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Stochastic patch structure Nicholson's blowflies system with mixed delays

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Abstract

This paper is devoted to studying a stochastic patch structure Nicholson's blowflies system with mixed delays which is a new model for the generalization of classic Nicholson's blowflies system. We examine stochastically ultimate boundedness and global asymptotic stability for the considered model by stochastic analysis technique. Finally, numerical simulations verify theoretical results of the present paper.

MSC: 34C25; 34K13

Keywords: Stochastic; Nicholson's blowflies system; Existence; Stability

1 Introduction

Deterministic Nicholson's blowflies models (including their various generalized forms) have been extensively studied, see e.g. [1–6]. In [7], Wang studied a class of impulsive stochastic Nicholson's blowflies models with patch structure and nonlinear harvesting terms on time scales and obtained the existence and exponential stability of piecewise mean-square almost periodic solutions for the model by using the contraction mapping principle and the Gronwall–Bellman inequality technique. The authors [8] considered a class of impulsive stochastic Nicholson's blowflies models. By applying Cauchy matrix, they obtained the existence and exponential stability of square-mean almost periodic solutions for the model with multiple nonlinear harvesting terms and delays. In recent years, the research for stochastic Nicholson's blowflies models has gradually become a hot topic. In 2019, Wang et al. [9] studied a stochastic Nicholson's blowflies delayed differential equation. After that, Zhu et al. [10] generalized the equation in [9] to the stochastic Nicholson's blowflies delay differential equation with regime switching.

Furthermore, behaviors of population model are influenced and determined by incorporating migration [11]. Hence, adding patch structure term in the Nicholson's blowflies models appears to be necessary. In 2001, Berezansky et al. [12] investigated a deterministic Nicholson's blowflies system with patch structure as follows:

$$\begin{cases} x_1'(t) = -(a_1 + b_2)x_1(t) + b_1 x_2(t) + p_1 x_1(t-\tau)e^{-\gamma_1 x_1(t)}, \\ x_2'(t) = -(a_2 + b_1)x_2(t) + b_2 x_1(t) + p_2 x_2(t-\tau)e^{-\gamma_2 x_2(t)}. \end{cases} \tag{1.1}$$

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Then, in 2019, Wang, Shi, and Chen [13], Yi and Liu [14] generalized Nicholson's blowflies system in [12] to the following two-dimensional stochastic system:

$$
\begin{cases}
dx_1(t) = [-(a_1 + b_2)x_1(t) + b_1 x_2(t) + p_1 x_1(t-\tau)e^{-\gamma_1 x_1(t)}]\,dt + \sigma_1 x_1(t)\,dB_1(t), \\
dx_2(t) = [-(a_2 + b_1)x_2(t) + b_2 x_1(t) + p_2 x_2(t-\tau)e^{-\gamma_2 x_2(t)}]\,dt + \sigma_2 x_2(t)\,dB_2(t).
\end{cases}
\tag{1.2}
$$

For more results about Nicholson's blowflies system, see e.g. [15–17].

Motivated by the above discussions, this paper is devoted to studying a stochastic Nicholson's blowflies system involving mixed delays and patch structure as follows:

$$
\begin{cases}
dx_1(t) = [-a_{11}(t) + b_{11}(t)e^{-x_1(t)} + a_{12}(t) - b_{12}(t)e^{-x_2(t)} \\
\qquad + \alpha_{11}(t)x_1(t-\tau_{11}(t))e^{-\beta_{11}(t)x_1(t-\gamma_{11}(t))} \\
\qquad + \alpha_{12}(t)x_1(t-\tau_{12}(t))e^{-\beta_{12}(t)x_1(t-\gamma_{12}(t))}]\,dt + \sigma_1(t)x_1(t)\,dB_1(t), \\
dx_2(t) = [-a_{22}(t) + b_{22}(t)e^{-x_2(t)} + a_{21}(t) - b_{21}(t)e^{-x_1(t)} \\
\qquad + \alpha_{21}(t)x_2(t-\tau_{21}(t))e^{-\beta_{21}(t)x_2(t-\gamma_{21}(t))} \\
\qquad + \alpha_{22}(t)x_2(t-\tau_{22}(t))e^{-\beta_{22}(t)x_2(t-\gamma_{22}(t))}] + \sigma_2(t)x_2(t)\,dB_1(t),
\end{cases}
\tag{1.3}
$$

