RESEARCH

Open Access



Global attractivity of Leslie–Gower predator-prey model incorporating prey cannibalism

Qifa Lin^{1*}, Chulei Liu², Xiangdong Xie¹ and Yalong Xue¹

*Correspondence: lqfnd_118@163.com ¹ Department of Mathematics, Ningde Normal University, Ningde, China Full list of author information is available at the end of the article

Abstract

Leslie–Gower predator prey model with Holling II type cannibalism term is proposed and studied in this paper. Local and global stability properties of the system are investigated. Our study indicates that cannibalism has no influence on the local stability property of the equilibrium, which is quite different to the known results. Numeric simulations are carried out to show the feasibility of the main results.

Keywords: Leslie–Gower predator prey model; Cannibalism; Stability

1 Introduction

The aim of this paper is to investigate the dynamic behaviors of the following Leslie–Gower predator prey model with prey cannibalism:

$$\frac{dH}{dt} = (r_1 + c_1 - a_1 P - b_1 H)H - \frac{fH^2}{d + H},$$

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right)P,$$
(1.1)

where *H* and *P* are the density of prey species and the predator species at time *t*, respectively. r_i , i = 1, 2, are the intrinsic growth rate of the prey and predator species, respectively. r_1/b_1 is the environment carrying capacity of the prey species, *f* is the cannibalism rate. $C(H) = f \times H \times \frac{H}{H+d}$ is the generic cannibalism term. c_1H is the new offsprings due to the cannibalism. Obviously, $c_1 < f$, since it takes depredation of a number of prey by the cannibal to produce one new offspring.

During the last decades, mathematics biology has become one of the important research areas [1–40]. Specially, many scholars investigated the dynamic behaviors of the ecosystem with cannibalism (see [28–34] and the references cited therein). Cannibalism often occurs in plankton, fishes, spiders [28], and social insect populations [29]. It is a behavior that consumes the same species and helps to provide food sources.

© The Author(s) 2020. This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit http://creativecommons.org/licenses/by/4.0/.



. . . .

Leslie [35] introduced the following predator-prey model where the "carrying capacity" of the predator's environment is proportional to the number of prey:

$$\frac{dH}{dt} = (r_1 - a_1 P - b_1 H)H,$$

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right)P,$$
(1.2)

where H and P are the density of prey species and the predator species at time t, respectively. Obviously, the above system admits a unique coexisting fixed point

$$H^* = \frac{r_1 a_2}{a_1 r_2 + a_2 b_1}, \qquad P^* = \frac{r_1 r_2}{a_1 r_2 + a_2 b_1}.$$
(1.3)

By applying linear analysis, one could easily show that this fixed point is stable. By constructing some suitable Lyapunov function, Korobeinikov [36] showed that the fixed point is globally stable.

Chen [37] extended model (1.2) by incorporating a refuge protecting *mH* of the prey, where $m \in [0, 1)$ is constant. This leaves (1 - m)H of the prey available to the predator, and modifying system (1.2) to the following system:

$$\frac{dH}{dt} = (r_1 - b_1 H)H - a_1(1 - m)HP,
\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{(1 - m)H}\right)P,$$
(1.4)

where $m \in [0, 1)$ and $r_i, a_i, i = 1, 2, b_1$ are all positive constants. He showed that prey refuge has no influence on the persistence property of the system.

Liang and Pan [38] proposed the following ratio-dependent Holling-Tanner model:

$$\frac{dx}{dt} = rx\left(1 - \frac{x}{k}\right) - \frac{mx}{Ay + x}y,$$

$$\frac{dy}{dt} = y\left[s\left(1 - h\frac{y}{x}\right)\right],$$
(1.5)

where r, k, m, A, s, h are all positive constants. Sufficient conditions which ensure the global stability of the positive equilibrium and the existence of a unique limit cycle have been obtained, respectively.

In 2016, Basheer et al. [31] proposed the prey-predator model with prey cannibalism as follows:

$$\frac{du}{dt} = u(1+c_1-u) - \frac{uv}{u+\alpha v} - c\frac{u^2}{u+d},$$

$$\frac{dv}{dt} = \delta v \left(\beta - \frac{v}{u}\right),$$
(1.6)

where $c_1 < c$, u and v represent the densities of prey and predator at time t, respectively. The parameters c_1 , α , c, d, δ , and β are all nonnegative constants. Here the generic cannibalism term C(u) is added in the prey equation and is given by

$$C(u)=c\times u\times \frac{u}{u+d},$$

where c is the cannibalism rate. This term has a clear gain of energy to the cannibalistic prey, and this leads to the increase in reproduction in the prey, modeled via adding a c_1u term to the prey equation. The authors showed that prey cannibalism cannot stabilize the unstable interior equilibrium, in certain parameter regime, but can destabilize the stable interior equilibrium, leading to a stable limit cycle. It brings to our attention that the authors only investigated the local stability property of the equilibrium and did not investigate the global stability property of the positive equilibrium, nor did they investigate the uniqueness of the limit cycle.

