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Stabilization of the weakly coupled plate equations with a locally distributed damping

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Abstract

In this paper, we study the indirect stabilization of a system of plate equations which are weakly coupled and locally damped. By virtue of the general results due to Burq in the study of asymptotic behavior of solutions, we prove that the semigroup associated to the system is logarithmically stable under some assumptions on the damping and the coupling terms. For this purpose, we adopt an approach based on the growth of the resolvent on the imaginary axis, which can be obtained by some Carleman estimates.

Keywords: Coupled plate equations; Indirect damping; Logarithmic decay; Carleman estimate

1 Introduction

Let Ω be a bounded domain in $\mathbb{R}^n$ with a C^4 boundary Γ. Consider the following two weakly coupled plate equations:

$$\begin{cases} y_{tt} + \Delta^2 y + c(x)z + d(x)y_t = 0 & \text{in } \mathbb{R}^+ \times \Omega, \\ z_{tt} + \Delta^2 z + c(x)y = 0 & \text{in } \mathbb{R}^+ \times \Omega, \\ y = \Delta y = z = \Delta z = 0 & \text{on } \mathbb{R}^+ \times \Gamma, \\ (y(0), y_t(0)) = (y^0, y^1), (z(0), z_t(0)) = (z^0, z^1) & \text{in } \Omega, \end{cases} \tag{1.1}$$

where $c(\cdot) \in L^\infty(\Omega;\mathbb{R})$ is the coupling function, and $d(\cdot) \in L^\infty(\Omega;\mathbb{R})$ is the damping function. Both $c(\cdot)$ and $d(\cdot)$ are nonnegative.

In system (1.1), the damping which is distributed locally in the domain under consideration acts through one of the equations only, and its effect is transmitted to the other equation through the coupling. Thus, system (1.1) is a special case of the general framework proposed by Russell (see [28]) for the indirect damping problem in elastic systems. Motivated by Russell's work, the indirect stabilization problem for all kinds of coupled systems have been extensively studied (see e.g. [1–4, 10, 16, 18, 23, 27, 30] and the references therein).

It is well known that a single wave equation is exponentially stable if and only if the geometric control condition (*GCC* for short) is satisfied (see [6]). When the *GCC* failed, Lebeau [22] first derived the logarithmic decay result for wave equations where no geometric restriction was imposed on the damping regions. After that, the logarithmic decay result was extended to many other problems (see [5, 9, 19] for a single wave or plate) with internal or boundary damping. However, for the indirect stabilization for weakly coupled wave-type equations (wave–wave, wave–plate, etc.), even under some geometric conditions, the exponential stability does not hold (see [3]). There are many results related to the polynomial decay. In this respect, we refer to [2–4] for the indirect stabilization for weakly coupled systems of wave–wave or wave–Petrowsky type, and [17, 24, 25, 29] for the weakly coupled plate–plate equations. Further, we refer to [21] for the uniform decay rates of the coupled wave equation and plate equation with the coupling on the interface, and [15] for the polynomial decay estimate for a single plate equation with local degenerated dissipations.

It should be pointed out that there are few references addressing the logarithmic decay rate for the indirect stabilization for weakly coupled wave-type equations. We refer to [11] for the logarithmic decay rates of the energy of a hyperbolic–parabolic system coupled by an interface, [12] for the logarithmic decay result of the weakly coupled hyperbolic equations, [13] for the logarithmic decay result of the weakly coupled wave–plate equations. As far as we know, there is no reference addressing the asymptotic behavior of the system (1.1).

In this paper, we will show the logarithmic decay property for solutions of the system (1.1). Due to Burq's [8] general results in the study of asymptotic behavior of solutions, it suffices to show some high-frequency estimates with exponential loss on the resolvent. To this aim, we borrow some ideas in [12, 13]. In [12], to get the energy decay for a system coupled by two wave equations, one is required to establish an interpolation inequality for a system coupled by two elliptic equations. In [13], to get the energy decay for a system coupled by wave–plate equations, one is required to establish an interpolation inequality for a system coupled by one elliptic and two parabolic equations. In our case, we consider the coupled plate–plate equations, noting that the plate operator "$\partial_t^2 + \Delta^2$" can be decomposed as two conjugate Schrödinger ones "$\partial_t^2 + \Delta^2 = (i\partial_t + \Delta)(-i\partial_t + \Delta)$", therefore, we have to get an interpolation inequality for a system coupled by four parabolic-type equations (see (4.5)). Since there is no elliptic-type equation in our situation, the interpolation inequality we obtain here differs from [12, 13]. See Sect. 3 for more details.

The rest of this paper is organized as follows. In Sect. 2, we give the main results in this paper. Section 3 is addressed to proving an interpolation inequality by virtue of Carleman estimates for the parabolic equations. At last, in Sect. 4, we prove our main results.

2 Statement of the main results

Let $m^* = \inf_{y\in H^2(\Omega)\cap H_0^1(\Omega)} \|\Delta y\|^2_{L^2(\Omega)}/\|y\|^2_{L^2(\Omega)}$. Throughout this paper, we assume that $c(\cdot)$ and $d(\cdot)$ satisfy

$$\begin{cases} c(x) \ge c_0 > 0 \quad \text{in } \omega_c, \\ \|c\|_{L^\infty(\Omega)} < m^*, \end{cases} \tag{2.1}$$

and

$$d(x) \ge d_0 > 0 \quad \text{in } \omega_d, \tag{2.2}$$

where ω_c and ω_d are arbitrary non-empty open subsets of Ω. The solutions y and z to system (1.1) are complex-valued functions. In what follows, we shall use $C = C(\Omega, \omega_c, \omega_d, c_0, d_0)$ to denote generic positive constants which may vary from line to line.