with the initial condition

$$
x_i(t) = \phi_i(t) \in C\big([-\tau, 0], \mathbb{R}^+\big), \quad t \in [-\tau, 0], i = 1, 2,
\tag{1.4}
$$

which has the maximum norm $\|\cdot\|$, where $\mathbb{R}^+ = (0, \infty)$, $\tau = \max_{t \geq 0}\{\tau_{ij}(t), \gamma_{ij}(t), i, j = 1, 2\}$. For $i, j = 1, 2$, a_{ij}, b_{ij}, α_{ij}, β_{ij}, $\sigma_i(t)$, $\tau_{ij}(t)$, and $\gamma_{ij}(t)$, are all positive continuous functions. When the case $\tau_{ij} = \gamma_{ij}$ ($i, j = 1, 2$), there exist lots of results, see [9–12]. For $i, j = 1, 2$, $a_{ii}(t) - b_{ii}(t)e^{-x_i(t)}$ is a density-dependent mortality term in ith patch; $\alpha_{ij}(t)x_i(t-\tau_{ij}(t))e^{-\beta_{ij}(t)x_i(t-\gamma_{ij}(t))}$ is a birth function with maturation delay τ_{ij} and feedback delay γ_{ij}; $B_i(t)$ is the standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t \geq 0}$. Throughout this paper, denote $f^+ = \sup_{t \geq 0} f(t)$, $f^- = \inf_{t \geq 0} f(t)$.

The main highlights list as follows:

(1) We study a new stochastic model which is a generalization of the classic Nicholson's blowflies system;

(2) We develop a stochastic analysis technique for studying dynamic properties of the stochastic Nicholson's blowflies system.

The following sections are organized as follows: In Sect. 2, we obtain some sufficient conditions for global existence and uniqueness to a positive solution of system (1.3). In Sect. 3, we obtain stochastically ultimate boundedness positive solution of system (1.3). Section 4 gives global asymptotic stability of system (1.3). In Sect. 5, a numerical example verifies the accuracy of the results in the present paper. Section 6 contains some conclusions of this paper and further research for the topic of this paper.

2 Existence and uniqueness of global positive solution

Theorem 2.1 *For any given initial data* (1.4), *there is a unique positive solution* $(x_1(t), x_2(t))^\top$ *on* $[-\tau, \infty)$ *and the solution will remain in* $\mathbb{R}^+ \times \mathbb{R}^+$ *with probability one.*

Proof The proof method of this theorem comes from Theorem 2.1 in Sect. 11.2 of [18]. Since the coefficients of (1.3) satisfy the local Lipschitz condition, then for any given initial

condition (1.4), there exists a unique local solution $x_1(t)$ and $x_2(t)$ on $[\tau, \tau_b)$, where τ_b is explosion time. For showing this solution is global, we shall prove $\tau_b = \infty$ a.s. Let $k_0 > 0$ be sufficiently large for initial values (1.4) in $(\frac{1}{k_0}, k_0)$. For each $k > k_0$, define the stopping time

$$\tau_n = \inf\left\{ t \in [0, \tau_b), x_i(t) \notin \left(\frac{1}{k_0}, k_0 \right), i = 1, 2 \right\}.$$

