact of beneficiary spreading rate on hesitators and forwarders. The parameters are chosen as $\rho_1 = 0.2$, $\rho_2 = 0.2$, $\delta_1 = 0.15$, $\delta_2 = 0.15$, $\beta = 0.24$, $\gamma_1 = 0.25$, $\alpha_1 = 0.5$, $\alpha_2 = 0.5$, and ρ_3 is chosen as 0.05, 0.1, 0.2. It shows that the beneficiary spreading rate has little effect on the hesitators, but it has a great influence on the forwarders. Hence,



increasing the acknowledged comment and positive feedback of the beneficiaries can help to increase the number of forwarders.

7 Conclusion

In this paper, we propose a novel *AHFB* model to describe the dynamic behavior of preferential information spreading on scale-free networks. We obtain the basic reproductive number R_0 with the next generation matrix method, which is closely related to the topology of the underlying networks and some model parameters. Interestingly, the basic reproductive number R_0 has no relationship with the degree-dependent new register $b(k)$. More specifically, by using the comparison principle and Lyapunov function, we prove that the information-eliminate equilibrium E_0 is globally asymptotically stable when $R_0 < 1$; the preferential information is uniformly persistent on the network when $R_0 > 1$. In all cases, the effects of attractiveness in preferential information and the hesitation psychology of customers on the information spreading dynamics have been discussed. It seems that increasing the attractiveness parameter α or decreasing the hesitation parameter δ

can improve preferential information spreading. Moreover, we study the preferential information spreading model with competitive mechanism and analyze corresponding dynamics behavior, then we draw a conclusion that the degree of preferential for products in the process of commodity competition is a core competitiveness factor. We have found that the degree of merchandise preference is 0.001 or 0.01 more than that of competitors and the number of forwarders will increase significantly. At the same time, increasing the spreading rate of beneficiaries, that is, the acknowledged comment and positive feedback of the beneficiaries can help promote the sale of products. The study has valuable guiding significance in effectively managing and controlling preferential information spreading on scale-free networks.

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Competing interests

The authors declare that they have no competing interests.

Authors' contributions

The authors contributed equally to this work. All authors read and approved the final manuscript.

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