Now let us introduce the energy space $\mathcal{H}$ over the field $\mathbb{C}$ as follows:

$$\mathcal{H} = \big(H^2(\Omega) \cap H_0^1(\Omega)\big) \times L^2(\Omega) \times \big(H^2(\Omega) \cap H_0^1(\Omega)\big) \times L^2(\Omega).$$

For any $U = (y, u, z, v)$, $\tilde{U} = (\tilde{y}, \tilde{u}, \tilde{z}, \tilde{v}) \in \mathcal{H}$, their inner product is

$$(U, \tilde{U}) \overset{\triangle}{=} \int_\Omega \big(\Delta y \Delta \bar{\tilde{y}} + \Delta z \Delta \bar{\tilde{z}} + c(x) y \bar{\tilde{z}} + c(x) z \bar{\tilde{y}}\big)\, dx + \int_\Omega (u \bar{\tilde{u}} + v \bar{\tilde{v}})\, dx. \tag{2.3}$$

Define a linear unbounded operator $\mathcal{A} : D(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ by

$$\begin{cases} D(\mathcal{A}) = \{U = (y, u, z, v) \in \mathcal{H} : \mathcal{A}U \in \mathcal{H}, y|_\Gamma = \Delta y|_\Gamma = z|_\Gamma = \Delta z|_\Gamma = 0\}, \\ \mathcal{A}U = (u, -\Delta^2 y - c(x) z - d(x) u, v, -\Delta^2 z - c(x) y). \end{cases} \tag{2.4}$$

In fact, if $U = (y, u, z, v) \in D(\mathcal{A})$, we have $y \in H^4(\Omega)$, $u \in H^2(\Omega) \cap H_0^1(\Omega)$, $z \in H^4(\Omega)$ and $v \in H^2(\Omega) \cap H_0^1(\Omega)$.

Let $U_0 = (y^0, y^1, z^0, z^1)$. Then system (1.1) can be rewritten as

$$\frac{dU}{dt} = \mathcal{A}U, \quad U(0) = U_0.$$

Proposition 2.1 *Assume $\|c\|_{L^\infty(\Omega)} < m^*$. Then $\mathcal{A}$ is the infinitesimal generator of a C_0 semigroup of contractions $\{e^{t\mathcal{A}}\}_{t \ge 0}$ on $\mathcal{H}$. Furthermore, $\mathcal{A}$ has a compact resolvent.*

Proof Since $\overline{D(\mathcal{A})} = \mathcal{H}$, by the Lumer–Phillips theorem (see [26]), $\mathcal{A}$ generates a C_0 semigroup of contractions if $\mathcal{A}$ is dissipative and $0 \in \rho(\mathcal{A})$, where $\rho(\mathcal{A})$ denotes the resolvent set of $\mathcal{A}$. For any $U = (y, u, z, v) \in D(\mathcal{A})$, noting the inner product (2.3) we define on $\mathcal{H}$, it is easy to show that

$$\operatorname{Re}(\mathcal{A}U, U) = -\int_\Omega d(x)|u|^2\, dx \le 0. \tag{2.5}$$

On the other hand, given $F = (f^0, f^1, g^0, g^1) \in \mathcal{H}$, let us consider the problem $\mathcal{A}U = F$, which is equivalent to

$$\begin{cases} u = f^0 & \text{in } \Omega, \\ v = g^0 & \text{in } \Omega, \end{cases} \tag{2.6}$$

and

$$\begin{cases} -\Delta^2 y - c(x) z = d(x) f^0 + f^1 & \text{in } \Omega, \\ -\Delta^2 z - c(x) y = g^1 & \text{in } \Omega, \\ y = \Delta y = z = \Delta z = 0 & \text{on } \Gamma. \end{cases} \tag{2.7}$$

Let $\mathcal{G} = (H^2(\Omega) \cap H_0^1(\Omega)) \times (H^2(\Omega) \cap H_0^1(\Omega))$. In the following, we shall prove that the problem (2.6)–(2.7) has a unique solution $U = (y,u,z,v) \in D(\mathcal{A})$. Introduce a sesquiliner form Λ over the product space $\mathcal{G} \times \mathcal{G}$ as follows:

$$\Lambda(V,\tilde{V}) = \int_\Omega \big(\Delta y \Delta \bar{\tilde{y}} + \Delta z \Delta \bar{\tilde{z}} + c(x) y \bar{\tilde{z}} + c(x) z \bar{\tilde{y}}\big)\, dx, \tag{2.8}$$

where $V = (y,z)$, $\tilde{V} = (\tilde{y},\tilde{z}) \in \mathcal{G}$. It is easy to see that Λ is continuous and coercive. Thus, by the Lax–Milgram theorem, for $(-d(x)f^0 - f^1, -g^1) \in \mathcal{G}'$, the system (2.7) has a unique weak solution $(y,z) \in \mathcal{G}$ such that, for any $(\tilde{y},\tilde{z}) \in \mathcal{G}$,

$$\int_\Omega \big(\Delta y \Delta \bar{\tilde{y}} + \Delta z \Delta \bar{\tilde{z}} + c(x) y \bar{\tilde{z}} + c(x) z \bar{\tilde{y}}\big)\, dx = -\int_\Omega \big(d(x) f^0 \bar{\tilde{y}} + f^1 \bar{\tilde{y}} + g^1 \bar{\tilde{z}}\big)\, dx. \tag{2.9}$$

Denote by $\mathcal{D}'(\Omega)$ the space of all distributions on Ω. By the equality above, one has

$$\Delta^2 y + c(x) z = -d(x) f^0 - f^1 \quad \text{in } \mathcal{D}'(\Omega), \tag{2.10}$$

which implies $\Delta^2 y = -c(x)z - d(x)f^0 - f^1 \in L^2(\Omega)$. Let $\tilde{f} = c(x)z + d(x)f^0 + f^1$. Then there exists a unique solution $\vartheta \in H^2(\Omega) \cap H_0^1(\Omega)$ such that $\Delta\vartheta = \tilde{f}$ in Ω and $\vartheta = 0$ on Γ. Now, noting (2.9), for any $\tilde{y} \in H^2(\Omega) \cap H_0^1(\Omega)$ we have $\int_\Omega \vartheta \Delta \tilde{y}\, dx = \int_\Omega \tilde{f} \tilde{y}\, dx = -\int_\Omega \Delta y \Delta \tilde{y}\, dx$. Since the mapping $-\Delta : H^2(\Omega) \cap H_0^1(\Omega) \to L^2(\Omega)$ is surjective, it follows that $\Delta y = -\vartheta \in H^2(\Omega) \cap H_0^1(\Omega)$. Then, by the elliptic regularity theory, we have $y \in H^4(\Omega)$ and the following estimate holds:

$$\|y\|_{H^4(\Omega)} \le C\big(\|\Delta y\|_{H^2(\Omega)} + \|y\|_{L^2(\Omega)}\big). \tag{2.11}$$

Thus, combining $\Delta(\Delta y) \in L^2(\Omega)$ with $\Delta y|_\Gamma = 0$, by the elliptic regularity theory, once again we have

$$\|\Delta y\|_{H^2(\Omega)} \le C\big(\|\Delta y\|_{L^2(\Omega)} + \big\|c(x)z + d(x)u + f^1\big\|_{L^2(\Omega)}\big). \tag{2.12}$$