Recently, stimulated by the works of Basheer et al. [31, 32], based on the traditional Lotka–Volterra type predator prey system, Deng et al. [33] investigated the dynamic behaviors of the following predator-prey model with cannibalism for predator:

$$\frac{dx}{dt} = x(b - \alpha x - my),$$

$$\frac{dy}{dt} = y(-\beta + c_1 + nx) - \frac{cy^2}{y + d},$$
(1.7)

where $c_1 < c$, x and y are the density of the prey and predator at time t, respectively. The authors showed that cannibalism has both positive and negative effects on the stability of the system, it depends on the dynamic behaviors of the original system. If the predator species in the system without cannibalism is extinct, then suitable cannibalism may lead to the coexistence of both species; in this case, cannibalism stabilizes the system. If the two species coexist in the stable state in the original system, then predator cannibalism may lead to the extinction of the prey species. In this case, cannibalism has an unstable effect. Also, unlike Basheer et al. [31, 32], by constructing some suitable Lyapunov function, Deng et al. [33] showed that if system (1.7) has a positive equilibrium, it then is globally asymptotically stable.

Stimulated by the works of Basheer et al. [31, 32] and Deng et al. [33], based on model (1.2), we propose the Leslie–Gower predator prey model with prey cannibalism, i.e., system (1.1). As far as system (1.1) is concerned, two interesting issues are proposed: Can we obtain sufficient conditions to ensure the existence of a unique globally stable positive equilibrium? Can we give some positive answer on the influence of the cannibalism on the dynamic behaviors of the system?

The rest of the paper is arranged as follows. In the next section, we investigate the existence and local stability of the equilibrium of system (1.1). In Sect. 3, we discuss the global stability of the equilibrium by using the iterative method. Numeric simulations are presented in Sect. 4 to show the feasibility of the main results. We end this paper with a brief discussion.

2 The existence and local stability of the equilibria of system (1.1)

Concerned with the existence of the equilibria of system (1.1), we have the following result.

Theorem 2.1 System (1.1) admits the boundary equilibrium $A(H_0, 0)$ and the unique positive equilibrium $B(H^*, P^*)$, where

$$H_0 = \frac{-A_2 + \sqrt{A_2^2 - 4A_1A_3}}{2A_1},$$

$$H^{*} = \frac{-B_{2} + \sqrt{B_{2}^{2} - 4B_{1}B_{3}}}{2B_{1}}, P^{*} = \frac{r_{2}H^{*}}{a_{2}},$$

$$A_{1} = b_{1} > 0,$$

$$A_{2} = b_{1}d - c_{1} + f - r_{1},$$

$$A_{3} = -c_{1}d - dr_{1} < 0,$$

$$B_{1} = a_{1}r_{2} + a_{2}b_{1} > 0,$$

$$B_{2} = a_{1}dr_{2} + a_{2}b_{1}d - a_{2}c_{1} + a_{2}f - a_{2}r_{1},$$

$$B_{3} = -a_{2}c_{1}d - a_{2}dr_{1} < 0.$$
(2.1)

Proof The equilibria of system (1.1) satisfy the equation

$$(r_1 + c_1 - a_1 P - b_1 H)H - \frac{fH^2}{d + H} = 0,$$

 $\left(r_2 - a_2 \frac{P}{H}\right)P = 0.$ (2.2)

From the second equation of (2.2), one has P = 0 or $P = \frac{r_2 H}{a_2}$. Substituting P = 0 to the first equation of (2.2) leads to

$$(r_1 + c_1 - b_1 H) - \frac{fH}{d + H} = 0.$$
(2.3)

Equation (2.3) is equivalent to

$$A_1H^2 + A_2H + A_3 = 0, (2.4)$$

where A_1, A_2, A_3 are defined by (2.1). (2.4) has a unique positive solution H_0 , hence, system (1.1) has the boundary equilibrium $A(H_0, 0)$.

Substituting $P = \frac{r_2 H}{a_2}$ to the first equation of (2.2) leads to

$$H\left(-Hb_1 - \frac{Ha_1r_2}{a_2} + c_1 + r_1\right) - \frac{fH^2}{d+H} = 0.$$
 (2.5)

Equation (2.5) is equivalent to

$$B_1 H^2 + B_2 H + B_3 = 0, (2.6)$$

where B_1, B_2, B_3 are defined by (2.1). (2.6) has a unique positive solution H^* , hence, system (1.1) has the boundary equilibrium $B(H^*, P^*)$.

This ends the proof of Theorem 2.1.

Theorem 2.2 $A(H_0, 0)$ is unstable equilibrium, and $B(H^*, P^*)$ is locally asymptotically stable.