Obviously, τ_n is increasing as $n \to \infty$ and $\tau_\infty \le \tau_b$ a.s. Set $\inf \emptyset = \infty$, where $\emptyset$ is an empty set. We claim that if $\tau_\infty = \infty$, then $\tau_b = \infty$ and $x_i(t) \in \mathbb{R}^+$ for $i = 1, 2, t \ge 0$. If not, there exist constants $T > 0$ and $\varepsilon \in (0, 1)$ such that $\mathbb{P}(\tau_\infty \le T) > \varepsilon$. Thus, there exists a number $n_1 > n_0$ such that $\mathbb{P}(\tau_\infty \le T) > \varepsilon$ for $n > n_1$. Define a C^2-function $V : \mathbb{R}^+ \times \mathbb{R}^+ \to \mathbb{R}^+$ by $V(x_1, x_2) = \sum_{i=1}^{2}(x_i - 1 - \ln x_i)$. Itô's formula shows that

$$dV(x_1, x_2) = \left[\left(1 - \frac{1}{x_1} \right) F(x_1, x_2) + \frac{\sigma_1^2}{2} \right] dt + \sigma_1(x_1 - 1)\, dB_1(t)$$
$$+ \left[\left(1 - \frac{1}{x_2} \right) G(x_1, x_2) + \frac{\sigma_2^2}{2} \right] dt + \sigma_2(x_2 - 1)\, dB_2(t), \tag{2.1}$$

where

$$
\begin{aligned}
F(x_1, x_2) &= -a_{11}(t) + b_{11}(t)e^{-x_1(t)} + a_{12}(t) - b_{12}(t)e^{-x_2(t)} \\
&\quad + \alpha_{11}(t)x_1\left(t - \tau_{11}(t) \right)e^{-\beta_{11}(t)x_1(t - \gamma_{11}(t))} \\
&\quad + \alpha_{12}(t)x_1\left(t - \tau_{12}(t) \right)e^{-\beta_{12}(t)x_1(t - \gamma_{12}(t))}, \\
G(x_1, x_2) &= -a_{22}(t) + b_{22}(t)e^{-x_2(t)} + a_{21}(t) - b_{21}(t)e^{-x_1(t)} \\
&\quad + \alpha_{21}(t)x_2\left(t - \tau_{21}(t) \right)e^{-\beta_{21}(t)x_2(t - \gamma_{21}(t))} \\
&\quad + \alpha_{22}(t)x_2\left(t - \tau_{22}(t) \right)e^{-\beta_{22}(t)x_2(t - \gamma_{22}(t))}.
\end{aligned}
\tag{2.2}
$$

Compute the term in (2.1):

$$
\begin{aligned}
&\left(1 - \frac{1}{x_1} \right) F(x_1, x_2) \\
&= \left(1 - \frac{1}{x_1} \right)\left[-a_{11}(t) + b_{11}(t)e^{-x_1(t)} + a_{12}(t) - b_{12}(t)e^{-x_2(t)} \right. \\
&\quad \left. + \alpha_{11}(t)x_1\left(t - \tau_{11}(t) \right)e^{-\beta_{11}(t)x_1(t - \gamma_{11}(t))} + \alpha_{12}(t)x_1\left(t - \tau_{12}(t) \right)e^{-\beta_{12}(t)x_1(t - \gamma_{12}(t))} \right] \\
&\le \left(1 - \frac{1}{x_1} \right)\left[-a_{11} + a_{12} + b_{11}e^{-x_1} - b_{12}e^{-x_2} \right. \\
&\quad \left. + \alpha_{11}\left(\|\phi_1\| + x_1 \right)e^{-\beta_{11}(t)x_1} + \alpha_{12}\left(\|\phi_1\| + x_1 \right)e^{-\beta_{12}(t)x_1} \right] \\
&\le a_{12}^+ + b_{11}^+ + \alpha_{11}^+\|\phi_1\| + \frac{\alpha_{11}^+}{\beta_{11}^- e} + \alpha_{12}^+\|\phi_1\| + \frac{\alpha_{12}^+}{\beta_{12}^- e} := K_1,
\end{aligned}
\tag{2.3}
$$

where we use $\sup_{x \in \mathbb{R}^+} xe^{-x} = \frac{1}{e}$. Similar to the above proof, we gain

$$\left(1 - \frac{1}{x_2} \right) G(x_1, x_2) \le a_{21}^+ + b_{22}^+ + \alpha_{21}^+\|\phi_2\| + \frac{\alpha_{21}^+}{\beta_{21}^- e} + \alpha_{22}^+\|\phi_2\| + \frac{\alpha_{22}^+}{\beta_{22}^- e} := K_2. \tag{2.4}$$

