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Dynamics analysis on a class of delayed neural networks involving inertial terms

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Abstract

This paper explores a class of unbounded distributed delayed inertial neural networks. By introducing some new differential inequality analysis and abandoning the traditional order reduction technique, some new assertions are derived to verify the global exponential stability of the addressed networks, which improve and generalize some recently published articles. Finally, two cases of numerical examples and simulations are given to illustrate these analytical conclusions.

Keywords: Distributed delayed inertial neural networks; Non-reduced order technique; Exponential stability

1 Introduction

In dynamic systems of neural networks, the existence of time delays is inevitable and is always a source of instability, chaos, and oscillation of the network system. In particular, on account of the occurrence of a lot of parallel routes with a series of different axon sizes and lengths, it is desired to account for the dynamical behaviors of neural networks by involving unbounded and continuously distributed delays. Meanwhile, there have existed many papers dealing with the dynamics studies of distributed delayed neural networks, which include stability analysis [1–4], almost periodic oscillation [5–9], and anti-periodic oscillation [10–12].

On the other hand, during the last two decades, by using reduced order technique, the famous inertial neural networks with constant delays and bounded time-varying delays have been widely investigated by many authors [13–37]. To reveal the rate of convergence, the exponential stability of inertial neural networks models with bounded delays has been extensively researched in [15–21, 29, 37] by changing the addressed models into the first order systems with some variable substitutions. Most recently, the authors in [36, 37] pointed out that the above transformation will generate new parameters and increase the dimension of the addressed systems, which is difficult to be achieved in practical problems. In addition, abandoning the traditional reduced-order technique, the authors of [36, 37] gained some new criteria to verify the synchronization and stability of the constant delayed inertial neural network involving a new Lyapunov functional, which are complementary to some known ones in [13, 35]. Unfortunately, there is no published paper touching upon the exponential stability on continuously distributed delayed inertial neural network models.

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Based on the above arguments, we shall use the non-reduced order technique to establish the global exponential stability for the following unbounded continuously distributed delayed inertial neural networks:

$$\begin{aligned}
 x_i''(t) = & -a_i x_i'(t) - b_i x_i(t) + \sum_{j=1}^n c_{ij} P_j(x_j(t)) + \sum_{j=1}^n d_{ij} \int_0^{+\infty} K_{ij}(u) \\
 & \times Q_j(x_j(t-u)) du + J_i(t), \quad t \geq 0, i \in D := \{1, 2, \dots, n\},
 \end{aligned}
 \tag{1.1}$$

where $x_i''(t)$ is labeled as an inertial term of the state vector $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ in (1.1), the constants $a_i > 0, b_i > 0, c_{ij}$, and d_{ij} are the connection weight parameters, the bounded external input $J_i \in C(\mathbb{R}, \mathbb{R})$ is continuous on \mathbb{R} , the delay kernel $K_{ij} \in C([0, +\infty), \mathbb{R})$, P_j and Q_j are the activation functions with Lipschitz constants L_j^P and L_j^Q obeying

$$|P_j(u) - P_j(v)| \leq L_j^P |u - v|, \quad |Q_j(u) - Q_j(v)| \leq L_j^Q |u - v|, \quad \forall u, v \in \mathbb{R}, j \in D.
 \tag{1.2}$$

In (1.1), we define

$$x_i(s) = \varphi_i(s), \quad x_i'(s) = \psi_i(s), \quad -\infty < s \leq 0, i \in D,
 \tag{1.3}$$

where φ_i and ψ_i are bounded and continuous initial values on $(-\infty, 0]$.

For the purpose of obtaining our main results in this paper, we presume the assumptions as follows:

- (T₁) For $i, j \in D$, there is $\kappa \in (0, +\infty)$ such that $|K_{ij}(t)|e^{\kappa t}$ is integrable on $[0, +\infty)$.
- (T₂) For $i \in D$, constants $\beta_i > 0$ and $\alpha_i \geq 0, \gamma_i \geq 0$ can be present to agree with that

$$E_i < 0, \quad 4E_i F_i > H_i^2,$$

where

$$\begin{cases}
 E_i = \alpha_i \gamma_i - a_i \alpha_i^2 + \frac{1}{2} \sum_{j=1}^n (|c_{ij}| L_j^P + |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| du) \alpha_i^2, \\
 F_i = -b_i \alpha_i \gamma_i + \frac{1}{2} \sum_{j=1}^n \alpha_j^2 (|c_{ji}| L_i^P + |d_{ji}| L_i^Q \int_0^{+\infty} |K_{ji}(u)| du) \\
 \quad + \frac{1}{2} \sum_{j=1}^n (|c_{ji}| L_i^P + |d_{ji}| L_i^Q \int_0^{+\infty} |K_{ji}(u)| du) |\alpha_i \gamma_j| \\
 \quad + \frac{1}{2} \sum_{j=1}^n (|c_{ij}| L_j^P + |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| du) |\alpha_i \gamma_i|, \\
 H_i = \beta_i + \gamma_i^2 - \alpha_i^2 b_i - a_i \gamma_i \alpha_i.
 \end{cases}$$