Proof The Jacobian matrix of system (1.1) is calculated as follows:

$$J(H,P) = \begin{pmatrix} A_{11} & -a_1H\\ \frac{P^2a_2}{H^2} & r_2 - 2\frac{a_2P}{H} \end{pmatrix},$$
(2.7)

where

$$A_{11} = -2Hb_1 - Pa_1 + c_1 + r_1 - 2\frac{Hf}{d+H} + \frac{fH^2}{(d+H)^2}$$

Then the Jacobian matrix of system (1.1) about the equilibrium $A(H_0, 0)$ is

$$J(A(H_0, 0)) = \begin{pmatrix} -2H_0b_1 + c_1 + r_1 - 2\frac{H_0f}{d+H_0} + \frac{fH_0^2}{(d+H_0)^2} & -a_1H_0 \\ 0 & r_2 \end{pmatrix}$$
$$= \begin{pmatrix} -H_0b_1 - \frac{dfH_0}{(d+H_0)^2} & -a_1H_0 \\ 0 & r_2 \end{pmatrix}.$$
(2.8)

The eigenvalues of J(A) are $\lambda_1 = r_2 > 0$, $\lambda_2 = -H_0b_1 - \frac{dfH_0}{(d+H_0)^2} < 0$. Thus, $A(H_0, 0)$ is a saddle. The Jacobian matrix of system (1.1) about the equilibrium $B(H^*, P^*)$ is

$$J(B(H^*, P^*)) = \begin{pmatrix} -H^*b_1 - \frac{dfH^*}{(d+H^*)^2} & -a_1H^* \\ r_2\frac{P^*}{H^*} & -r_2 \end{pmatrix}.$$
 (2.9)

Then we have

$$\operatorname{Det} J(B(H^*, P^*) = r_2 \left(H^* b_1 + \frac{df H^*}{(d+H^*)^2} \right) + a_1 H^* r_2 \frac{P^*}{H^*} > 0$$

and

$$\operatorname{Tr} J(B(H^*, P^*)) = -H^*b_1 - \frac{dfH^*}{(d+H^*)^2} - r_2 < 0.$$

So that both eigenvalues of $J(B(H^*, P^*))$ have negative real parts [38], and $B(H^*, P^*)$ is locally asymptotically stable.

This ends the proof of Theorem 2.2.

3 Global attractivity

The aim of this section is to investigate the global attractivity of the positive equilibrium of system (1.1). To do so, we need the following lemma.

Lemma 3.1 If a > 0, b > 0 and $\dot{x} \ge x(b - ax)$, when $t \ge 0$ and x(0) > 0, we have

$$\liminf_{t\to+\infty} x(t) \geq \frac{b}{a}.$$

If a > 0, b > 0 and $\dot{x} \le x(b - ax)$, when $t \ge 0$ and x(0) > 0, we have

$$\limsup_{t\to+\infty} x(t) \le \frac{b}{a}.$$

The above lemma is a direct corollary of Lemma 2.2 of Chen [40], so we omit the detailed proof here.

Concerned with the global attractivity of the positive equilibrium, we have the following result.

Theorem 3.1 The positive equilibrium $B(H^*, P^*)$ is globally attractive provided that

$$a_1 r_2 < a_2 b_1 \tag{3.1}$$

holds.

Proof From (3.1), we could choose $\varepsilon > 0$ small enough such that

$$r_1 + c_1 - a_1 \frac{r_2}{a_2} \left(\frac{r_1 + c_1}{b_1} + \varepsilon \right) - \left(2 + \frac{a_2}{r_2} \right) \varepsilon > 0.$$
(3.2)

Indeed, inequality (3.1) is equivalent to

$$r_1 + c_1 > a_1 \frac{r_2}{a_2} \frac{(r_1 + c_1)}{b_1},$$

hence, for $\varepsilon > 0$, which satisfies

$$\varepsilon < \frac{r_1 + c_1 - a_1 \frac{r_2}{a_2} \frac{(r_1 + c_1)}{b_1}}{a_1 \frac{r_2}{a_2} + 2 + \frac{a_2}{r_2}},$$

inequality (3.2) holds.

Let (H(t), P(t)) be any positive solution of system (1.1). From the first equation of (1.1), we have

$$\frac{dH}{dt} = (r_1 + c_1 - a_1 P - b_1 H)H - \frac{fH^2}{d + H} \\
\leq (r_1 + c_1 - b_1 H)H.$$
(3.3)

Applying Lemma 3.1 to (3.3) leads to

$$\limsup_{t \to +\infty} H(t) \le \frac{r_1 + c_1}{b_1}.$$
(3.4)

For $\varepsilon > 0$ small enough, which satisfies (3.2), it follows from (3.4) that there exists $T_{11} > 0$ such that

$$H(t) < \frac{r_1 + c_1}{b_1} + \varepsilon \stackrel{\text{def}}{=} M_1^{(1)}.$$
(3.5)

From the second equation of (1.1), for $t > T_{11}$, we have

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right) P$$

$$\leq \left(r_2 - a_2 \frac{P}{M_1^{(1)}}\right) P.$$
(3.6)