Let

$$LV(x_1, x_2) = \left(1 - \frac{1}{x_1}\right)F(x_1, x_2) + \frac{\sigma_1^2}{2} + \left(1 - \frac{1}{x_2}\right)G(x_1, x_2) + \frac{\sigma_2^2}{2}.$$

By (2.3) and (2.4), there is a constant $K > 0$ such that

$$LV(x_1, x_2) \leq K. \tag{2.5}$$

Integrating both sides of (2.5) from $-\tau$ to $\tau_n \wedge T$ and taking the expectation $\mathbb{E}$, it follows by (2.1) that

$$\mathbb{E}V\big(x_1(\tau_n \wedge T), x_2(\tau_n \wedge T)\big) \leq V\big(x_1(-\tau), x_2(-\tau)\big) + K(t + \tau).$$

Let $\Theta_n = \{\tau_n \leq T\}$, then $\mathbb{P}(\Theta_n) \geq \varepsilon$. Then, for each $\vartheta \in \Theta_n$, $x_1(\tau_n, \vartheta)$ or $x_2(\tau_n, \vartheta)$ equals n or $\frac{1}{n}$ Thus, $V(x_1(\tau_n \wedge T), x_2(\tau_n \wedge T))$ is no less than $(n - 1 - \ln n) \wedge (\frac{1}{n} - \ln n)$. Thus,

$$
\begin{aligned}
V\big(x_1(-\tau), x_2(-\tau)\big) + K(t + \tau) &\geq \mathbb{E}\big[1_{\Theta_n} V\big(x_1(\tau_n), x_2(\tau_n)\big)\big] \\
&\geq \mathbb{P}(\Theta_n)\left[(n - 1 - \ln n) \wedge \left(\frac{1}{n} - \ln n\right)\right] \\
&\geq \varepsilon\left[(n - 1 - \ln n) \wedge \left(\frac{1}{n} - \ln n\right)\right],
\end{aligned}
$$

where 1_{Θ_n} is the indicator function of Θ_n. Letting $n \to \infty$ leads to $\infty > V(x_1(-\tau), x_2(-\tau)) + KT = \infty$, which is a contradiction. So we gain $\tau_\infty = \infty$ a.s. The proof is finished. $\qquad\square$

3 Stochastically ultimate boundedness

Definition 3.1 System (1.3) is said to be stochastically ultimately bounded if, for $\varepsilon \in (0, 1)$, there exists a constant $L(\varepsilon)$ such that, for any initial data (1.4), the solution $x = (x_1, x_2)^\top$ of (1.3) satisfies

$$\lim_{t \to \infty} \sup \mathbb{P}\big\{|x(t)| \leq L\big\} \geq 1 - \varepsilon.$$

Lemma 3.1 *Let $\gamma \in (0, 1)$. There is a constant $M(\gamma) > 0$ which is independent initial data* (1.4) *such that the solution $x = (x_1, x_2)^\top$ of* (1.3) *satisfies* $\lim_{t \to \infty} \sup \mathbb{E}|x(t)|^\gamma \leq M$.

Proof Let $V(x_1, x_2) = x_1^\gamma + x_2^\gamma$ for $\gamma \in (0, 1)$, $x_1, x_2 > 0$. Itô's formula shows that

$$dV(x_1, x_2) = \mathcal{L}V(x_1, x_2)\, dt + \sigma_1 \gamma x_1^\gamma\, dB_1(t) + \sigma_2 \gamma x_2^\gamma\, dB_2(t), \tag{3.1}$$

where

$$
\begin{aligned}
\mathcal{L}V(x_1, x_2) &= \gamma x_1^{\gamma-1} F(x_1, x_2) + 0.5\gamma(\gamma - 1)\sigma_1^2 x_1^\gamma \\
&\quad + \gamma x_2^{\gamma-1} G(x_1, x_2) + 0.5\gamma(\gamma - 1)\sigma_2^2 x_2^\gamma, \tag{3.2}
\end{aligned}
$$