Remark 1.1 Since (1.1) can be converted into the first order functional differential equations, from (1.2), (T₁), and the basic theory on infinite delayed differential equation in [38], one can show the existence and uniqueness of every solution of (1.1) and (1.3) on $[0, +\infty)$.

Remark 1.2 Assumption (T₂) means that a_i and b_i are large enough and satisfy the above matrix inequalities which are adopted to guarantee the stability of system (1.1). Clearly, the above matrix inequalities are weaker than those used in [15–21], where $|a_i - b_i|$ is assumed to be small enough.

The main principle of this article is by employing differential inequality analysis to establish the global exponential stability of the addressed networks. To do that, our contributions are based on four aspects: (1) Propose a general inertial neural that has unbounded continuously distributed delays and is more general than the ones considered in [15–37]. (2) Establish sufficient conditions to ensure the global exponential stability of system (1.1). This is the first time to derive such a result for this type of unbounded continuously distributed delay inertial neural networks. (3) The results obtained in this article are original and complete those obtained previously in [17–21, 36, 37]. (4) The theoretical results play an important role in the design of the electrical implementation of the unbounded delayed inertial neural networks and in the processing of the transmission of its signals.

In the rest of this paper, Sect. 2 gives the global exponential convergence of all solutions with their derivatives of networks (1.1) under conditions (1.2), (T_1) , and (T_2) . Section 3 shows numerical figures. Conclusions are drawn in the last section.

2 Global exponential stability

Theorem 2.1 *Under (1.2), (T_1) , and (T_2) , system (1.1) is globally exponentially stable. More precisely, label $x(t) = (x_1(t), x_2(t), \dots, x_n(t))$ and $y(t) = (y_1(t), y_2(t), \dots, y_n(t))$ as two solutions of system (1.1) satisfying*

$$x_i(s) = \varphi_i^x(s), \quad x'_i(s) = \psi_i^x(s), \quad y_i(s) = \varphi_i^y(s), \quad y'_i(s) = \psi_i^y(s), \quad i \in D, \tag{2.1}$$

where $\varphi_i^x, \psi_i^x, \varphi_i^y$, and ψ_i^y are bounded and continuous on $(-\infty, 0]$. Then, one can take two positive constants λ and $\Lambda = \Lambda_{\varphi^x, \psi^x, \varphi^y, \psi^y}$ such that

$$|x_i(t) - y_i(t)| \leq \Lambda e^{-\lambda t}, \quad |x'_i(t) - y'_i(t)| \leq \Lambda e^{-\lambda t}, \quad \forall t \geq 0, i \in D.$$

Remark 2.1 In Theorem 2.1, $x(t)$ and $x'(t)$ are exponentially convergent to $y(t)$ and $y'(t)$, respectively. This suggests that the stability on system (1.1) is in accordance with the exponential stability definition adopted in [15–21, 29].

Proof of Theorem 2.1 Label $w_i(t) = y_i(t) - x_i(t) (i \in D)$, then, for $i \in D$,

$$w''_i(t) = -a_i w'_i(t) - b_i w_i(t) + \sum_{j=1}^n c_{ij} \tilde{P}_j(w_j(t)) + \sum_{j=1}^n d_{ij} \int_0^{+\infty} K_{ij}(u) \tilde{Q}_j(w_j(t-u)) du, \tag{2.2}$$

where

$$\tilde{P}_j(w_j(t)) = P_j(y_j(t)) - P_j(x_j(t)), \quad \tilde{Q}_j(w_j(t-u)) = Q_j(y_j(t-u)) - Q_j(x_j(t-u)), \quad j \in D.$$

