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A stochastic predator–prey model for integrated pest management

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Abstract

This paper studies a stochastic predator–prey model for integrated pest management. It shows that the system has a positive solution that exists globally. The long time behavior of the system is investigated, and a condition for the pest to go extinct is given. Then the numerical simulations are carried out to illustrate our theoretical results and facilitate their interpretation.

Keywords: Integrated pest management; Predator–prey model; Pest extinction; Impulsive effects

1 Introduction

As an important area of research, pest control has generated an increasing interest in recent years [1–12]. Pest control is a complex issue in real applications, where a key objective is to reduce harm caused by pests to plants, animals, and humans. Traditional method for pest control is the seasonal or state-dependent spraying of chemical pesticide, which can reduce the pest population considerably. In fact, nowadays in most cropping systems, insecticides are still the principal means of controlling pests once the economic threshold has been reached [7, 8, 11, 12]. Chemical spraying is useful and sometimes could be the only feasible method in preventing economic loss. However, it may create both human and environmental risks by applying broad-spectrum pesticides.

A better and effective strategy for controlling pest and preventing pest damage is to combine different methods, which is known as *Integrated Pest Management* (IPM) [4–6, 8–10]. The goal of IPM is to manage pest damage by the most economical means and with the least possible hazard to the environment. To achieve such a goal, one needs to have sufficient information on the pest and its control methods. One of the environmentally friendly pest control methods is to reduce the pest population by its natural enemies, which is often an important component of an IPM strategy. In this approach, human beings play an active role by increasing the number of natural enemies at critical times. It is usually through mass releases of the natural enemies in a field or greenhouse. However, other pest control methods could be used at the same time when there are not enough natural enemies to decrease pest populations.

On the other hand, in the real world, the growth of species often suffers from disturbances due to some natural and man-made factors such as drought, flooding, harvesting, fire, earthquake, and so on. Such disturbances often occur in a relatively short time interval and cause sharp changes in the population which are often modeled as impulses at some sequence of discrete times.

Inspired by the above discussion, we shall propose, in this paper, a stochastic predator–prey IPM model. Initially, we shall show that our model has a positive solution, and then investigate the long time behavior of the system. We shall establish some sufficient conditions for the pest to go extinct. Moreover, we shall give some numerical examples and carry out computer simulations to illustrate our theoretical results and their biological implications.

2 The stochastic IPM model

In [8], the authors first take into account the simplest case where in each impulsive period T there is a pesticide application, so the killing efficiency rate function can be formulated by the exponentially decaying piecewise periodic function. Further, in each impulsive period T, Q ($Q \geq 0$) is the constant number of natural enemies added to the population during each impulsive event. These assumptions result in the following pest-natural enemy model:

$$\begin{cases} \left.\begin{aligned} &\frac{dx(t)}{dt} = rx(t)(1-\frac{x(t)}{K}) - \beta x(t)y(t), \\ &\frac{dy(t)}{dt} = \lambda\beta x(t)y(t) - \eta y(t), \end{aligned}\right\} \quad t \neq nT, \\ y(nT^+) = y(nT) + Q, \quad t = nT, \end{cases} \tag{2.1}$$

where $x(t)$ and $y(t)$ are the population density of the pest and the natural enemy at time t, respectively, r represents the intrinsic growth rate, K is the carrying capacity parameter, β denotes the attack rate of the predator, λ represents conversion efficiency, and η is the predator mortality rate. The same model (2.1) without any residual effects of the pesticides on the pest (i.e., only an instantaneous killing efficiency was considered) has been investigated, see [5] for details.