Applying Lemma 3.1 to (3.6) leads to

$$\limsup_{t \to +\infty} P(t) \le \frac{r_2 M_1^{(1)}}{a_2}.$$
(3.7)

For $\varepsilon > 0$ small enough, which satisfies (3.2), it follows from (3.7) that there exists $T_{12} > 0$ such that

$$P(t) < \frac{r_2 M_1^{(1)}}{a_2} + \varepsilon \stackrel{\text{def}}{=} M_2^{(1)}.$$
(3.8)

From the first equation of (1.1) and (3.8), for $t \ge T_{12}$, we also have

$$\frac{dH}{dt} = (r_1 + c_1 - a_1 P - b_1 H)H - \frac{fH^2}{d + H} \\
\geq \left(r_1 + c_1 - a_1 M_2^{(1)} - \left(b_1 + \frac{f}{d}\right)H\right)H.$$
(3.9)

From (3.2) we could see that inequality (3.1) implies that

$$r_1 + c_1 - a_1 M_2^{(1)} > 0. (3.10)$$

Hence, applying Lemma 3.1 to (3.9) leads to

$$\liminf_{t \to +\infty} H(t) \ge \frac{r_1 + c_1 - a_1 M_2^{(1)}}{b_1 + \frac{f}{d}}.$$
(3.11)

For $\varepsilon > 0$, which satisfies (3.2), it follows from (3.11) that there exists $T_{13} > T_{12}$ such that

$$H(t) > \frac{r_1 + c_1 - a_1 M_2^{(1)}}{b_1 + \frac{f}{d}} - \varepsilon \stackrel{\text{def}}{=} m_1^{(1)}.$$
(3.12)

From (3.12) and the second equation of (1.1), for $t \ge T_{13}$, we have

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right)P$$

$$\geq \left(r_2 - a_2 \frac{P}{m_1^{(1)}}\right)P.$$
(3.13)

Applying Lemma 3.1 to (3.13) leads to

$$\liminf_{t \to +\infty} P(t) \ge \frac{r_2 m_1^{(1)}}{a_2}.$$
(3.14)

For $\varepsilon > 0$, which satisfies (3.2), it follows from (3.14) that there exists $T_{14} > T_{13}$ such that

$$P(t) > \frac{r_2 m_1^{(1)}}{a_2} - \varepsilon \stackrel{\text{def}}{=} m_2^{(1)}.$$
(3.15)

From the first equation of (1.1), for $t > T_{14}$, we have

$$\frac{dH}{dt} = (r_1 + c_1 - a_1 P - b_1 H)H - \frac{fH^2}{d + H} \\
\leq \left(r_1 + c_1 - a_1 m_2^{(1)} - \left(b_1 + \frac{f}{d + M_1^{(1)}}\right)H\right)H.$$
(3.16)

Applying Lemma 3.1 to (3.16) leads to

$$\limsup_{t \to +\infty} H(t) \le \frac{r_1 + c_1 - a_1 m_2^{(1)}}{b_1 + \frac{f}{d + M_1^{(1)}}}.$$
(3.17)

For $\varepsilon > 0$ small enough, which satisfies (3.2), it follows from (3.17) that there exists $T_{21} > 0$ such that

$$H(t) < \frac{r_1 + c_1 - a_1 m_2^{(1)}}{b_1 + \frac{f}{d + M_1^{(1)}}} + \frac{\varepsilon}{2} \stackrel{\text{def}}{=} M_1^{(2)}.$$
(3.18)

Obviously,

$$M_1^{(2)} = \frac{r_1 + c_1 - a_1 m_2^{(1)}}{b_1 + \frac{f}{d + M_1^{(1)}}} + \frac{\varepsilon}{2} < \frac{r_1 + c_1}{b_1} + \varepsilon = M_1^{(1)}.$$
(3.19)

From the second equation of (1.1), for $t > T_{21}$, we have

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right) P$$

$$\leq \left(r_2 - a_2 \frac{P}{M_1^{(2)}}\right) P.$$
(3.20)

Applying Lemma 3.1 to (3.20) leads to

$$\limsup_{t \to +\infty} P(t) \le \frac{r_2 M_1^{(2)}}{a_2}.$$
(3.21)

For $\varepsilon > 0$ small enough, which satisfies (3.2), it follows from (3.21) that there exists $T_{22} > 0$ such that

$$P(t) < \frac{r_2 M_1^{(2)}}{a_2} + \varepsilon \stackrel{\text{def}}{=} M_2^{(2)}.$$
(3.22)

From (3.8), (3.19), and (3.22), one has

$$M_2^{(2)} < M_2^{(1)}. \tag{3.23}$$

From the first equation of (1.1) and (3.8), for $t \ge T_{12}$, we also have

$$\frac{dH}{dt} = (r_1 + c_1 - a_1 P - b_1 H)H - \frac{fH^2}{d+H}$$

$$\geq \left(r_1 + c_1 - a_1 M_2^{(2)} - \left(b_1 + \frac{f}{d + m_1^{(1)}}\right)H\right)H.$$
(3.24)