In the light of (T_2) , one can select a constant $\lambda \in (0, \frac{1}{2}\kappa]$ to agree with that

$$E_i^\lambda < 0, \quad 4E_i^\lambda F_i^\lambda > (H_i^\lambda)^2, \quad i \in D, \tag{2.3}$$

where

$$\begin{cases} E_i^\lambda = \lambda\alpha_i^2 + \alpha_i\gamma_i - a_i\alpha_i^2 + \frac{1}{2}\alpha_i^2 \sum_{j=1}^n (|c_{ij}|L_j^P + |d_{ij}|L_j^Q \int_0^{+\infty} |K_{ij}(u)| du), \\ F_i^\lambda = -b_i\alpha_i\gamma_i + \lambda\beta_i + \lambda\gamma_i^2 + \frac{1}{2} \sum_{j=1}^n (|c_{ij}|L_j^P + |d_{ij}|L_j^Q \int_0^{+\infty} |K_{ij}(u)| du) |\alpha_i\gamma_i| \\ \quad + \frac{1}{2} \sum_{j=1}^n \alpha_j^2 (|c_{ji}|L_i^P + |d_{ji}|L_i^Q \int_0^{+\infty} |K_{ji}(u)| e^{2\lambda u} du) \\ \quad + \frac{1}{2} \sum_{j=1}^n (|c_{ji}|L_i^P + |d_{ji}|L_i^Q \int_0^{+\infty} |K_{ji}(u)| e^{2\lambda u} du) |\alpha_j\gamma_j|, \\ H_i^\lambda = \beta_i + \gamma_i^2 + 2\lambda\alpha_i\gamma_i - b_i\alpha_i^2 - a_i\alpha_i\gamma_i. \end{cases}$$

Designate

$$\begin{aligned} \Pi(t) &= \frac{1}{2} \sum_{i=1}^n \beta_i w_i^2(t) e^{2\lambda t} + \frac{1}{2} \sum_{i=1}^n (\alpha_i w_i'(t) + \gamma_i w_i(t))^2 e^{2\lambda t} \\ &\quad + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |d_{ij}| + |\alpha_i\gamma_i| |d_{ij}|) L_j^Q \int_0^{+\infty} |K_{ij}(u)| \int_{t-u}^t w_j^2(s) e^{2\lambda(u+s)} ds du. \end{aligned}$$

Differentiating $\Pi(t)$ on solutions along system (2.2) leads to

$$\begin{aligned} \Pi'(t) &= 2\lambda \left[\frac{1}{2} \sum_{i=1}^n \beta_i w_i^2(t) e^{2\lambda t} + \frac{1}{2} \sum_{i=1}^n (\alpha_i w_i'(t) + \gamma_i w_i(t))^2 e^{2\lambda t} \right] \\ &\quad + \sum_{i=1}^n \beta_i w_i(t) w_i'(t) e^{2\lambda t} + \sum_{i=1}^n (\alpha_i w_i'(t) + \gamma_i w_i(t)) (\alpha_i w_i''(t) + \gamma_i w_i'(t)) e^{2\lambda t} \\ &\quad + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |d_{ij}| + |\alpha_i\gamma_i| |d_{ij}|) L_j^Q \int_0^{+\infty} |K_{ij}(u)| [w_j^2(t) e^{2\lambda(u+t)} \\ &\quad - w_j^2(t-u) e^{2\lambda t}] du \\ &= 2\lambda \left[\frac{1}{2} \sum_{i=1}^n \beta_i w_i^2(t) e^{2\lambda t} + \frac{1}{2} \sum_{i=1}^n (\alpha_i w_i'(t) + \gamma_i w_i(t))^2 e^{2\lambda t} \right] \\ &\quad + \sum_{i=1}^n (\beta_i + \gamma_i^2) w_i(t) w_i'(t) e^{2\lambda t} + \sum_{i=1}^n \alpha_i (\alpha_i w_i'(t) + \gamma_i w_i(t)) e^{2\lambda t} \\ &\quad \times \left[-a_i w_i'(t) - b_i w_i(t) + \sum_{j=1}^n c_{ij} \tilde{P}_j(w_j(t)) + \sum_{j=1}^n d_{ij} \int_0^{+\infty} K_{ij}(u) \tilde{Q}_j(w_j(t-u)) du \right] \\ &\quad + \sum_{i=1}^n \alpha_i \gamma_i (w_i'(t))^2 e^{2\lambda t} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |d_{ij}| + |\alpha_i\gamma_i| |d_{ij}|) L_j^Q \\ &\quad \times \left[\int_0^{+\infty} |K_{ij}(u)| e^{2\lambda u} du w_j^2(t) e^{2\lambda t} - \int_0^{+\infty} |K_{ij}(u)| w_j^2(t-u) du e^{2\lambda t} \right] \\ &\leq e^{2\lambda t} \left\{ \sum_{i=1}^n (\beta_i + \gamma_i^2 + 2\lambda\alpha_i\gamma_i - a_i\alpha_i\gamma_i - b_i\alpha_i^2) w_i(t) w_i'(t) \right. \\ &\quad \left. + \sum_{i=1}^n (\lambda\alpha_i^2 + \alpha_i\gamma_i - a_i\alpha_i^2) (w_i'(t))^2 - \sum_{i=1}^n (b_i\alpha_i\gamma_i - \lambda\beta_i - \lambda\gamma_i^2) w_i^2(t) \right\} \end{aligned}$$