When considering the interference of external factors, model (2.1) is not applicable. In order to describe these phenomena more accurately, some authors considered the stability of *Stochastic Differential Equations* (SDE) [13–24]. When a pesticide kills a pest instantly, impulsive differential equations (hybrid dynamical systems) can provide a natural description of pulse-like actions. Based on the above discussion, we establish the stochastic predator–prey impulsive pest management model

$$\begin{cases} \left.\begin{aligned} &dx(t) = [rx(t)(1-\frac{x(t)}{K}) - \beta x(t)y(t)]\,dt + \alpha_1 x(t)\,dB_1(t), \\ &dy(t) = \lambda\beta x(t)y(t)\,dt - \eta y(t)\,dt + \alpha_2 y(t)\,dB_2(t), \end{aligned}\right\} \quad t \neq nT, \\ \left.\begin{aligned} &x(t^+) = (1-p_n)x(t), \\ &y(t^+) = (1+q_n)y(t), \end{aligned}\right\} \quad t = nT, \end{cases} \tag{2.2}$$

where α_1 and α_2 are the coefficients of the effects of environmental stochastic perturbations on the pest and the natural enemy, $B_i(t), i = 1, 2$, is the standard Brownian motion, $0 \leq p_n < 1$ is the proportion by which the pest density is reduced by killing the number of pests at time $t = nT$, and q_n denotes the proportion of natural enemies released at time nT.

3 Dynamic behavior of the model

In this section, we will state and prove our main results.

Theorem 3.1 *For any given initial value* $(x_0, y_0) \in R_+^2$, *model* (2.2) *has a unique solution* $(x(t), y(t))$ *defined for all* $t \in [0, \infty)$.

Proof Consider the following SDE without impulse:

$$\begin{cases} dx_1(t) = [rx_1(t)[1 - \prod_{0<nT<t}(1-p_n)\frac{x_1(t)}{K}] - \beta \prod_{0<nT<t}(1+q_n)y_1(t)]\,dt \\ \qquad + \alpha_1 x_1(t)\,dB_1(t), \\ dy_1(t) = \lambda\beta \prod_{0<nT<t}(1-p_n)x_1(t)y_1(t)\,dt - \eta y_1(t)\,dt + \alpha_2 y_1(t)\,dB_2(t), \end{cases} \tag{3.1}$$

with the initial value $(x_{10}, y_{10}) = (x_0, y_0)$. According to the classical theory of SDE without impulse, Eq. (3.1) has a unique global positive solution.

Let

$$\begin{cases} x(t) = \prod_{0<nT<t}(1-p_n)x_1(t), \\ y(t) = \prod_{0<nT<t}(1+q_n)y_1(t), \end{cases}$$

with the initial value $(x_{10}, y_{10}) = (x_0, y_0)$. In fact, $x(t)$ and $y(t)$ are continuous on each interval $t \in (nT, (n+1)T]$, here $n \in \mathcal{Z}^+ = \{0, 1, 2, \ldots\}$.

For $x(t)$, we have

$$\begin{aligned} dx(t) &= d\Big[\prod_{0<nT<t}(1-p_n)x_1(t)\Big] = \prod_{0<nT<t}(1-p_n)\,dx_1(t) \\ &= \prod_{0<nT<t}(1-p_n)\Big(rx_1(t)\Big[1 - \prod_{0<nT<t}(1-p_n)\frac{x_1(t)}{K}\Big] \\ &\quad - \beta\prod_{0<nT<t}(1+q_n)y_1(t)\Big)\,dt + \prod_{0<nT<t}(1-p_n)\alpha_1 x_1(t)\,dB_1(t) \\ &= \Big(rx(t)\Big[1 - \frac{x(t)}{K}\Big] - \beta x(t)y(t)\Big)\,dt + \alpha_1 x(t)\,dB_1(t) \end{aligned}$$

for each $n \in \mathcal{N}$ and $t \neq nT$. Meanwhile,

$$\begin{aligned} x(nT^+) &= \lim_{t\to nT^+} x(t) = \lim_{t\to nT^+} \prod_{0<iT<t}(1-p_i)x_1(t) \\ &= \lim_{t\to nT^+} \prod_{0<iT\leq nT}(1-p_i)x_1(nT^+) \\ &= (1-p_n)\prod_{0<iT<nT}(1-p_i)x_1(nT) \\ &= (1-p_n)x(nT) \end{aligned}$$

for each $n \in \mathcal{N}$. Besides

$$\begin{aligned} x(nT^-) &= \lim_{t\to nT^-} x(t) = \lim_{t\to nT^-} \prod_{0<iT<t}(1-p_i)x_1(t) \\ &= \lim_{t\to nT^+} \prod_{0<iT<nT}(1-p_i)x_1(nT^-) \\ &= \prod_{0<iT<nT}(1-p_i)x_1(nT) = (1-p_n)x(nT). \end{aligned}$$