From (3.2) and (3.23) we could see that

$$r_1 + c_1 - a_1 M_2^{(2)} > 0. (3.25)$$

Hence, applying Lemma 3.1 to (3.24) leads to

$$\liminf_{t \to +\infty} H(t) \ge \frac{r_1 + c_1 - a_1 M_2^{(2)}}{b_1 + \frac{f}{d + m_1^{(1)}}}.$$
(3.26)

For $\varepsilon > 0$, which satisfies (3.2), it follows from (3.26) that there exists $T_{23} > T_{22}$ such that

$$H(t) > \frac{r_1 + c_1 - a_1 M_2^{(2)}}{b_1 + \frac{f}{d + m_1^{(1)}}} - \frac{\varepsilon}{2} \stackrel{\text{def}}{=} m_1^{(2)}.$$
(3.27)

From (3.12), (3.23), and (3.27) one has

$$m_1^{(2)} > m_1^{(1)}$$
. (3.28)

From (3.28) and the second equation of (1.1), for $t \ge T_{23}$, we have

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right)P$$

$$\geq \left(r_2 - a_2 \frac{P}{m_1^{(2)}}\right)P.$$
(3.29)

Applying Lemma 3.1 to (3.29) leads to

$$\liminf_{t \to +\infty} P(t) \ge \frac{r_2 m_1^{(2)}}{a_2}.$$
(3.30)

For $\varepsilon > 0$, which satisfies (3.2), it follows from (3.30) that there exists $T_{24} > T_{23}$ such that

$$P(t) > \frac{r_2 m_1^{(2)}}{a_2} - \varepsilon \stackrel{\text{def}}{=} m_2^{(2)}.$$
(3.31)

From (3.15), (3.28), and (3.31) one has

$$m_2^{(2)} > m_2^{(1)}$$
 (3.32)

It follows from (3.18), (3.22), (3.27), and (3.31) that, for all $t \ge T_{24}$,

$$0 < m_1^{(1)} < m_1^{(2)} < x(t) < M_1^{(2)} < M_1^{(1)},$$

$$0 < m_2^{(1)} < m_2^{(2)} < y(t) < M_2^{(2)} < M_2^{(1)}.$$
(3.33)

Repeating the above procedure, we get four sequences $M_i^{(n)}$, $m_i^{(n)}$, i = 1, 2, n = 1, 2, ..., such that

$$M_1^{(n)} = \frac{r_1 + c_1 - a_1 m_2^{(n-1)}}{b_1 + \frac{f}{d + M_1^{(n-1)}}} + \frac{\varepsilon}{n},$$
(3.34)

$$m_1^{(n)} = \frac{r_1 + c_1 - a_1 M_2^{(n)}}{b_1 + \frac{f}{d + m_1^{(n-1)}}} - \frac{\varepsilon}{n},$$
(3.35)

$$M_2^{(n)} = \frac{r_2 M_1^{(n)}}{a_2} + \frac{\varepsilon}{n},$$
(3.36)

$$m_2^{(n)} = \frac{r_2 m_1^{(n)}}{a_2} - \frac{\varepsilon}{n}.$$
(3.37)

Now, we will show that the sequences $M_i^{(n)}$ are strictly decreasing, and the sequences $m_i^{(n)}$ are strictly increasing for i = 1, 2 by induction. Firstly, from (3.33), we have

$$m_i^{(1)} < m_i^{(2)}, \qquad M_i^{(2)} < M_i^{(1)}, \quad i = 1, 2.$$
 (3.38)

Let us suppose that

$$m_i^{(n-1)} < m_i^{(n)}, \qquad M_i^{(n)} < M_i^{(n-1)}, \quad i = 1, 2.$$
 (3.39)

It then follows from (3.34) and (3.39) that

$$M_1^{(n)} > M_1^{(n+1)}. ag{3.40}$$

By using (3.40), it follows from (3.36) that

$$M_2^{(n)} > M_2^{(n+1)}. ag{3.41}$$

It then follows from (3.35), (3.39), and (3.41) that

$$m_1^{(n+1)} > m_1^{(n)}$$
. (3.42)

From (3.37) and (3.42), we have

$$m_2^{(n+1)} > m_2^{(n)}$$
. (3.43)

Therefore, we have

$$0 < m_1^{(1)} < m_1^{(2)} < \dots < m_1^{(n)} < x(t) < M_1^{(n)} < \dots < M_1^{(2)} < M_1^{(1)},$$

$$0 < m_2^{(1)} < m_2^{(2)} < \dots < m_2^{(n)} < y(t) < M_2^{(n)} < \dots < M_2^{(2)} < M_2^{(1)}.$$
(3.44)