$F(x_1, x_2)$ and $G(x_1, x_2)$ are defined by (2.2). Compute the term in (3.2):

$$
\begin{aligned}
\gamma x_1^{\gamma-1} F(x_1, x_2) = \gamma x_1^{\gamma-1} \big[&-a_{11}(t) + b_{11}(t)e^{-x_1(t)} + a_{12}(t) - b_{12}(t)e^{-x_2(t)} \\
&+ \alpha_{11}(t)x_1\big(t - \tau_{11}(t)\big)e^{-\beta_{11}(t)x_1(t-\gamma_{11}(t))} \\
&+ \alpha_{12}(t)x_1\big(t - \tau_{12}(t)\big)e^{-\beta_{12}(t)x_1(t-\gamma_{12}(t))} \big] \\
\leq \gamma b_{11}^+ &+ \gamma a_{12}^+ + \gamma \alpha_{11}^+ \|\phi_1\| + \frac{\gamma \alpha_{11}^+}{\beta_{11}^- e} + \gamma \alpha_{12}^+ \|\phi_1\| + \frac{\gamma \alpha_{12}^+}{\beta_{12}^- e} := M_1,
\end{aligned}
\tag{3.3}
$$

where we use $\sup_{x \in \mathbb{R}^+} x e^{-x} = \frac{1}{e}$. Similar to the above proof, we have

$$
\gamma x_1^{\gamma-1} G(x_1, x_2) \leq \gamma b_{22}^+ + \gamma a_{21}^+ + \gamma \alpha_{21}^+ \|\phi_1\| + \frac{\gamma \alpha_{21}^+}{\beta_{21}^- e} + \gamma \alpha_{22}^+ \|\phi_1\| + \frac{\gamma \alpha_{22}^+}{\beta_{22}^- e} := M_2.
\tag{3.4}
$$

In view of (3.1)–(3.4), we have

$$
dV(x_1, x_2) = (M_1 + M_2)\, dt + \sigma_1 \gamma x_1^\gamma \, dB_1(t) + \sigma_2 \gamma x_2^\gamma \, dB_2(t).
\tag{3.5}
$$

By (3.5) we gain

$$
\begin{aligned}
d\big[e^t V(x_1, x_2)\big] &= e^t \big[V(x_1, x_2)\, dt + dV(x_1, x_2) \big] \\
&\leq e^t(M_1 + M_2)\, dt + e^t \sigma_1 \gamma x_1^\gamma \, dB_1(t) + e^t \sigma_2 \gamma x_2^\gamma \, dB_2(t),
\end{aligned}
$$

and $e^t \mathbb{E} V(x_1, x_2) \leq V(x_1(0), x_2(0)) + (M_1 + M_2)e^t - M_1 - M_2$, which implies $\lim_{t \to \infty} \sup \mathbb{E} V(x_1, x_2) \leq M_1 + M_2$. Furthermore, $|x|^2 \leq 2(\max\{x_1, x_2\})^2$, which implies

$$
|x|^\gamma \leq 2^{\frac{\gamma}{2}} \big(\max\{x_1, x_2\}\big)^\gamma \leq 2^{\frac{\gamma}{2}} V(x_1, x_2).
$$

Thus,

$$
\lim_{t \to \infty} \sup \mathbb{E}|x|^\gamma \leq 2^{\frac{\gamma}{2}} (M_1 + M_2) := M. \qquad \square
$$

Theorem 3.1 *System* (1.3) *is stochastically ultimately bounded.*

Proof By Lemma 3.1, there exists a positive constant $K > 0$ such that

$$
\lim_{t \to \infty} \sup \mathbb{E}|x|^{\frac{1}{2}} \leq K.
$$

Let $H = \frac{K^2}{\varepsilon^2}$. Then, for any $\varepsilon > 0$, by Chebyshev's inequality, we have

$$
\mathbb{P}\big\{ |x(t)| > H \big\} \leq \frac{\mathbb{E}|x|^{\frac{1}{2}}}{H^{\frac{1}{2}}} = \varepsilon.
$$