Similarly, for $y(t)$, we can get that

$$\begin{aligned} dy(t) &= d\Biggl[\prod_{0<nT<t}(1+q_n)y_1(t)\Biggr] = \prod_{0<nT<t}(1+q_n)\,dy_1(t) \\ &= \lambda\beta\prod_{0<nT<t}(1-p_n)\prod_{0<nT<t}(1+q_n)y_1(t)x_1(t)\,dt \\ &\quad - d\prod_{0<nT<t}(1+q_n)y_1(t)\,dt + \alpha_2\prod_{0<nT<t}(1+q_n)y_1(t)\,dB_2(t) \\ &= \lambda\beta x(t)y(t)\,dt - \eta y(t)\,dt + \alpha_2 y(t)\,dB_2(t) \end{aligned}$$

for each $n \in \mathcal{N}$ and $t \neq nT$, and

$$\begin{aligned} y\bigl(nT^+\bigr) &= \lim_{t\to nT^+} y(t) = \lim_{t\to nT^+}\prod_{0<iT<t}(1+q_i)y_1(t) \\ &= \lim_{t\to nT^+}\prod_{0<iT\le nT}(1+q_i)y_1\bigl(nT^+\bigr) \\ &= (1+q_n)\prod_{0<iT<nT}(1+q_i)y_1(nT) \\ &= (1+q_n)y(nT) \end{aligned}$$

for each $n \in \mathcal{N}$. Moreover,

$$\begin{aligned} y\bigl(nT^-\bigr) &= \lim_{t\to nT^-} y(t) = \lim_{t\to nT^-}\prod_{0<iT<t}(1+q_i)y_1(t) \\ &= \lim_{t\to nT^+}\prod_{0<iT<nT}(1+q_i)y_1\bigl(nT^-\bigr) \\ &= \prod_{0<iT<nT}(1+q_i)y_1(nT) = (1+q_n)y(nT) \end{aligned}$$

for each $n \in \mathcal{N}$. This completes the proof. □

The above theorem proves that there exists a positive solution of model (2.2). Then we give some numerical simulations to illustrate the above conclusions, see Fig. 1. In Fig. 1(i) we fix $\alpha_1 = \alpha_2 = 0$, the time sequence diagram and phase diagram corresponding to model (2.2) are drawn. That is, the time sequence diagram and phase diagram of the deterministic system corresponding to model (2.2). In Fig. 1(ii) we fix $\alpha_1 = \alpha_2 = 0.05$. In Fig. 1(iii) we fix $\alpha_1 = \alpha_2 = 0.1$. In Fig. 1(iv) we fix $\alpha_1 = \alpha_2 = 0.3$, the time sequence diagram and phase diagram corresponding to model (2.2) are drawn. From Fig. 1(ii), (iii) we know that the smaller the outside interference is, the more obvious the phenomenon is. In Fig. 1(iv), α_1 and α_2 over value. Chaos may occur in the system, and the pulse phenomenon is covered. So impulses cannot be produced clearly in Fig. 1(iv).

Theorem 3.2 *For any initial value* $(x_0, y_0) \in R^2_+$, *there exist functions* $e(t)$, $E(t)$, $g(t)$, *and* $G(t)$, *such that the positive solution of model* (2.2) *satisfies the following inequalities*:

$$e(t) \le x(t) \le E(t), \qquad g(t) \le y(t) \le G(t), \quad t \ge 0, \ a.s. \tag{3.2}$$