Hence, the limits of $M_i^{(n)}$ and $m_i^{(n)}$, i = 1, 2, n = 1, 2, ..., exist. Denote that

$$\lim_{n \to +\infty} M_1^{(n)} = \overline{H}, \qquad \lim_{n \to +\infty} m_1^{(n)} = \underline{H}, \qquad \lim_{n \to +\infty} M_2^{(n)} = \overline{P}, \qquad \lim_{n \to +\infty} m_2^{(n)} = \underline{P}.$$
 (3.45)

Then $\overline{H} \ge \underline{H}, \overline{P} \ge \underline{P}$. Letting $n \to +\infty$ in (3.34)–(3.37), we obtain

$$\overline{H} = \frac{r_1 + c_1 - a_1 \underline{P}}{b_1 + \frac{f}{d + \overline{H}}},\tag{3.46}$$

$$\underline{H} = \frac{r_1 + c_1 - a_1 \overline{P}}{b_1 + \frac{f}{d + \underline{H}}},$$
(3.47)

$$\overline{P} = \frac{r_2 \overline{H}}{a_2},\tag{3.48}$$

$$\underline{P} = \frac{r_2 \underline{H}}{a_2}.\tag{3.49}$$

Substituting (3.48) and (3.49) to (3.46) and (3.47), it then follows

$$b_1\overline{H} + \frac{f\overline{H}}{d+\overline{H}} = r_1 + c_1 - a_1\frac{r_2\underline{H}}{a_2},$$
(3.50)

$$b_1\underline{H} + \frac{f\underline{H}}{d+\underline{H}} = r_1 + c_1 - a_1\frac{r_2\overline{H}}{a_2}.$$
(3.51)

Subtracting the above two equalities leads to

$$b_1(\overline{H} - \underline{H}) + \frac{f\overline{H}}{d + \overline{H}} - \frac{f\underline{H}}{d + \underline{H}} = a_1 \frac{r_2}{a_2} (\overline{H} - \underline{H}), \qquad (3.52)$$

which is equivalent to

$$\left(b_1 - a_1 \frac{r_2}{a_2}\right) (\overline{H} - \underline{H}) + \frac{df(\overline{H} - \underline{H})}{(d + \overline{H})(d + \underline{H})} = 0.$$
(3.53)

Since condition (3.1) implies that

$$b_1 - a_1 \frac{r_2}{a_2} > 0. (3.54)$$

Also

$$df > 0.$$
 (3.55)

Hence, it follows from (3.53) that

$$\overline{H} = \underline{H}.$$
(3.56)

From (3.48), (3.49), and (3.56), we have

 $\overline{P} = \underline{P}.$

Under the assumption of Theorem 3.1, system (1.1) admits a unique positive solution (H^*, P^*) , hence $\overline{H} = \underline{H} = H^*, \overline{P} = \underline{P} = P^*$. That is to say,

$$\lim_{t \to +\infty} H(t) = H^*, \qquad \lim_{t \to +\infty} P(t) = P^*.$$
(3.57)

This ends the proof of Theorem 3.1.





4 Numeric simulations

Example 4.1 Now let us consider the following model:

$$\frac{dH}{dt} = (r_1 + c_1 - a_1 P - b_1 H) H - \frac{fH^2}{d + H},$$

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right) P.$$
(4.1)

Here, corresponding to system (1.1), we take $r_1 = b_1 = f = d = r_2 = a_2 = 1$, $c_1 = 0.5$, $a_1 = 0.2$, then one could see that

$$a_1r_2 = 0.2 < a_2b_1 = 1.$$

Hence, it follows from Theorem 3.1 that system (4.1) admits a unique positive equilibrium, which is globally attractive. Numeric simulations (Figs. 1 and 2) also support this assertion.



Example 4.2 Now let us consider the following model:

$$\frac{dH}{dt} = (r_1 + c_1 - a_1 P - b_1 H)H - \frac{fH^2}{d + H},$$

$$\frac{dP}{dt} = \left(r_2 - a_2 \frac{P}{H}\right)P.$$
(4.2)

Here, corresponding to system (1.1), we take $r_1 = b_1 = f = d = a_2 = 1$, $c_1 = 0.5$, $a_1 = 2$, $r_2 = 2$. Then one could see that

$$a_1r_2 = 4 > a_2b_1 = 1.$$

Hence, inequality (3.1) does not hold, one could make a conclusion from Theorem 2.1 that system (4.2) admits a unique positive equilibrium, which is locally asymptotically stable. However, one could not draw any conclusion about the global asymptotic stability of the positive equilibrium. Numeric simulation (Fig. 3) shows that the positive equilibrium is globally asymptotically stable.