Similarly, we also have $z \in H^4(\Omega)$, $\Delta z \in H_0^1(\Omega)$, and the following estimates hold:

$$\|z\|_{H^4(\Omega)} \le C\big(\|\Delta z\|_{H^2(\Omega)} + \|z\|_{L^2(\Omega)}\big), \tag{2.13}$$

and

$$\|\Delta z\|_{H^2(\Omega)} \le C\big(\|\Delta z\|_{L^2(\Omega)} + \big\|c(x)y + g^1\big\|_{L^2(\Omega)}\big). \tag{2.14}$$

On the other hand, taking $(\tilde{y},\tilde{z}) = (y,z)$ in (2.9), it is easy to find that

$$\|\Delta y\|_{L^2(\Omega)} + \|\Delta z\|_{L^2(\Omega)} \le C\|F\|_{\mathcal{H}}. \tag{2.15}$$

Finally, combining (2.6) with (2.11)–(2.15) we conclude that $0 \in \rho(\mathcal{A})$ and

$$\|y\|_{H^4(\Omega)} + \|u\|_{H^2(\Omega)} + \|z\|_{H^4(\Omega)} + \|v\|_{H^2(\Omega)} \le C\|F\|_{\mathcal{H}}, \tag{2.16}$$

which implies that $\mathcal{A}^{-1}$ is compact. □

The energy of system (1.1) at time t is given by

$$E(t) = \frac{1}{2}\int_\Omega \left(\left|\Delta y(t)\right|^2 + \left|y_t(t)\right|^2 + \left|\Delta z(t)\right|^2 + \left|z_t(t)\right|^2\right) dx + \int_\Omega c(x)\,\mathrm{Re}(y\bar{z})\,dx. \tag{2.17}$$

When $d \equiv 0$, $E(\cdot)$ is obviously conservative. Otherwise, we have

$$E(t_2) - E(t_1) = -\int_{t_1}^{t_2}\int_\Omega d(x)|y_t|^2\,dx\,dt, \quad \forall t_2 \geq t_1 \geq 0,$$

which implies the energy is nonincreasing and system (1.1) is dissipative.

Our main result is stated as follows.

Theorem 2.1 *Let $c(\cdot)$ and $d(\cdot)$ satisfy (2.1) and (2.2), respectively. Suppose that $\omega_c \cap \omega_d \neq \emptyset$. Then there exists a constant $C > 0$ such that for all $(y^0, y^1, z^0, z^1) \in D(\mathcal{A})$ the solution $e^{t\mathcal{A}}(y^0, y^1, z^0, z^1) \in C(\mathbb{R}^+; D(\mathcal{A})) \cap C^1(\mathbb{R}^+; \mathcal{H})$ to system (1.1) satisfies*

$$\left\| e^{t\mathcal{A}}\left(y^0, y^1, z^0, z^1\right)\right\|_{\mathcal{H}} \leq \frac{C}{\ln(2+t)}\left\|\left(y^0, y^1, z^0, z^1\right)\right\|_{D(\mathcal{A})}, \quad \forall t \geq 0. \tag{2.18}$$

In order to prove Theorem 2.1, let us recall the following result:

Lemma 2.1 ([7, Theorem A]) *Assume that $\mathcal{B}$ is the infinitesimal generator of a bounded C_0 semigroup $\{e^{t\mathcal{B}}\}_{t\in\mathbb{R}^+}$ on Hilbert space $\tilde{\mathcal{H}}$. Let $\rho(\mathcal{B})$ denote the resolvent set of $\mathcal{B}$. If $i\mathbb{R} \subset \rho(\mathcal{B})$ and there exists a positive constant C such that*

$$\sup_{|\tau|\leq\xi}\left\|(i\tau - \mathcal{B})^{-1}\right\|_{\mathcal{L}(\tilde{\mathcal{H}})} \leq Ce^{C\xi}, \quad \forall \xi \geq 0,$$

then, for any $k \in \mathbb{N}^*$, there exists a positive constant C_k such that

$$\sup_{s\geq t}\left\|e^{s\mathcal{B}}(1-\mathcal{B})^{-k}\right\|_{\mathcal{L}(\tilde{\mathcal{H}})} \leq \frac{C_k}{\ln^k(2+t)}, \quad \forall t \geq 0,$$

where $\mathbb{N}^* = \mathbb{N} \setminus \{0\}$.

According to Lemma 2.1, Theorem 2.1 is reduced to the following resolvent estimate of the operator $\mathcal{A}$.

Theorem 2.2 *Under the assumptions of Theorem 2.1, we have $i\mathbb{R} \subset \rho(\mathcal{A})$ and there exists a constant $C > 0$ such that, for every $\beta \in \mathbb{R}$,*

$$\left\|(i\beta - \mathcal{A})^{-1}\right\|_{\mathcal{L}(\mathcal{H})} \leq Ce^{C|\beta|}.$$

Remark 2.1 In this paper, we assume that $\omega_c \cap \omega_d \neq \emptyset$. It would be quite interesting to consider the case that $\omega_c \cap \omega_d = \emptyset$. Some results are obtained in [4] for one dimensional coupled wave equations. For the multi-dimensional case, as far as we know, it is an unsolved problem.

3 An interpolation inequality for the coupled parabolic system

In this section, we shall prove an interpolation inequality for the following coupled parabolic system:

$$\begin{cases} p_s + \Delta p = w & \text{in } Q, \\ w_s - \Delta w - id(x)p_s - c(x)q = w^0 & \text{in } Q, \\ q_s + \Delta q = z & \text{in } Q, \\ z_s - \Delta z - c(x)p = z^0 & \text{in } Q, \\ p = \Delta p = q = \Delta q = 0 & \text{on } \Sigma. \end{cases} \tag{3.1}$$

Here $Q = (-2,2) \times \Omega$, $\Sigma = (-2,2) \times \Gamma$ and $w^0, z^0 \in L^2(Q)$.

Set

$$X \triangleq (-1,1) \times \Omega, \qquad \widetilde{X} \triangleq (-2,2) \times \omega, \qquad \omega = \omega_c \cap \omega_d.$$

We have the following interpolation inequality for the system (3.1).