Thus,

$$
\lim_{t \to \infty} \sup \mathbb{P}\big\{ |x(t)| \leq H \big\} \geq 1 - \varepsilon. \qquad \square
$$

4 Global asymptotic stability

Definition 4.1 For any two positive solutions $(x_1(t), x_2(t))^\top$ and $(y_1(t), y_2(t))^\top$ of system (1.3), system (1.3) is said to be globally asymptotically stable if

$$\lim_{t \to \infty} |x_1(t) - x_2(t)| = \lim_{t \to \infty} |y_1(t) - y_2(t)| = 0 \quad a.s.$$

Lemma 4.1 *Let* $(x_1(t), x_2(t))^\top$ *be a solution of* (1.3) *with initial data* (1.4). *Then there exist constants* $L(p), G(p) > 0$ *such that*

$$\mathbb{E}[x_1^p(t)] \leq L(p) \quad and \quad \mathbb{E}[x_2^p(t)] \leq G(p) \quad for all p > 1, t \geq -\tau. \tag{4.1}$$

Furthermore, each solution $(x_1(t), x_2(t))^\top$ *of system* (1.3) *is uniformly continuous on* $t \geq 0$.

Proof Define $V(u) = u^p$ for $u > 0$ and $p > 1$. Itô's formula shows that

$$dV(x_1) = \left[px_1^{p-1} F(x_1, x_2) + 0.5\sigma_1^2 p(p-1)x_1^p \right] dt + \sigma_1 px_1^p \, dB_1(t), \tag{4.2}$$

where $F(x_1, x_2)$ is defined by (2.2). Use Itô's formula again to $e^t V(x_1)$ and (4.2), then

$$d[e^t V(x_1)] = e^t V(x_1) \, dt + e^t \, dV(x_1)$$
$$= \left[e^t x_1^p + e^t px_1^{p-1} F(x_1, x_2) + 0.5e^t \sigma_1^2 p(p-1)x_1^p \right] dt + e^t \sigma_1 px_1^p \, dB_1(t). \tag{4.3}$$

Integrate both sides of (4.3) from $-\tau$ to t and and take expectations, then

$$\mathbb{E}[e^t x_1^p] \leq \phi_1^p(-\tau) + \mathbb{E} \int_{-\tau}^t e^s x_1^p(s) \Big[1 + pb_{11}e^{-x_1} + pa_{12} + p\alpha_{11}(\|\phi_1\| + x_1)e^{-\beta_{11}x_1}$$
$$+ p\alpha_{12}(\|\phi_1\| + x_1)e^{-\beta_{12}x_1} + 0.5p(p-1)\sigma_1^2 \Big] ds$$
$$\leq \phi_1^p(-\tau) + \mathbb{E} \int_{-\tau}^t e^s x_1^p(s) \Big[1 + pb_{11}^+ + pa_{12}^+ + p\alpha_{11}^+ \|\phi_1\| + \frac{p\alpha_{11}^+}{\beta_{11}^- e}$$
$$+ p\alpha_{12}^+ \|\phi_1\| + \frac{p\alpha_{12}^+}{\beta_{12}^- e} + 0.5p(p-1)(\sigma_1^2)^+ \Big] ds.$$

Thus,

$$\mathbb{E}[e^t x_1^p] \leq \phi_1^p(-\tau) + \int_{-\tau}^t L_1(p)\mathbb{E}[e^s x_1^p] \, ds,$$

where

$$L_1(p) = 1 + pb_{11}^+ + pa_{12}^+ + p\alpha_{11}^+ \|\phi_1\| + \frac{p\alpha_{11}^+}{\beta_{11}^- e} + p\alpha_{12}^+ \|\phi_1\| + \frac{p\alpha_{12}^+}{\beta_{12}^- e} + 0.5p(p-1)(\sigma_1^2)^+.$$