5 Conclusion

Based on the traditional Leslie–Gower predator prey model and the works of Basheer et al. [31, 32] and Deng et al. [33], we proposed a Leslie–Gower predator prey model incorporating the nonlinear cannibalism. Already Basheer et al. [31] incorporated the cannibalism to the Holling–Tanner model with ratio-dependent functional response (i.e., system (1.2)). They showed that cannibalism in the prey cannot stabilize the unstable interior equilibrium in the ODE case, but can destabilize the stable interior equilibrium, leading to a stable limit cycle. In this paper, we focus our attention on the most simple Leslie–Gower predator prey model. Our study shows that the system with cannibalism always admits a positive equilibrium and a predator free equilibrium, the predator free equilibrium is unstable, while the positive equilibrium is locally asymptotically stable. Compared with the Leslie–Gower predator prey system without cannibalism (i.e., system (1.2)), our result shows that cannibalism has no influence on the local stability property of the positive equilibrium, this is quite different to the results of Basheer et al. [31, 32].

are also different to those of Deng et al. [33]: for the system they considered, cannibalism may have both positive or negative effects on the stability of the system. Also, under some very simple conditions, we could also prove that the positive equilibrium is globally attractive. It brings to our attention that inequality (3.1) is independent of the coefficients of the cannibalism term. However, for the system without cannibalism (i.e., system (1.2)), the positive equilibrium is globally attractive without any restriction on the coefficients, and numeric simulation (Fig. 3) also shows that (3.1) could be dropped out. Hence we have a conjecture:

Conjecture Condition (3.1) is not needed to ensure the positive equilibrium of system (1.1) to be globally attractive.

However, at present, we have difficulty to prove this conjecture, we leave this for future investigation.

Acknowledgements

The authors would like to thank Dr. Hang Deng for bringing our attention to the paper of A. Basheer.

Funding

The research was supported by the Natural Science Foundation of Fujian Province (2019J01841).

Availability of data and materials

Data sharing not applicable to this article as no datasets were generated or analysed during the current study.

Ethics approval and consent to participate Not applicable.

Competing interests

The authors declare that there is no conflict of interests.

Consent for publication

Not applicable.

Authors' contributions

All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

Author details

¹Department of Mathematics, Ningde Normal University, Ningde, China. ²College of Mathematics and Computer Science, Fuzhou University, Fuzhou, China.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 2 February 2020 Accepted: 27 March 2020 Published online: 08 April 2020

References

- 1. Chen, F.D., Chen, W.L., et al.: Permanence of a stage-structured predator-prey system. Appl. Math. Comput. 219(17), 8856–8862 (2013)
- Chen, F.D., Xie, X.D., et al.: Partial survival and extinction of a delayed predator-prey model with stage structure. Appl. Math. Comput. 219(8), 4157–4162 (2012)
- Chen, F.D., Wang, H.N., Lin, Y.H., Chen, W.L.: Global stability of a stage-structured predator-prey system. Appl. Math. Comput. 223, 45–53 (2013)
- Li, T.T., Chen, F.D., et al.: Stability of a mutualism model in plant-pollinator system with stage-structure and the Beddington–DeAngelis functional response. J. Nonlinear Funct. Anal. 2017, Article ID 50 (2017)
- Yu, S.: Global stability of a modified Leslie–Gower model with Beddington–DeAngelis functional response. Adv. Differ. Equ. 2014, Article ID 84 (2014)
- Li, Z., Han, M.A., et al.: Global stability of stage-structured predator-prey model with modified Leslie–Gower and Holling-type II schemes. Int. J. Biomath. 6, Article ID 1250057 (2012)
- Li, Z., Han, M., et al.: Global stability of a predator-prey system with stage structure and mutual interference. Discrete Contin. Dyn. Syst., Ser. B 19(1), 173–187 (2014)
- 8. Lin, X., Xie, X., et al.: Convergences of a stage-structured predator-prey model with modified Leslie-Gower and Holling-type II schemes. Adv. Differ. Equ. 2016, Article ID 181 (2016)