$$\begin{aligned}
 & + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |d_{ij}| + |\alpha_i \gamma_i| |d_{ij}|) L_j^Q \int_0^{+\infty} |K_{ij}(u)| e^{2\lambda u} du w_j^2(t) \\
 & - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |d_{ij}| + |\alpha_i \gamma_i| |d_{ij}|) L_j^Q \int_0^{+\infty} |K_{ij}(u)| w_j^2(t-u) du \\
 & + \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |w_i'(t)| + |\alpha_i \gamma_i| |w_i(t)|) |c_{ij}| |\tilde{P}_j(w_j(t))| \\
 & + \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |w_i'(t)| + |\alpha_i \gamma_i| |w_i(t)|) |d_{ij}| \int_0^{+\infty} |K_{ij}(u)| |\tilde{Q}_j(w_j(t-u))| du \Big\} \\
 = & e^{2\lambda t} \left\{ \sum_{i=1}^n (\beta_i + \gamma_i^2 + 2\lambda \alpha_i \gamma_i - a_i \alpha_i \gamma_i - b_i \alpha_i^2) w_i(t) w_i'(t) \right. \\
 & + \sum_{i=1}^n (\lambda \alpha_i^2 + \alpha_i \gamma_i - a_i \alpha_i^2) (w_i'(t))^2 \\
 & + \sum_{i=1}^n \left[-b_i \alpha_i \gamma_i + \lambda \beta_i + \lambda \gamma_i^2 + \frac{1}{2} \sum_{j=1}^n (\alpha_j^2 |d_{ji}| + |\alpha_j \gamma_j| |d_{ji}|) L_i^Q \right. \\
 & \times \left. \int_0^{+\infty} |K_{ji}(u)| e^{2\lambda u} du \right] w_i^2(t) \\
 & - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |d_{ij}| + |\alpha_i \gamma_i| |d_{ij}|) L_j^G \int_0^{+\infty} |K_{ij}(u)| w_j^2(t-u) du \\
 & + \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |w_i'(t)| + |\alpha_i \gamma_i| |w_i(t)|) |c_{ij}| |\tilde{P}_j(w_j(t))| \\
 & + \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |w_i'(t)| + |\alpha_i \gamma_i| |z_i(t)|) |d_{ij}| \\
 & \times \left. \int_0^{+\infty} |K_{ij}(u)| |\tilde{Q}_j(w_j(t-u))| du \right\}, \quad \forall t \in [0, +\infty). \tag{2.4}
 \end{aligned}$$

With the help of (1.2) and the fact that $AB \leq \frac{1}{2}(A^2 + B^2)$ ($A, B \in \mathbb{R}$), one can see

$$\begin{aligned}
 & \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |w_i'(t)| + |\alpha_i \gamma_i| |w_i(t)|) |c_{ij}| |\tilde{P}_j(w_j(t))| \\
 & \leq \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i^2 |c_{ij}| L_j^P ((w_i'(t))^2 + w_j^2(t)) \\
 & + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n |\alpha_i \gamma_i| |c_{ij}| L_j^P (w_i^2(t) + w_j^2(t)) \\
 = & \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i^2 |c_{ij}| L_j^P (w_i'(t))^2 + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (|\alpha_i \gamma_i| |c_{ij}| L_j^P + \alpha_j^2 |c_{ji}| L_i^P \\
 & + |\alpha_j \gamma_j| |c_{ji}| L_i^P) w_i^2(t)
 \end{aligned}$$