By Gronwall's inequality, we have $\mathbb{E}[e^t x_1^p] \leq \phi_1^p(-\tau)e^{L_1(p)(t+\tau)}$. Hence, there exists a constant $T > 0$ such that $\mathbb{E}[x_1^p(t)] \leq 2e^{L_1(p)}$ for all $t \geq T$. Due to $\mathbb{E}[x_1^p(t)]$ is continuous, there exists a constant $L_2 > 0$ such that $\mathbb{E}[x_1^p(t)] \leq L_2$ for all $t \in [-\tau, T]$. Let $L(p) = \max\{2e^{L_1(p)}, L_2, \|\phi_1\|\}$, then $\mathbb{E}[x_1^p(t)] \leq L(p)$ for all $t \geq -\tau$. Similar to the above proof, there exists a constant $G(p) > 0$ such that $\mathbb{E}[x_2^p(t)] \leq G(p)$ for all $t \geq -\tau$. □

Lemma 4.2 ([19]) *Assume that an n-dimensional stochastic process $\mathcal{X}$ on $t \geq 0$ satisfies the condition*

$$\mathbb{E}|\mathcal{X}(t) - \mathcal{X}(s)|^{k_1} \leq c|t - s|^{1+k_2} \quad \text{for } t, s \geq 0,$$

where k_1, k_2, and c are positive constants. Then there is a continuous modification $\overline{\mathcal{X}}(t)$ of $\mathcal{X}(t)$ such that

$$\mathbb{P}\left\{\omega: \sup_{0<|t-s|<h(\omega)} \frac{\overline{\mathcal{X}}(t) - \mathcal{X}(t)}{|t-s|^\theta} \leq \frac{2}{1 - 2^{-\theta}}\right\} = 1,$$

where $h(\omega)$ is a positive random variable, $\theta \in (0, \frac{k_2}{k_1})$. In other words, almost every sample path of $\overline{\mathcal{X}}(t)$ is locally but uniformly Hölder continuous with exponent θ.

Lemma 4.3 *Let $(x_1(t), x_2(t))^\top$ be a solution of (1.3) with initial data (1.4). Then almost every sample path of $(x_1(t), x_2(t))^\top$ is uniformly continuous on $t \geq 0$.*

Proof The first equation of system (1.3) is equivalent to the following stochastic integer equation:

$$\begin{aligned}
x_1(t) = \phi_1(0) + \int_0^t &\Big[-a_{11}(s) + b_{11}(s)e^{-x_1(s)} + a_{12}(s) - b_{12}(s)e^{-x_2(s)} \\
&+ \alpha_{11}(s)x_1\big(s - \tau_{11}(s)\big)e^{-\beta_{11}(s)x_1(s-\gamma_{11}(s))} \\
&+ \alpha_{12}(s)x_1\big(s - \tau_{12}(s)\big)e^{-\beta_{12}(s)x_1(s-\gamma_{12}(s))}\Big] ds + \int_0^t \sigma_1(s)x_1(s)\, dB_1(s). \quad (4.4)
\end{aligned}$$

By (4.1), computing the term in (4.4), we gain

$$\begin{aligned}
\mathbb{E}|F(x_1, x_2)|^p &\leq \mathbb{E}|a_{11}^+ + b_{11}^+ + a_{12}^+ + b_{12}^+ + \alpha_{11}^+\|\phi_1\| + \alpha_{11}^+ x_1 + \alpha_{12}^+\|\phi_1\| + \alpha_{12}^+ x_1|^p \\
&\leq 2^{p-1}\Big[\big(a_{11}^+ + b_{11}^+ + a_{12}^+ + b_{12}^+ + \alpha_{11}^+\|\phi_1\| + \alpha_{12}^+\|\phi_1\|\big)^p + \big(\alpha_{11}^+ + \alpha_{12}^+\big)\mathbb{E}x_1^p\Big] \\
&\leq \mathbb{E}|a_{11}^+ + b_{11}^+ + a_{12}^+ + b_{12}^+ + \alpha_{11}^+\|\phi_1\| + \alpha_{11}^+ x_1 + \alpha_{12}^+\|\phi_1\| + \alpha_{12}^+ x_1|^p \\
&\leq 2^{p-1}\Big[\big(a_{11}^+ + b_{11}^+ + a_{12}^+ + b_{12}^+ + \alpha_{11}^+\|\phi_1\| + \alpha_{12}^+\|\phi_1\|\big)^p + \big(\alpha_{11}^+ + \alpha_{12}^+\big)L(p)\Big] \\
&:= L_1(p). \quad (4.5)
\end{aligned}$$