Theorem 3.1 *Under the assumptions in Theorem 2.1, there exists a constant $C > 0$ such that, for any $\varepsilon > 0$, any solution (p, w, q, z) of the system (3.1) satisfies*

$$\begin{aligned} &\|p\|_{L^2(X)} + \|w\|_{L^2(X)} + \|q\|_{L^2(X)} + \|z\|_{L^2(X)} \\ &\quad \le Ce^{C/\varepsilon}\big(\|w^0\|_{L^2(Q)} + \|z^0\|_{L^2(Q)} + \|p\|_{L^2(\widetilde{X})} + \|dp_s\|_{L^2(Q)}\big) \\ &\qquad + Ce^{-1/\varepsilon}\big(\|p\|_{L^2(Q)} + \|w\|_{L^2(Q)} + \|q\|_{L^2(Q)} + \|z\|_{L^2(Q)}\big). \end{aligned} \tag{3.2}$$

3.1 Some preliminaries

The proof of Theorem 3.1 is based on the Carleman estimates for the parabolic operators $\pm\partial_s + \Delta$. In this subsection, we collect some known results we need.

As we know, Carleman estimate can be regarded as a weighted energy estimate. To begin with, we first give the choice of the weight functions. Let ω_0 be a non-empty subdomain of Ω such that $\omega_0 \subset \overline{\omega}_0 \subset \omega$. By [14, Lemma 1.1] we know that there exists a function $\hat{\psi} \in C^2(\overline{\Omega}; \mathbb{R})$ such that

$$\hat{\psi} > 0 \quad \text{in } \Omega, \qquad \hat{\psi} = 0 \quad \text{on } \partial\Omega, \qquad |\nabla\hat{\psi}| > 0 \quad \text{in } \overline{\Omega \setminus \omega_0}. \tag{3.3}$$

With such choice of $\hat{\psi}$ and $\lambda, \mu > 1$, we define

$$\begin{cases} \psi = \psi(s,x) \triangleq \frac{\hat{\psi}(x)}{\|\hat{\psi}\|_{L^\infty(\Omega)}} + b^2 - s^2, & x \in \overline{\Omega}, s \in \mathbb{R}, \\ \theta = e^{\ell}, \qquad \ell = \lambda\phi, \qquad \phi = e^{\mu\psi}, \end{cases} \tag{3.4}$$

where $b \in (1,2)$ will be given later.

Choose a cut-off function $\eta \in C_0^\infty(\omega)$ such that

$$\begin{cases} 0 \le \eta(x) \le 1, & x \in \omega, \\ \eta(x) = 1, & x \in \omega_0. \end{cases} \tag{3.5}$$

For fixed μ, proceeding with exactly the same analysis as [13], we have the following result.

Lemma 3.1 ([13, Lemma 3.2]) *Let $\gamma \in \mathbb{R}$ and ℓ be given by (3.4). Let k be a positive integer and $k \geq 2$. Then, for a fixed μ, there is a constant $\lambda_0 > 0$ such that, for all $\lambda \geq \lambda_0$, one can find a constant $C > 0$ such that, for all $z \in H_0^2(-b, b; H^2(\Omega) \cap H_0^1(\Omega))$ and for any $\beta \geq 2$,*

$$\begin{aligned}&\int_{-b}^{b}\int_{\Omega}\theta^2\eta^k|\nabla z|^2\,dx\,ds\\&\quad\leq\frac{1}{\lambda^\beta}\int_{-b}^{b}\int_{\Omega}\theta^2\eta^k|\gamma z_s+\Delta z|^2\,dx\,ds+C\lambda^\beta\int_{-b}^{b}\int_{\Omega}\theta^2\eta^{k-2}|z|^2\,dx\,ds.\end{aligned}\tag{3.6}$$

Further, we recall the following well-known Carleman estimate for the parabolic operator $\gamma z_s + \Delta z$.

Lemma 3.2 ([13, Lemma 3.4]) *Let $\gamma \in \mathbb{R}$ and ℓ be given by (3.4). Then there is a constant $\mu_1 > 0$ such that, for all $\mu \geq \mu_1$, one can find two constants $C > 0$ and $\lambda_1 = \lambda_1(\mu) > 0$ so that, for any $\lambda \geq \lambda_1$, for all $z \in H_0^2(-b, b; H^2(\Omega) \cap H_0^1(\Omega))$,*

$$\begin{aligned}&\lambda\mu^2\int_{-b}^{b}\int_{\Omega}\theta^2\phi\big(|\nabla z|^2+\lambda^2\mu^2\phi^2|z|^2\big)\,dx\,ds\\&\quad\leq C\bigg(\int_{-b}^{b}\int_{\Omega}\theta^2|\gamma z_s+\Delta z|^2\,dx\,ds+\lambda^3\mu^4\int_{-b}^{b}\int_{\omega_0}\theta^2\phi^3|z|^2\,dx\,ds\bigg).\end{aligned}\tag{3.7}$$

3.2 Proof of Theorem 3.1

To prove the interpolation inequality (3.2), first we apply the Carleman estimates in Lemma 3.2 to p, w, q and z, respectively, to get (3.13). Then the main difficulty is to estimate the energy of $(\hat{p}, \hat{q}, \hat{w}, \hat{z})$ localized on ω_0 by only the energy of p localized on ω, which can be solved by using their coupling relations and the multiplier technique. Since the proof is long, we divide it into several steps.

Step 1. There is no boundary conditions for p, w, q and z at $s = \pm 2$ in the system (3.1). Thus, we need to introduce a cut-off function $\varphi = \varphi(s) \in C_0^\infty(-b, b)$ such that

$$\begin{cases}0\leq\varphi(s)\leq 1, & |s|<b,\\ \varphi(s)=1, & |s|\leq b_0.\end{cases}\tag{3.8}$$

Here $1 < b_0 < b \leq 2$ are given as follows:

$$b\triangleq\sqrt{1+\frac{1}{\mu}\ln\big(2+e^\mu\big)},\qquad b_0\triangleq\sqrt{b^2-\frac{1}{\mu}\ln\bigg(\frac{1+e^\mu}{e^\mu}\bigg)},\tag{3.9}$$

where μ is the parameter appeared in Lemma 3.2 and is large enough. Put

$$\hat{p}=\varphi p,\qquad\hat{w}=\varphi w,\qquad\hat{q}=\varphi q,\qquad\hat{z}=\varphi z.\tag{3.10}$$

Noting that φ does not depend on x, it follows from (3.1) that

$$\begin{cases} \hat{p}_s + \Delta\hat{p} = F_1 & \text{in } Q, \\ \hat{w}_s - \Delta\hat{w} = F_2 & \text{in } Q, \\ \hat{q}_s + \Delta\hat{q} = F_3 & \text{in } Q, \\ \hat{z}_s - \Delta\hat{z} = F_4 & \text{in } Q, \\ \hat{p} = \Delta\hat{p} = \hat{q} = \Delta\hat{q} = 0 & \text{on } \Sigma, \end{cases} \tag{3.11}$$

where

$$\begin{cases} F_1 \stackrel{\triangle}{=} \hat{w} + \varphi_s p, \\ F_2 \stackrel{\triangle}{=} id\hat{p}_s + c(x)\hat{q} + \varphi_s w - id\varphi_s p + \varphi w^0, \\ F_3 \stackrel{\triangle}{=} \hat{z} + \varphi_s q, \\ F_4 = c(x)\hat{p} + \varphi_s z + \varphi z^0. \end{cases} \tag{3.12}$$