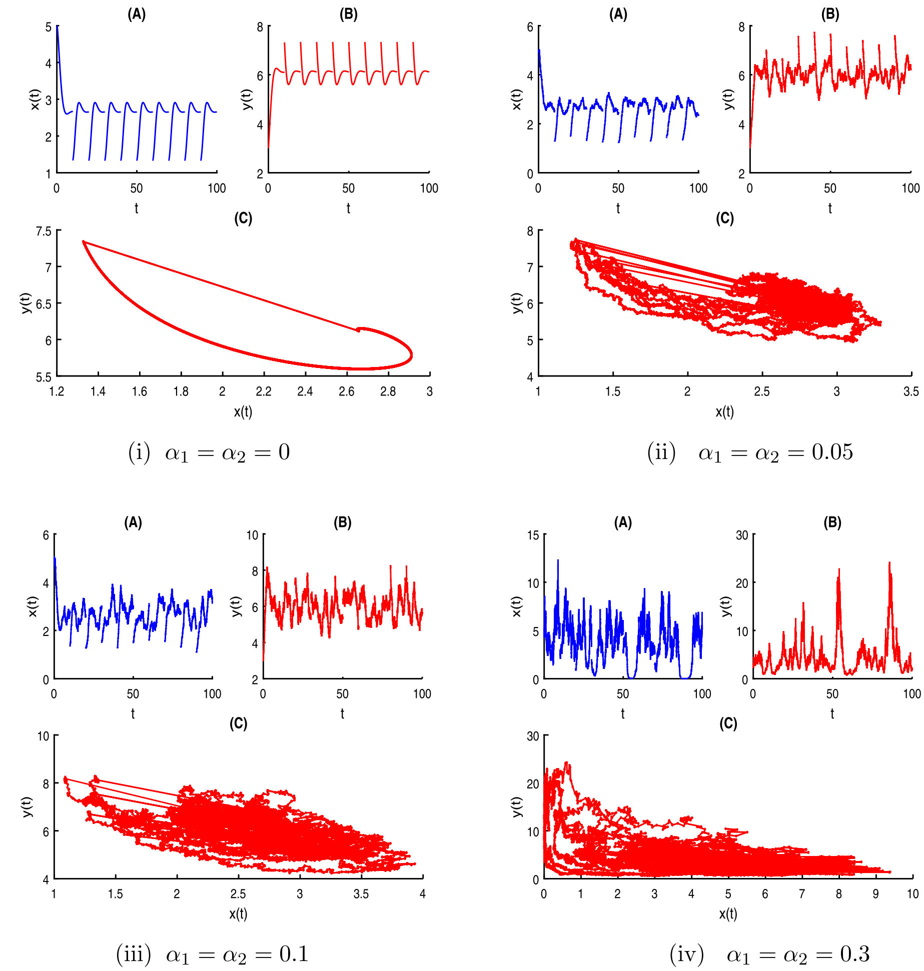

Figure 1 Time series graphs and phase diagrams for model (2.2): The parameter values are fixed as follows: initial value $(x_0, y_0) = (5, 3)$, $r = 3$, $K = 12$, $\beta = 0.3$, $\lambda = 0.5$, $\eta = 0.4$

Proof Since the solution of model (2.2) is positive, we have

$$dx(t) \le \left(rx(t)\left[1 - \frac{x(t)}{K}\right]\right) dt + \alpha_1 x(t)\, dB_1(t).$$

We construct the following equations:

$$\begin{cases} dE(t) = (rE(t)[1 - \frac{E(t)}{K}])\, dt + \alpha_1 E(t)\, dB_1(t), & t \neq nT, \\ E(t^+) = (1 - p_n)E(t), & t = nT, \\ E(0) = x_0. \end{cases} \tag{3.3}$$

Obviously, model (2.2) has a global continuous positive solution with x_0 as the initial value

$$x(t) = \frac{\prod_{0<nT<t}(1 - p_n) \exp[\int_0^t (r - 0.5\alpha_1^2)\, ds + \alpha_1 \int_0^t dB(s)]}{\frac{1}{x_0} + \int_0^t \prod_{0<nT<s}(1 - p_n)\frac{r}{K} \exp[\int_0^s (r - 0.5\alpha_1^2)\, d\tau + \alpha_1 \int_0^s dB(\tau)]\, ds}.$$