and

$$\begin{aligned}
 & \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |w'_i(t)| + |\alpha_i \gamma_i| |w_i(t)|) |d_{ij}| \int_0^{+\infty} |K_{ij}(u)| |\tilde{Q}_j(z_j(t-u))| du \\
 & \leq \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i^2 |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| ((w'_i(t))^2 + w_j^2(t-u)) du \\
 & \quad + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n |\alpha_i \gamma_i| |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| (w_i^2(t) + z_j^2(t-u)) du \\
 & = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n \alpha_i^2 |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| du (w'_i(t))^2 \\
 & \quad + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n |\alpha_i \gamma_i| |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| du w_i^2(t) \\
 & \quad + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n (\alpha_i^2 |d_{ij}| L_j^Q + |\alpha_i \gamma_i| |d_{ij}| L_j^Q) \int_0^{+\infty} |K_{ij}(u)| w_j^2(t-u) du,
 \end{aligned}$$

which, along with (2.3) and (2.4), results in

$$\begin{aligned}
 \Pi'(t) & \leq e^{2\lambda t} \left\{ \sum_{i=1}^n (\beta_i + \gamma_i^2 + 2\lambda \alpha_i \gamma_i - a_i \alpha_i \gamma_i - b_i \alpha_i^2) w_i(t) w'_i(t) \right. \\
 & \quad + \sum_{i=1}^n \left[\lambda \alpha_i^2 + \alpha_i \gamma_i - a_i \alpha_i^2 \right. \\
 & \quad \left. + \frac{1}{2} \alpha_i^2 \sum_{j=1}^n \left(|c_{ij}| L_j^P + |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| du \right) \right] (w'_i(t))^2 \\
 & \quad + \sum_{i=1}^n \left[-b_i \alpha_i \gamma_i + \lambda \beta_i + \lambda \gamma_i^2 + \frac{1}{2} \sum_{j=1}^n \left(|c_{ij}| L_j^P + |d_{ij}| L_j^Q \int_0^{+\infty} |K_{ij}(u)| du \right) |\alpha_i \gamma_i| \right. \\
 & \quad \left. + \frac{1}{2} \sum_{j=1}^n \alpha_j^2 \left(|c_{ji}| L_i^P + |d_{ji}| L_i^Q \int_0^{+\infty} |K_{ji}(u)| e^{2\lambda u} du \right) \right. \\
 & \quad \left. + \frac{1}{2} \sum_{j=1}^n \left(|c_{ji}| L_i^P + |d_{ji}| L_i^Q \int_0^{+\infty} |K_{ji}(u)| e^{2\lambda u} du \right) |\alpha_i \gamma_j| \right] w_i^2(t) \Big\} \\
 & = e^{2\lambda t} \left\{ \sum_{i=1}^n (E_i^\lambda (z'_i(t))^2 + F_i^\lambda w_i^2(t) + H_i^\lambda w_i(t) w'_i(t)) \right\} \\
 & = e^{2\lambda t} \left\{ \sum_{i=1}^n E_i^\lambda \left(w'_i(t) + \frac{H_i^\lambda}{2E_i^\lambda} w_i(t) \right)^2 + \sum_{i=1}^n \left(F_i^\lambda - \frac{(H_i^\lambda)^2}{4E_i^\lambda} \right) w_i^2(t) \right\} \\
 & \leq 0, \quad \forall t \in [0, +\infty).
 \end{aligned} \tag{2.5}$$

This indicates that $\Pi(t) \leq \Pi(0)$ on $[0, +\infty)$ and

$$\frac{1}{2} \sum_{i=1}^n \beta_i z_i^2(t) e^{2\lambda t} + \frac{1}{2} \sum_{i=1}^n (\alpha_i z_i'(t) + \gamma_i z_i(t))^2 e^{2\lambda t} \leq U(0), \quad t \in [0, +\infty).$$

Moreover, by using the Cauchy–Schwarz inequality,

$$(\alpha_i w_i'(t) e^{\lambda t} + \gamma_i w_i(t) e^{\lambda t})^2 = (\alpha_i w_i'(t) + \gamma_i w_i(t))^2 e^{2\lambda t}$$

and

$$\alpha_i |w_i'(t)| e^{\lambda t} \leq |\alpha_i w_i'(t) e^{\lambda t} + \gamma_i w_i(t) e^{\lambda t}| + |\gamma_i w_i(t) e^{\lambda t}|$$

imply that one can take a positive constant $\Lambda_{\varphi, \psi} > 0$ satisfying

$$|w_i'(t)| \leq \Lambda_{\varphi, \psi} e^{-\lambda t}, \quad |w_i(t)| \leq \Lambda_{\varphi, \psi} e^{-\lambda t}, \quad \forall t \geq 0, i \in D,$$

which finishes the proof of Theorem 2.1. □

Corollary 2.1 Under (1.2), define $J_i(t) \equiv J$ as a constant, and

$$\beta_i = \alpha_i^2 b_i + a_i \gamma_i \alpha_i - \gamma_i^2 > 0, \quad E_i < 0, \quad F_i < 0, \quad i \in D.$$

Then the equilibrium point in (1.1) is globally exponentially stable.