By using Lemma 3.2 for $\gamma = \pm 1$, we conclude that there is a $\mu_1 > 0$ such that, for all $\mu \geq \mu_1$, one can find two constants $C = C(\mu) > 0$ and $\lambda_1 = \lambda_1(\mu)$ so that, for all $\lambda \geq \lambda_1$,

$$\begin{aligned} &\lambda\mu^2 \int_{-b}^{b}\int_{\Omega} \theta^2\phi\big(|\nabla\hat{p}|^2 + |\nabla\hat{w}|^2 + |\nabla\hat{q}|^2 + |\nabla\hat{z}|^2 \\ &\quad + \lambda^2\mu^2\phi^2\big(|\hat{p}|^2 + |\hat{w}|^2 + |\hat{q}|^2 + |\hat{z}|^2\big)\big)\,dx\,ds \\ &\quad \leq C\bigg[\int_{-b}^{b}\int_{\Omega} \theta^2\big(|F_1|^2 + |F_2|^2 + |F_3|^2 + |F_4|^2\big)\,dx\,ds \\ &\quad + \lambda^3\mu^4 \int_{-b}^{b}\int_{\omega_0} \theta^2\phi^3\big(|\hat{p}|^2 + |\hat{w}|^2 + |\hat{q}|^2 + |\hat{z}|^2\big)\,dx\,ds\bigg]. \end{aligned} \tag{3.13}$$

Step 2. Let us estimate $\int_{-b}^{b}\int_{\omega_0} \theta^2|\hat{z}|^2\,dx\,ds$.

Recall that $\eta \in C_0^\infty(\omega)$ satisfying $\eta = 1$ in ω_0. By (3.11) and (3.12), we have

$$\begin{aligned} &\theta^2\eta^{12}|\hat{z}|^2 \\ &\quad = \theta^2\eta^{12}\bar{\hat{z}}(\hat{q}_s + \Delta\hat{q}) - \theta^2\eta^{12}\bar{\hat{z}}\varphi_s q \\ &\quad = -\theta^2\eta^{12}\hat{q}(\bar{\hat{z}}_s - \Delta\bar{\hat{z}}) + \big(\theta^2\eta^{12}\bar{\hat{z}}\hat{q}\big)_s - \big(\theta^2\eta^{12}\big)_s\bar{\hat{z}}\hat{q} - \theta^2\eta^{12}\bar{\hat{z}}\varphi_s q \\ &\quad + \sum_{j=1}^{n}\big[\theta^2\eta^{12}(\bar{\hat{z}}\hat{q}_{x_j} - \bar{\hat{z}}_{x_j}\hat{q})\big]_{x_j} + \sum_{j=1}^{n}\big(\theta^2\eta^{12}\big)_{x_j}(\bar{\hat{z}}_{x_j}\hat{q} - \bar{\hat{z}}\hat{q}_{x_j}). \end{aligned} \tag{3.14}$$

Integrating (3.14) on $(-b, b) \times \Omega$, noting that $\hat{z}(-b) = \hat{z}(b) = 0$ in Ω, $\hat{q} = \hat{z} = 0$ on the boundary, by (3.11)–(3.12), we find that

$$\begin{aligned} &\int_{-b}^{b}\int_{\Omega} \theta^2\eta^{12}|\hat{z}|^2\,dx\,ds \\ &\quad \leq C\bigg[\int_{-b}^{b}\int_{\Omega} \theta^2\big|z^0\big|^2\,dx\,ds + \int_{-b}^{b}\int_{\omega} \theta^2|\hat{p}|^2\,dx\,ds \end{aligned}$$

$$+ \int_{(-b,-b_0)\cup(b_0,b)} \int_\Omega \theta^2\big(|q|^2 + |z|^2\big)\,dx\,ds + \lambda^5 \int_{-b}^{b}\int_\Omega \theta^2\eta^{11}|\hat{q}|^2\,dx\,ds$$
$$+ \lambda^2 \int_{-b}^{b}\int_\Omega \theta^2\eta^{10}|\nabla\hat{q}|^2\,dx\,ds + \frac{1}{\lambda^3}\int_{-b}^{b}\int_\Omega \theta^2|\nabla\hat{z}|^2\bigg]. \tag{3.15}$$

Next, taking $\beta = 3$ in Lemma 3.1, we have

$$\int_{-b}^{b}\int_\Omega \theta^2\eta^{10}|\nabla\hat{q}|^2\,dx\,ds$$
$$\le \frac{1}{\lambda^3}\int_{-b}^{b}\int_\Omega \theta^2\eta^{10}|F_3|^2\,dx\,ds + C\lambda^3\int_{-b}^{b}\int_\Omega \theta^2\eta^8|\hat{q}|^2\,dx\,ds. \tag{3.16}$$

Combining (3.15) and (3.16) we obtain

$$\int_{-b}^{b}\int_\Omega \theta^2\eta^{12}|\hat{z}|^2\,dx\,ds$$
$$\le C\bigg[\int_{-b}^{b}\int_\Omega \theta^2\big|z^0\big|^2\,dx\,ds + \int_{-b}^{b}\int_\omega \theta^2|\hat{p}|^2\,dx\,ds$$
$$+ \int_{(-b,-b_0)\cup(b_0,b)}\int_\Omega \theta^2\big(|q|^2 + |z|^2\big)\,dx\,ds$$
$$+ \lambda^5\int_{-b}^{b}\int_\Omega \theta^2\eta^8|\hat{q}|^2\,dx\,ds + \frac{1}{\lambda^3}\int_{-b}^{b}\int_\Omega \theta^2\big(|\nabla\hat{z}|^2 + \lambda^2|\hat{z}|^2\big)\,dx\,ds\bigg]. \tag{3.17}$$