According to the comparison theorem for stochastic equations, we get

$$x(t) \le E(t), \quad t \in \big[0, t^*\big), \text{ a.s.}$$

Besides, the following inequalities can be obtained from the second equations of model (2.2):

$$dy(t) = \lambda\beta x(t)y(t)\,dt - \eta y(t)\,dt + \alpha_2 y(t)\,dB_2(t) \ge -\eta y(t)\,dt + \alpha_2 y(t)\,dB_2(t).$$

Obviously,

$$g(t) = \frac{\prod_{0<nT<t}(1+q_n)\exp[-\frac{\alpha_2^2}{2}t + \alpha_2 B_2(t)]}{\frac{1}{y_0} + \eta\int_0^t \prod_{0<nT<t}(1+q_n)\exp[-\frac{\alpha_2^2}{2}s + \alpha_2 B_2(s)]\,ds}$$

is a solution of the equation

$$\begin{cases} dg(t) = -\eta y(t)\,dt + \alpha_2 y(t)\,dB_2(t), & t \ne nT, \\ g(t^+) = (1+q_n)g(t), & t = nT, \\ g(0) = y_0 \end{cases} \tag{3.4}$$

and $y(t) \ge g(t)$, $t \in [0, t^*)$, a.s. From the second equations of model (2.2), we have

$$dy(t) \le \lambda\beta E(t)y(t)\,dt - \eta y(t)\,dt + \alpha_2 y(t)\,dB_2(t).$$

Similarly, we get that

$$y(t) \le G(t), \quad t \in \big[0, t^*\big), \text{ a.s.}$$

Here,

$$G(t) = \frac{\prod_{0<nT<t}(1+q(nT))\exp[-\frac{\alpha_2^2}{2}t + \alpha_2 B_2(t)]}{\frac{1}{y_0} + \lambda\beta\int_0^t \prod_{0<nT<t}(1+q(nT))\exp[-\frac{\alpha_2^2}{2}s + \alpha_2 B_2(s)]E(s)\,ds}.$$

It follows from the first equations of model (2.2) that

$$dx(t) \ge \left[rx(t)\left(1 - \frac{x(t)}{K}\right) - h(t)x(t) - \beta x(t)E(t)\right]dt + \alpha_1 x(t)\,dB_1(t).$$

According to the comparison theorem for stochastic equations, we get

$$\begin{aligned} x(t) &\ge e(t) \\ &= \frac{\prod_{0<nT<t}(1-p(t))\exp[(r-\frac{\alpha_1^2}{2})t - \beta\int_0^t (G(s)+h(s))\,ds + \alpha_1 B_1(t)]}{\frac{1}{x_0} + \frac{r}{K}\int_0^t \prod_{0<nT<t}(1-p(t))\exp[(r-\frac{\alpha_1^2}{2})s - \beta\int_0^s (G(\tau)+h(\tau))\,d\tau + \alpha_1 B_1(s)]\,ds} \end{aligned}$$

for $t \in [0, t^*)$, a.s.

In other words,

$$e(t) \le x(t) \le E(t), \qquad g(t) \le y(t) \le G(t), \quad t \in \big[0, t^*\big), \text{ a.s.}$$

We conclude that $e(t)$, $E(t)$, $g(t)$, and $G(t)$ all exist for $t \ge 0$ and satisfy the following inequalities:

$$e(t) \le x(t) \le E(t), \qquad g(t) \le y(t) \le G(t), \quad t \ge 0, \text{ a.s.} \qquad \square$$

Theorem 3.3 *If* $\lim_{t\to\infty} \frac{\sum_{0<nT<t} \ln(1-p_n)}{t} < 0.5\alpha_2^2 - r$, *then the pests of model* (2.2) *tend to extinction according to probability* 1.