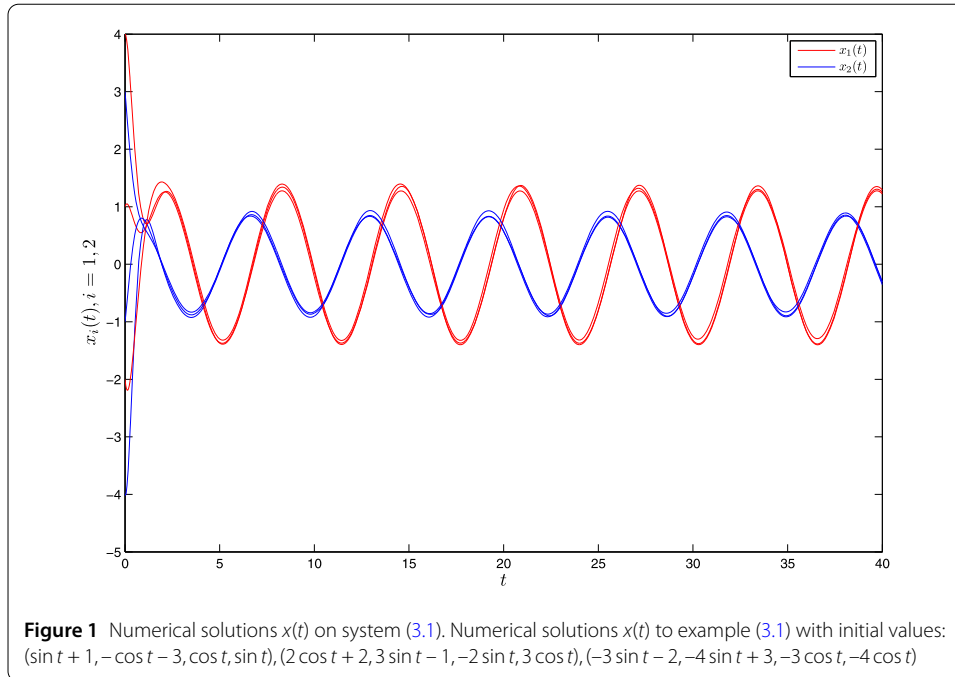
Proof With the aid of the proof of Corollary 2 in [36], one can reveal that (1.1) has exactly one equilibrium y^* . Then Remark 2.1 and Theorem 2.1 give that y^* is globally exponentially stable. This ends the proof. □

Remark 2.2 Since the authors in [36] have not considered the exponential stability on inertial neural networks by using non-reduced order method, our results in Theorem 2.1 and Corollary 2.1 not only improve the main conclusions of [36], but also generalize them. Furthermore, using an argument similar to the one adopted in Theorem 2.1, it is not difficult to obtain the global exponential synchronization of networks (1.1) subject to its driving system.

3 Some simulations

In this section, we give some numeric simulation results to verify our theoretical results. We choose the following models:

$$\begin{cases} x_1''(t) = -3.81x_1'(t) - 8.21x_1(t) + 1.21P_1(x_1(t)) + 1.51P_2(x_2(t)) \\ \quad - 0.81 \int_0^{+\infty} (\sin 2u) e^{-u} Q_1(x_1(t-u)) du \\ \quad + 1.91 \int_0^{+\infty} (\sin 3u) e^{-u} Q_2(x_2(t-u)) du + 10 \sin t, \\ x_2''(t) = -4.71x_2'(t) - 10.91x_2(t) - 0.91P_1(x_1(t)) - 1.71P_2(x_2(t)) \\ \quad - 2.51 \int_0^{+\infty} (\sin 4u) e^{-u} Q_1(x_1(t-u)) du \\ \quad + 2.11 \int_0^{+\infty} (\sin 5u) e^{-u} Q_2(x_2(t-u)) du + 10 \cos t, \\ P_i(u) = Q_i(u) = 0.25(|u+1| - |u-1|), \quad i = 1, 2, \end{cases} \tag{3.1}$$