Step 3. Let us estimate $\int_{-b}^{b}\int_\Omega \theta^2\eta^8|\hat{q}|^2\,dx\,ds$.
Multiplying the second equation of (3.11) by $\theta^2\eta^8\overline{\hat{q}}$, we get

$$c(x)\theta^2\eta^8|\hat{q}|^2$$
$$= \theta^2\eta^8\overline{\hat{q}}(\hat{w}_s - \Delta\hat{w}) - \theta^2\eta^8\overline{\hat{q}}\big(id\hat{p}_s + \varphi_s w - id\varphi_s p + \varphi w^0\big)$$
$$= -\theta^2\eta^8\hat{w}(\overline{\hat{q}}_s + \Delta\overline{\hat{q}}) + \big(\theta^2\eta^8\overline{\hat{q}}\hat{w}\big)_s - \big(\theta^2\eta^8\big)_s\overline{\hat{q}}\hat{w} - \theta^2\eta^8\overline{\hat{q}}\big(id\varphi p_s + \varphi_s w + \varphi w^0\big)$$
$$- \sum_{j=1}^{n}\big[\theta^2\eta^8(\overline{\hat{q}}\hat{w}_{x_j} - \overline{\hat{q}}_{x_j}\hat{w})\big]_{x_j} - \sum_{j=1}^{n}\big(\theta^2\eta^8\big)_{x_j}(\overline{\hat{q}}_{x_j}\hat{w} - \overline{\hat{q}}\hat{w}_{x_j}). \tag{3.18}$$

Now, integrating (3.18) on $(-b,b)\times\Omega$, recalling (2.1) for c_0, we find that

$$c_0\int_{-b}^{b}\int_\Omega \theta^2\eta^8|\hat{q}|^2\,dx\,ds$$
$$\le C\bigg[\int_{-b}^{b}\int_\Omega \theta^2\big|w^0\big|^2\,dx\,ds + \int_{-b}^{b}\int_\omega \theta^2|p_s|^2\,dx\,ds$$
$$+ \int_{(-b,-b_0)\cup(b_0,b)}\int_\Omega \theta^2\big(|w|^2 + |q|^2\big)\,dx\,ds$$
$$+ \lambda^{10}\int_{-b}^{b}\int_\Omega \theta^2\eta^7|\hat{w}|^2\,dx\,ds + \lambda^2\int_{-b}^{b}\int_\Omega \theta^2\eta^6|\nabla\hat{w}|^2\,dx\,ds$$
$$+ \frac{1}{\lambda^8}\int_{-b}^{b}\int_\Omega \theta^2\big(|\nabla\hat{q}|^2 + \lambda^2|\hat{z}|^2\big)\bigg]. \tag{3.19}$$

By Lemma 3.1 and taking $\beta = 8$, we have

$$\begin{aligned}&\int_{-b}^{b}\int_{\Omega}\theta^2\eta^6|\nabla\hat{w}|^2\,dx\,ds\\&\quad\le\frac{1}{\lambda^8}\int_{-b}^{b}\int_{\Omega}\theta^2\eta^6|F_2|^2\,dx\,ds+C\lambda^8\int_{-b}^{b}\int_{\Omega}\theta^2\eta^4|\hat{w}|^2\,dx\,ds.\end{aligned}\tag{3.20}$$

Substituting (3.20) into (3.19), it is easy to show

$$\begin{aligned}&\int_{-b}^{b}\int_{\Omega}\theta^2\eta^8|\hat{q}|^2\,dx\,ds\\&\quad\le C\biggl[\int_{-b}^{b}\int_{\Omega}\theta^2\bigl|w^0\bigr|^2\,dx\,ds+\int_{-b}^{b}\int_{\omega}\theta^2|p_s|^2\,dx\,ds\\&\qquad+\int_{(-b,-b_0)\cup(b_0,b)}\int_{\Omega}\theta^2\bigl(|w|^2+|q|^2\bigr)\,dx\,ds\\&\qquad+\lambda^{10}\int_{-b}^{b}\int_{\Omega}\theta^2\eta^4|\hat{w}|^2\,dx\,ds+\frac{1}{\lambda^8}\int_{-b}^{b}\int_{\Omega}\theta^2\bigl(|\nabla\hat{q}|^2+\lambda^2|\hat{z}|^2+\lambda^2|\hat{q}|^2\bigr)\biggr].\end{aligned}\tag{3.21}$$

Step 4. Let us estimate $\int_{-b}^{b}\int_{\Omega}\theta^2\eta^4|\hat{w}|^2\,dx\,ds$.
Similar to (3.14), we have

$$\begin{aligned}&\theta^2\eta^4|\hat{w}|^2\\&\quad=\theta^2\eta^4\overline{\hat{w}}(\hat{p}_s+\Delta\hat{p})-\theta^2\eta^4\overline{\hat{w}}\varphi_s p\\&\quad=-\theta^2\eta^4\hat{p}(\overline{\hat{w}}_s-\Delta\overline{\hat{w}})+\bigl(\theta^2\eta^4\overline{\hat{w}}\hat{p}\bigr)_s-\bigl(\theta^2\eta^4\bigr)_s\overline{\hat{w}}\hat{p}-\theta^2\eta^4\overline{\hat{w}}\varphi_s p\\&\qquad+\sum_{j=1}^{n}\bigl[\theta^2\eta^4(\overline{\hat{w}}\hat{p}_{x_j}-\overline{\hat{w}}_{x_j}\hat{p})\bigr]_{x_j}+\sum_{j=1}^{n}\bigl(\theta^2\eta^4\bigr)_{x_j}(\overline{\hat{w}}_{x_j}\hat{p}-\overline{\hat{w}}\hat{p}_{x_j}).\end{aligned}\tag{3.22}$$

Integrating (3.22) on $(-b,b)\times\Omega$, we find that

$$\begin{aligned}&\int_{-b}^{b}\int_{\Omega}\theta^2\eta^4|\hat{w}|^2\,dx\,ds\\&\quad\le C\biggl[\lambda^{20}\int_{-b}^{b}\int_{\omega}\theta^2|p|^2\,dx\,ds+\int_{-b}^{b}\int_{\omega}\theta^2|p_s|^2\,dx\,ds\\&\qquad+\int_{(-b,-b_0)\cup(b_0,b)}\int_{\Omega}\theta^2|w|^2\,dx\,ds\\&\qquad+\int_{-b}^{b}\int_{\Omega}\theta^2\bigl|w^0\bigr|^2\,dx\,ds+\lambda^2\int_{-b}^{b}\int_{\Omega}\theta^2\eta^2|\nabla\hat{p}|^2\,dx\,ds\\&\qquad+\frac{1}{\lambda^{18}}\int_{-b}^{b}\int_{\Omega}\theta^2\bigl(|\nabla\hat{w}|^2+\lambda^2|\hat{q}|^2\bigr)\biggr].\end{aligned}\tag{3.23}$$