Proof We make the following transformation:

$$x(t) = \prod_{0<nT<t} (1-p_n)\phi(t).$$

For the first equations of model (2.2), by using Itô's formula, we have

$$\begin{aligned} d\ln\phi(t) &= \frac{d\phi(t)}{\phi(t)} - \frac{(d\phi(t))^2}{2\phi^2(t)} \\ &\le \left[r - 0.5\alpha_1^2 - \frac{\prod_{0<nT<t}(1-p_n)\phi(t)}{K}\right] dt + \alpha_1\, dB(t) \\ &= \left[r - 0.5\alpha_1^2 - \frac{x(t)}{K}\right] dt + \alpha_1\, dB(t). \end{aligned} \tag{3.5}$$

Integrating both sides from 0 to t for Eq. (3.5),

$$\ln\phi(t) = \ln x_0 + \int_0^t \left[r - 0.5\alpha_1^2 - \frac{x(s)}{K}\right] ds + M_1(t), \tag{3.6}$$

where $M_1(t)$ is a local martingale

$$M_1(t) = \alpha_1 \int_0^t dB(s),$$

whose quadratic variation is

$$\big\langle M_1(t), M_1(t)\big\rangle = \alpha_1^2 t.$$

Using the strong law of large numbers for local martingales, we have

$$\lim_{t\to\infty} M_1(t)/t = 0.$$

On the other hand, it follows from (3.6) that

$$\sum_{0<nT<t} \ln(1-p_n) + \ln\phi(t) - \ln x_0 = \sum_{0<nT<t} \ln(1-p_n) + \int_0^t \left[r - 0.5\alpha_1^2 - \frac{x(s)}{K}\right] ds + M_1(t).$$

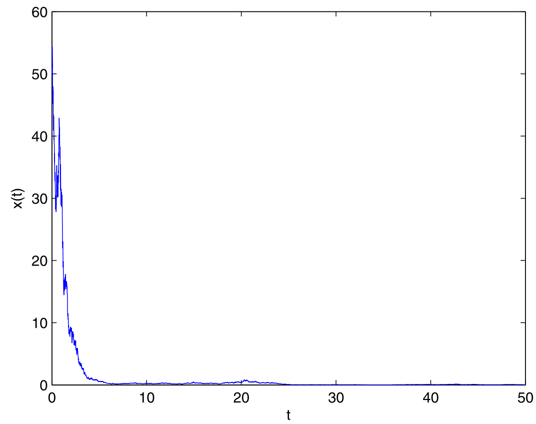

Figure 2 Pest population of model (2.2): The parameter values are fixed as follows: $x_0 = 50$, $r = 0.3$, $\alpha_1 = 0.6$, $K = 12$

In other words, we have that

$$\ln x(t) - \ln x_0 = \sum_{0<nT<t} \ln(1-p_n) + \int_0^t \left[r - 0.5\alpha_1^2 - \frac{x(s)}{K}\right] ds + M_1(t).$$

Therefore

$$\ln x(t) - \ln x_0 \le \sum_{0<nT<t} \ln(1-p_n) + \int_0^t \left[r - 0.5\alpha_1^2\right] ds + M_1(t).$$

Based on the hypothesis,

$$\lim_{t\to\infty} x(t) = 0. \qquad \square$$

In view of Theorem 3.3, we can get that the pest goes to extinction, as Fig. 2.

4 Conclusion

Noting that IPM is a long-term management strategy that uses a combination of biological, cultural, and chemical tactics to reduce pests to tolerable levels, control tactics must be taken once a critical density of pests (*Economic Threshold*, ET) is observed in the field so that the *Economic Injury Level* (EIL) is not exceeded. On the other hand, we only consider the impact of pesticides. So model (2.2) can be rewritten into the following form:

$$\begin{cases} \left.\begin{aligned} dx(t) &= [rx(t)(1-\tfrac{x(t)}{K}) - \beta x(t)y(t)]\,dt + \alpha_1 x(t)\,dB_1(t), \\ dy(t) &= \lambda\beta x(t)y(t)\,dt - \eta y(t)\,dt + \alpha_2 y(t)\,dB_2(t), \end{aligned}\right\} & x < ET, \\ \left.\begin{aligned} x(t^+) &= (1-p(t))x(t), \\ y(t^+) &= (1+q(t))y(t), \end{aligned}\right\} & x = ET. \end{cases} \tag{4.1}$$