and

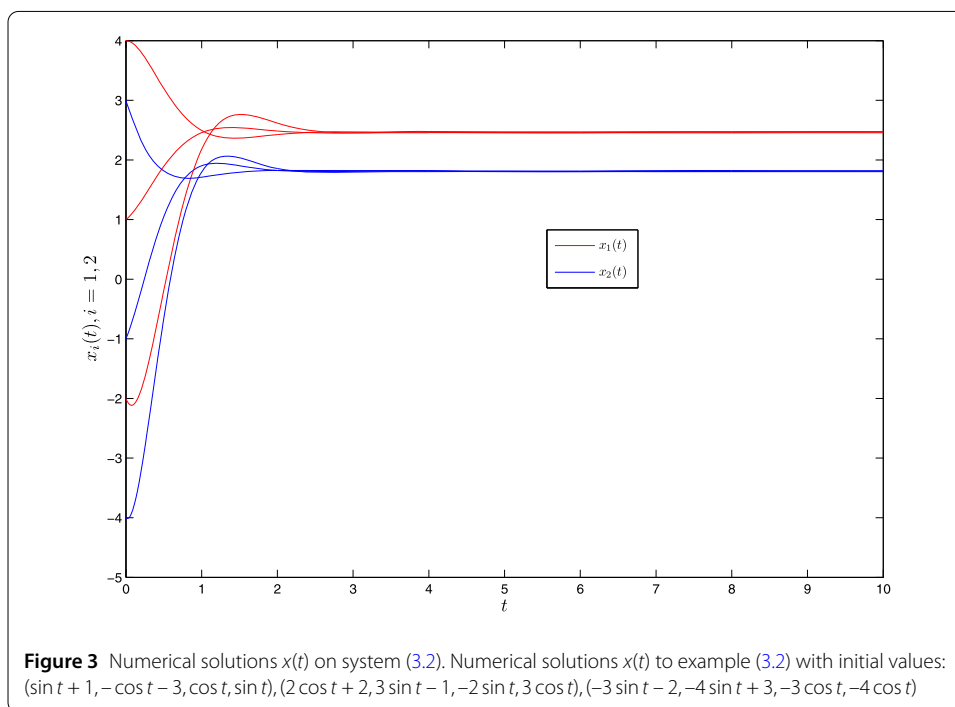
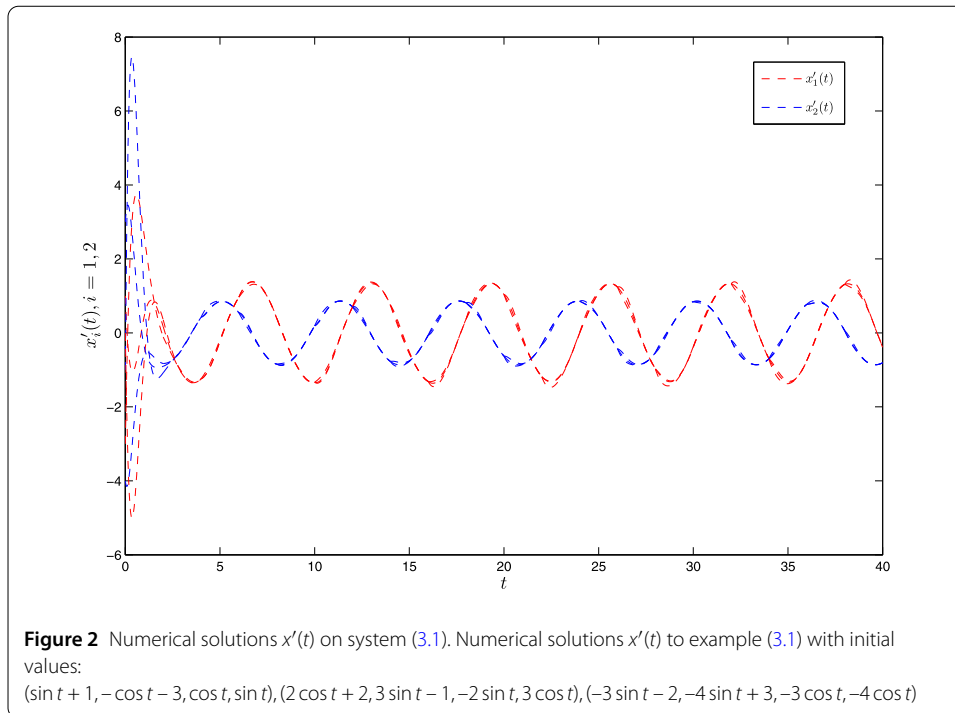
$$\begin{cases} x_1''(t) = -3.81x_1'(t) - 8.21x_1(t) + 1.21P_1(x_1(t)) + 1.51P_2(x_2(t)) \\ \quad - 0.81 \int_0^{+\infty} (\sin 2u)e^{-u} Q_1(x_1(t-u)) du \\ \quad + 1.91 \int_0^{+\infty} (\sin 3u)e^{-u} Q_2(x_2(t-u)) du + 20, \\ x_2''(t) = -4.71x_2'(t) - 10.91x_2(t) - 0.91P_1(x_1(t)) - 1.71P_2(x_2(t)) \\ \quad - 2.51 \int_0^{+\infty} (\sin 4u)e^{-u} Q_1(x_1(t-u)) du \\ \quad + 2.11 \int_0^{+\infty} (\sin 5u)e^{-u} Q_2(x_2(t-u)) du + 20, \\ P_i(u) = Q_i(u) = \frac{1}{4\pi} \arctan u, \quad i = 1, 2. \end{cases} \quad (3.2)$$