Applying Lemma 3.1 again, we deduce

$$\int_{-b}^{b}\int_{\Omega}\theta^2\eta^2|\nabla\hat{p}|^2\,dx\,ds\le\frac{1}{\lambda^{18}}\int_{-b}^{b}\int_{\Omega}\theta^2|\hat{w}|^2\,dx\,ds+C\lambda^{18}\int_{-b}^{b}\int_{\omega}\theta^2|p|^2\,dx\,ds.\tag{3.24}$$

By (3.23)–(3.24) we get

$$\begin{aligned}
&\int_{-b}^{b}\int_{\Omega}\theta^2\eta^4|\hat{w}|^2\,dx\,ds \\
&\quad\le C\biggl[\lambda^{20}\int_{-b}^{b}\int_{\omega}\theta^2|p|^2\,dx\,ds+\int_{-b}^{b}\int_{\omega}\theta^2|p_s|^2\,dx\,ds \\
&\qquad+\int_{(-b,-b_0)\cup(b_0,b)}\int_{\Omega}\theta^2|w|^2\,dx\,ds+\int_{-b}^{b}\int_{\Omega}\theta^2\bigl|w^0\bigr|^2\,dx\,ds \\
&\qquad+\frac{1}{\lambda^{18}}\int_{-b}^{b}\int_{\Omega}\theta^2\bigl(|\nabla\hat{w}|^2+\lambda^2|\hat{q}|^2+\lambda^2|\hat{w}|^2\bigr)\biggr].
\end{aligned} \tag{3.25}$$

Step 5. Combining (3.13), (3.17), (3.21) and (3.25), by (3.10), we have

$$\begin{aligned}
&\lambda\mu^2\int_{-1}^{1}\int_{\Omega}\theta^2\phi\bigl(|\nabla p|^2+|\nabla w|^2+|\nabla q|^2+|\nabla z|^2 \\
&\qquad+\lambda^2\mu^2\phi^2\bigl(|p|^2+|w|^2+|q|^2+|z|^2\bigr)\bigr)\,dx\,ds \\
&\quad\le Ce^{C\lambda}\biggl[\int_{-b}^{b}\int_{\Omega}\bigl(\bigl|z^0\bigr|^2+\bigl|w^0\bigr|^2+|dp_s|^2\bigr)\,dx\,ds+\int_{-b}^{b}\int_{\omega}|p|^2\,dx\,ds\biggr] \\
&\qquad+C\lambda^{18}\int_{(-b,-b_0)\cup(b_0,b)}\int_{\Omega}\theta^2\bigl(|p|^2+|w|^2+|q|^2+|z|^2\bigr)\,dx\,ds.
\end{aligned} \tag{3.26}$$

Recalling (3.4) and (3.9) for the definitions of ϕ, b and b_0, it is easy to see that

$$\begin{cases}
\phi(s,\cdot)\ge 2+e^{\mu}, & \text{for } |s|\le 1, \\
\phi(s,\cdot)\le 1+e^{\mu}, & \text{for } b_0\le |s|\le b.
\end{cases} \tag{3.27}$$

Fixing the parameter μ in (3.9), and using (3.27), one finds that

$$\begin{aligned}
&\lambda e^{2\lambda(2+e^{\mu})}\int_{-1}^{1}\int_{\Omega}\bigl(|\nabla p|^2+|\nabla w|^2+|\nabla q|^2+|\nabla z|^2+|p|^2+|w|^2+|q|^2+|z|^2\bigr)\,dx\,ds \\
&\quad\le Ce^{C\lambda}\biggl[\int_{-2}^{2}\int_{\Omega}\bigl(\bigl|z^0\bigr|^2+\bigl|w^0\bigr|^2+|dp_s|^2\bigr)\,dx\,ds+\int_{-2}^{2}\int_{\omega}|p|^2\,dx\,ds\biggr] \\
&\qquad+C\lambda^{18}e^{2\lambda(1+e^{\mu})}\int_{-2}^{2}\int_{\Omega}\bigl(|p|^2+|w|^2+|q|^2+|z|^2\bigr)\,dx\,ds,
\end{aligned} \tag{3.28}$$

which implies

$$\begin{aligned}
&\int_{-1}^{1}\int_{\Omega}\bigl(|p|^2+|w|^2+|q|+|z|^2\bigr)\,dx\,ds \\
&\quad\le Ce^{C\lambda}\biggl[\int_{-2}^{2}\int_{\Omega}\bigl(\bigl|z^0\bigr|^2+\bigl|w^0\bigr|^2+|dp_s|^2\bigr)\,dx\,ds+\int_{-2}^{2}\int_{\omega}|p|^2\,dx\,ds\biggr] \\
&\qquad+Ce^{-\lambda}\int_{-2}^{2}\int_{\Omega}\bigl(|p|^2+|w|^2+|q|^2+|z|^2\bigr)\,dx\,ds.
\end{aligned} \tag{3.29}$$

For every $\varepsilon>0$, by taking λ in (3.29) large enough, it follows that (3.2) holds. Thus we complete the proof of Theorem 3.1.

4 Proof of the main result

In this section, we shall give the proof of the logarithmic decay result. To this end, we only need to prove Theorem 2.2.

Proof of Theorem 2.2 We divide the proof into two steps.

Step 1. Fix $F = (f^0, f^1, g^0, g^1) \in \mathcal{H}$ and $U_0 = (y^0, y^1, z^0, z^1) \in D(\mathcal{A})$. Then

$$(i\beta - \mathcal{A})U_0 = F \tag{4.1}$$

is equivalent to

$$\begin{cases} i\beta y^0 - y^1 = f^0 & \text{in } \Omega, \\ \Delta^2 y^0 - \beta^2 y^0 + i\beta\, dy^0 + c(x)z^0 = f^1 + (i\beta + d)f^0 & \text{in } \Omega, \\ i\beta z^0 - z^1 = g^0 & \text{in } \Omega, \\ \Delta^2 z^0 - \beta^2 z^0 + c(x)y^0 = g^1 + i\beta g^0 & \text{in } \Omega, \\ y^0 = \Delta y^0 = z^0 = \Delta z^0 = 0 & \text{on } \Gamma. \end{cases} \tag{4.2}$$

Put

$$p = e^{\beta s} y^0, \qquad q = e^{\beta s} z^0, \qquad s \in \mathbb{R}. \tag{4.3}$$