- 9. Xiao, Z., Li, Z., Zhu, Z., et al.: Hopf bifurcation and stability in a Beddington–DeAngelis predator-prey model with stage structure for predator and time delay incorporating prey refuge. Open Math. **17**(1), 141–159 (2019)
- 10. Zhao, L., Qin, B., Chen, F.: Permanence and global stability of a may cooperative system with strong and weak cooperative partners. Adv. Differ. Equ. **2018**, Article ID 172 (2018)
- Yue, Q.: Permanence of a delayed biological system with stage structure and density-dependent juvenile birth rate. Eng. Lett. 27(2), 1–5 (2019)
- 12. Lei, C.: Dynamic behaviors of a stage-structured commensalism system. Adv. Differ. Equ. 2018, Article ID 301 (2018)
- Lei, C.Q.: Dynamic behaviors of a stage structure amensalism system with a cover for the first species. Adv. Differ. Equ. 2018, Article ID 272 (2018)
- Lin, Q., Xie, X., et al.: Dynamical analysis of a logistic model with impulsive Holling type-II harvesting. Adv. Differ. Equ. 2018, Article ID 112 (2018)
- 15. Xie, X., Xue, Y., et al.: Permanence and global attractivity of a nonautonomous modified Leslie–Gower predator-prey model with Holling-type II schemes and a prey refuge. Adv. Differ. Equ. **2016**, Article ID 184 (2016)
- Xie, X.D., Chen, F.D., et al.: Note on the stability property of a cooperative system incorporating harvesting. Discrete Dyn. Nat. Soc. 2014, Article ID 327823 (2014)
- Wu, R., Li, L: Permanence and global attractivity of the discrete predator-prey system with Hassell–Varley–Holling III type functional response. Discrete Dyn. Nat. Soc. 2013, Article ID 393729 (2013)
- Xue, Y., Xie, X., et al.: Global attractivity and extinction of a discrete competitive system with infinite delays and single feedback control. Discrete Dyn. Nat. Soc. 2018, Article ID 1893181 (2018)
- Xue, Y., Xie, X., et al.: Almost periodic solution of a discrete commensalism system. Discrete Dyn. Nat. Soc. 2015, Article ID 295483 (2015)
- 20. Lin, Q.: Allee effect increasing the final density of the species subject to the Allee effect in a Lotka–Volterra commensal symbiosis model. Adv. Differ. Equ. **2018**, Article ID 196 (2018)
- Lin, Q.: Stability analysis of a single species logistic model with Allee effect and feedback control. Adv. Differ. Equ. 2018, Article ID 190 (2018)
- 22. Chen, L., Wang, Y., et al.: Influence of predator mutual interference and prey refuge on Lotka–Volterra predator-prey dynamics. Commun. Nonlinear Sci. Numer. Simul. 18(11), 3174–3180 (2013)
- 23. He, M., Chen, F.: Extinction and stability of an impulsive system with pure delays. Appl. Math. Lett. 91, 128–136 (2019)
- Wu, R., Li, L., Zhou, X.: A commensal symbiosis model with Holling type functional response. J. Math. Comput. Sci. 16(3), 364–371 (2016)
- Yang, K., Miao, Z.S., et al.: Influence of single feedback control variable on an autonomous Holling-II type cooperative system. J. Math. Anal. Appl. 435(1), 874–888 (2016)
- Chen, F., Xie, X., et al.: Extinction in two species nonautonomous nonlinear competitive system. Appl. Math. Comput. 274, 119–124 (2016)
- Chen, B.: Dynamic behaviors of a non-selective harvesting Lotka–Volterra amensalism model incorporating partial closure for the populations. Adv. Differ. Equ. 2018, Article ID 111 (2018)
- Walters, C., Christensen, V., Fulton, B., et al.: Predictions from simple predator-prey theory about impacts of harvesting forage fishes. Ecol. Model. 337, 272–280 (2016)
- Kang, Y., Rodriguez-Rodriguez, M., Evilsizor, S.: Ecological and evolutionary dynamics of two-stage models of social insects with egg cannibalism. J. Math. Anal. Appl. 430(1), 324–353 (2015)
- Zhang, F., Chen, Y., Li, J.: Dynamical analysis of a stage-structured predator-prey model with cannibalism. Math. Biosci. 307, 33–41 (2019)
- Basheer, A., Quansah, E., Bhowmick, S., et al.: Prey cannibalism alters the dynamics of Holling–Tanner-type predator-prey models. Nonlinear Dyn. 85(4), 2549–2567 (2016)
- 32. Basheer, A., Parshad, R.D., Quansah, E., et al.: Exploring the dynamics of a Holling–Tanner model with cannibalism in both predator and prey population. Int. J. Biomath. 11(01), 1850010 (2018)
- Deng, H., Chen, F., Zhu, Z., et al.: Dynamic behaviors of Lotka–Volterra predator-prey model incorporating predator cannibalism. Adv. Differ. Equ. 2019, Article ID 359 (2019)
- Chen, F., Huang, X., Deng, H.: Dynamic behaviors of a stage structure single species model with cannibalism. Appl. Math. E-Notes 19, 469–475 (2019)
- 35. Leslie, P.H.: A stochastic model for studying the properties of certain biological systems by numerical methods. Biometrika **45**, 16–31 (1958)
- Korobeinikov, A.: A Lyapunov function for Leslie–Gower predator-prey models. Appl. Math. Lett. 14(6), 697–699 (2001)
- Chen, F., Chen, L., Xie, X.: On a Leslie–Gower predator-prey model incorporating a prey refuge. Nonlinear Anal., Real World Appl. 10(5), 2905–2908 (2009)
- Liang, Z.Q., Pan, H.W.: Qualitative analysis of a ratio-dependent Holling–Tanner model. J. Math. Anal. Appl. 334, 954–964 (2009)
- Merdan, H.: Stability analysis of a Lotka–Volterra type predator-prey system involving Allee effect. ANZIAM J. 52, 139–145 (2010)
- 40. Chen, F.D.: On a nonlinear nonautonomous predator-prey model with diffusion and distributed delay. J. Comput. Appl. Math. 180(1), 33–49 (2005)