It is easy to check that (3.1) and (3.2) satisfy all the conditions made in Theorem 2.1 and Corollary 2.1, respectively. Consequently, (3.1) and (3.2) are globally exponentially stable. The numeric simulations in Figs. 1–4 support the theoretical results in Sect. 2.

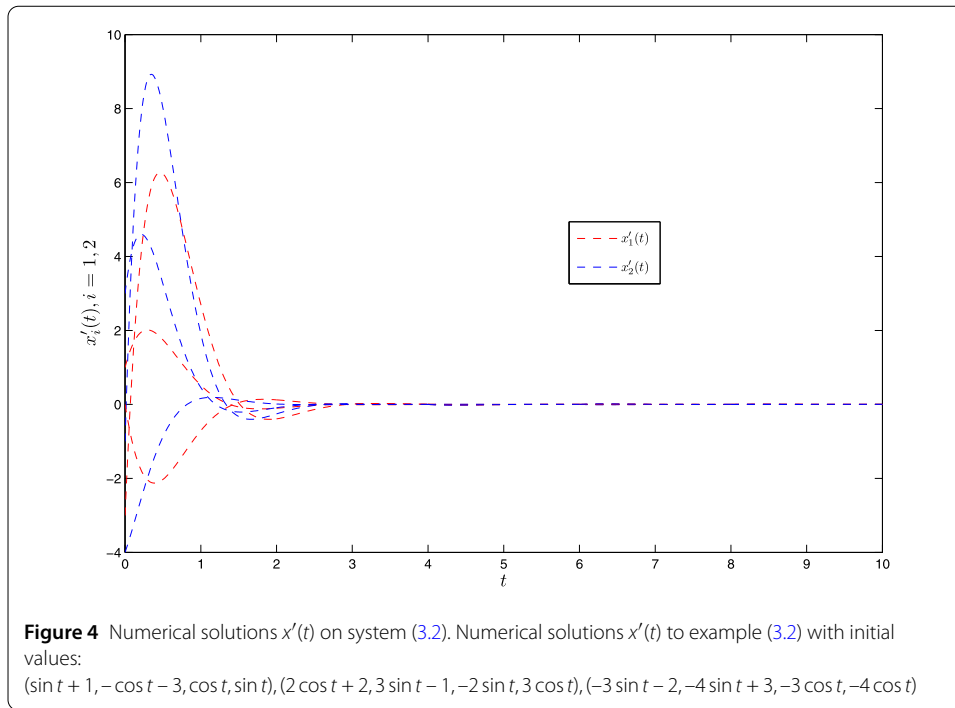
Remark 3.1 Because the exponential stability of unbounded distributed delayed inertial neural networks has never been touched by the aid of the non-reduced order method, it is clear to find that all results in the references [15–37, 39–78] cannot be straightly employed to reveal the exponential convergence on the solutions and their derivative for networks (3.1) and (3.2).

4 Conclusions

In this article, without utilizing the reduced order technique, the global exponential stability of unbounded continuously distributed delayed inertial neural networks has been considered. By combining Lyapunov function way with differential inequality analysis, some sufficient assertions have been gained to evidence the global exponential convergence on



all solution and their derivatives in the addressed networks. It should be pointed out that our assumptions are easily checked in practice by simple inequality technique, and the approach adopted in this paper provides a possible way to investigate the dynamic topic on other unbounded continuously distributed delayed inertial neural networks. We would like to extend our approach to study the periodicity and dissipativity for unbounded distributed delayed inertial neural network models.



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Availability of data and materials

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

CH and JZ contributed equally to this revised work. All authors read and approved the final manuscript.

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