Then (p, q) solves the following equation:

$$\begin{cases} \Delta^2 p - p_{ss} + idp_s + c(x)q = [f^1 + (i\beta + d)f^0]e^{\beta s} & \text{in } \mathbb{R} \times \Omega, \\ \Delta^2 q - q_{ss} + c(x)p = (g^1 + i\beta g^0)e^{\beta s} & \text{in } \mathbb{R} \times \Omega, \\ p = \Delta p = q = \Delta q = 0 & \text{on } \mathbb{R} \times \Gamma. \end{cases} \tag{4.4}$$

Set $w = p_s + \Delta p$ and $z = q_s + \Delta q$. Clearly, (p, w, q, z) solves the following equation:

$$\begin{cases} \Delta p + p_s = w & \text{in } \mathbb{R} \times \Omega, \\ \Delta w - w_s + idp_s + c(x)q = [f^1 + (i\beta + d)f^0]e^{\beta s} & \text{in } \mathbb{R} \times \Omega, \\ \Delta q + q_s = z & \text{in } \mathbb{R} \times \Omega, \\ \Delta z - z_s + c(x)p = (g^1 + i\beta g^0)e^{\beta s} & \text{in } \mathbb{R} \times \Omega, \\ p = w = q = z = 0 & \text{on } \mathbb{R} \times \Gamma. \end{cases} \tag{4.5}$$

Step 2. By (4.3), we have the following estimate:

$$\begin{cases} \|y^0\|_{H^2(\Omega)\cap H_0^1(\Omega)} + \|z^0\|_{H^2(\Omega)\cap H_0^1(\Omega)} \\ \quad \le Ce^{C|\beta|}(\|p\|_{L^2(X)} + \|w\|_{L^2(X)} + \|q\|_{L^2(X)} + \|z\|_{L^2(X)}), \\ \|p\|_{L^2(Q)} + \|w\|_{L^2(Q)} + \|q\|_{L^2(Q)} + \|z\|_{L^2(Q)} \\ \quad \le Ce^{C|\beta|}(\|y^0\|_{H^2(\Omega)\cap H_0^1(\Omega)} + \|z^0\|_{H^2(\Omega)\cap H_0^1(\Omega)}), \\ \|p\|_{L^2(\widetilde{X})} \le Ce^{C|\beta|}\|y^0\|_{L^2(\omega)}. \end{cases} \tag{4.6}$$

Applying Theorem 3.1 to Eq. (4.5), and by (4.6), we get

$$\begin{aligned} &\|y^0\|_{H^2(\Omega)\cap H_0^1(\Omega)} + \|z^0\|_{H^2(\Omega)\cap H_0^1(\Omega)} \\ &\quad \le Ce^{C|\beta|}\big(\|f^0\|_{L^2(\Omega)} + \|f^1\|_{L^2(\Omega)} + \|g^0\|_{L^2(\Omega)} + \|g^1\|_{L^2(\Omega)} + \|dy^0\|_{L^2(\Omega)}\big). \end{aligned} \tag{4.7}$$

Multiplying the second equation of (4.2) by $\overline{y^0}$ and integrating it on Ω, we have

$$\int_\Omega \big(|\Delta y^0|^2 - \beta^2|y^0|^2 + i\beta\, d|y^0|^2 + cz^0\overline{y^0}\big)\, dx = \int_\Omega \big[f^1 + (i\beta + d)f^0\big]\overline{y^0}\, dx. \tag{4.8}$$

Multiplying the fourth equation of (4.2) by $\overline{z^0}$ and integrating it on Ω, we find that

$$\int_\Omega \big(|\Delta z^0|^2 - \beta^2|z^0|^2 + cy^0\overline{z^0}\big)\, dx = \int_\Omega (g^1 + i\beta g^0)\overline{z^0}\, dx. \tag{4.9}$$

Taking the imaginary part in both sides of (4.8) and (4.9), we have

$$\begin{aligned} &|\beta| \int_\Omega d|y^0|^2\, dx \\ &\quad \le C\big(1 + |\beta|\big)\big(\|f^0\|_{L^2(\Omega)} + \|f^1\|_{L^2(\Omega)} + \|g^0\|_{L^2(\Omega)} + \|g^1\|_{L^2(\Omega)}\big) \\ &\qquad \times \big(\|y^0\|_{L^2(\Omega)} + \|z^0\|_{L^2(\Omega)}\big). \end{aligned} \tag{4.10}$$

As $0 \in \rho(\mathcal{A})$ (see the proof of Proposition 2.1), one can find a positive number $\delta > 0$ such that $(-\delta, \delta) \subset \rho(\mathcal{A})$. For $|\beta| < \delta$, Theorem 2.2 holds trivially. For $|\beta| \ge \delta$, combining (4.7) and (4.10), we derive that

$$\begin{aligned} &\|y^0\|_{H^2(\Omega)\cap H_0^1(\Omega)} + \|z^0\|_{H^2(\Omega)\cap H_0^1(\Omega)} \\ &\quad \le Ce^{C|\beta|}\big(\|f^0\|_{L^2(\Omega)} + \|f^1\|_{L^2(\Omega)} + \|g^0\|_{L^2(\Omega)} + \|g^1\|_{L^2(\Omega)}\big). \end{aligned} \tag{4.11}$$

Recalling that $y^1 = i\beta y^0 - f^0$, $z^1 = i\beta z^0 - g^0$, it follows

$$\begin{aligned} &\|y^1\|_{L^2(\Omega)} + \|z^1\|_{L^2(\Omega)} \\ &\quad \le \|f^0\|_{L^2(\Omega)} + |\beta|\|y^0\|_{L^2(\Omega)} + \|g^0\|_{L^2(\Omega)} + |\beta|\|z^0\|_{L^2(\Omega)} \\ &\quad \le Ce^{C|\beta|}\big(\|f^0\|_{L^2(\Omega)} + \|f^1\|_{L^2(\Omega)} + \|g^0\|_{L^2(\Omega)} + \|g^1\|_{L^2(\Omega)}\big). \end{aligned} \tag{4.12}$$

By (4.11)–(4.12), we know that there exists $C > 0$ such that

$$\|U_0\|_{\mathcal{H}} \le Ce^{C|\beta|}\big\|(i\beta - \mathcal{A})U_0\big\|_{\mathcal{H}}. \tag{4.13}$$

Since $\mathcal{A}$ has compact resolvents, $i\beta \in \rho(\mathcal{A})$ as long as $i\beta - \mathcal{A}$ is injective (see [20, Theorem 6.29]). Therefore, by (4.13) we have $i\mathbb{R} \subset \rho(\mathcal{A})$ and

$$\big\|(i\beta - \mathcal{A})^{-1}\big\|_{\mathcal{L}(\mathcal{H})} \le Ce^{C|\beta|}.$$

This completes the proof of Theorem 2.2. □

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