We have selected some parameters to simulate the time series and phase diagrams of model (4.1) pests and natural enemies. As shown in Fig. 3, we take different external interference intensity. The numerical results show that the greater the external interference

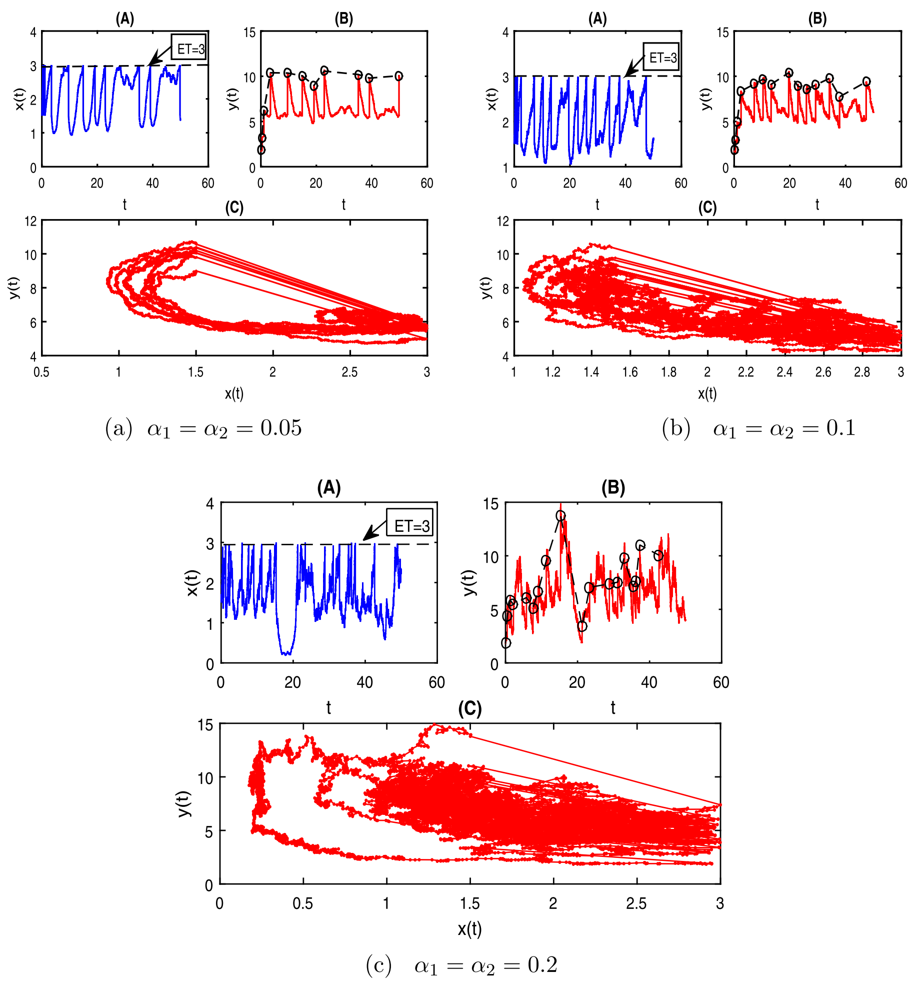

Figure 3 The change trend of solutions of model (4.1): The parameter values are fixed as follows: initial value $(x_0, y_0) = (3, 1)$, $r = 3$, $K = 12$, $\beta = 0.3$, $\lambda = 0.5$, $\eta = 0.4$, $ET = 3$

intensity is, the more complex the pest control is. Our results provide some theoretical basis and application value for the comprehensive pest management.

Obviously, these results indicate that the models proposed in this paper can help us to understand pest-natural enemy interactions, to design appropriate control strategies, and to make management decisions on insect pest control. We would like to mention here that an interesting but challenging problem associated with the studies of system (2.2) should be how to optimize the number of periodically released natural enemy and the dosage of spraying pesticides to reduce pests to tolerable levels with little economical cost and minimal effect on environmental stochastic perturbations. We leave this for future work.

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Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to each part of this manuscript. All authors read and approved the final manuscript.

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