Furthermore, by moment inequality for stochastic integrals, for $0 \leq t_1 \leq t_2$ and $p > 2$, we gain

$$\begin{aligned}
\mathbb{E}&\left|\int_{t_1}^{t_2} \sigma_1(t)x_1(t)\, dB_1(t)\right|^p \\
&\leq \big[(\sigma_1^2)^+\big]^p\big[0.5p(p-1)\big]^{\frac{p}{2}}(t_2 - t_1)^{\frac{p}{2}-1}\int_{t_1}^{t_2}\mathbb{E}|x_1(t)|^p\, dt \\
&\leq \big[(\sigma_1^2)^+\big]^p\big[0.5p(p-1)\big]^{\frac{p}{2}}(t_2 - t_1)^{\frac{p}{2}}L(p). \quad (4.6)
\end{aligned}$$

$\square$

In view of (4.4)–(4.6), for $0 \le t_1 \le t_2$, $t_2 - t_1 \le 1$, and $p > 2$, we have

$$
\mathbb{E}\big|x_1(t_2) - x_1(t_1)\big|^p = \mathbb{E}\left|\int_{t_1}^{t_2} F\big(x_1(t), x_2(t)\big)\, dt + \int_{t_1}^{t_2} \sigma_1(t)x_1(t)\, dB_1(t)\right|^p
$$

$$
\le 2^{p-1}\mathbb{E}\left|\int_{t_1}^{t_2} F\big(x_1(t), x_2(t)\big)\, dt\right|^p + 2^{p-1}\mathbb{E}\left|\int_{t_1}^{t_2} \sigma_1(t)x_1(t)\, dB_1(t)\right|^p
$$

$$
\le 2^{p-1}L_1(p)^p(t_2 - t_1)^p + 2^{p-1}\big[(\sigma_1^2)^+\big]^p\big[0.5p(p-1)\big]^{\frac{p}{2}}(t_2 - t_1)^{\frac{p}{2}}L(p)
$$

$$
\le \big(2^{p-1}L_1(p)^p + 2^{p-1}\big[(\sigma_1^2)^+\big]^p\big[0.5p(p-1)\big]^{\frac{p}{2}}L(p)\big)(t_2 - t_1)^{\frac{p}{2}}.
$$

By Lemma 4.2, almost every path of $x_1(t)$ is locally but uniformly Hölder continuous with exponent $\theta \in (0, \frac{p-2}{2p})$. Similar to the above proof, almost every path of $x_2(t)$ is locally but uniformly Hölder continuous.

Lemma 4.4 ([20]) *Let f be a nonnegative function defined on $[0, \infty)$ which is integrable and is uniformly continuous. Then $\lim_{t \to \infty} f(t) = 0$.*

Theorem 4.1 *If $(a_{11} - a_{12})^- - b_{11}^+ - \alpha_{11}^+\|\phi_1\| - \alpha_{11}^+ > 0$ and $(a_{22} - a_{21})^- - b_{22}^+ - \alpha_{21}^+\|\phi_2\| - \alpha_{21}^+ > 0$, then system (1.3) is globally asymptotically stable.*

Proof For any two positive solutions $(x_1(t), x_2(t))^\top$ and $(y_1(t), y_2(t))^\top$ of (1.3), define $\mathcal{V}(t)$ on $t \ge 0$ by $\mathcal{V}(t) = |\ln x_1(t) - \ln x_2(t)| + |\ln y_1(t) - \ln y_2(t)|$. Compute the right differential of $\mathcal{V}(t)$

$$
d^+\mathcal{V}(t) = \mathrm{sgn}(x_1 - x_2)\left[\left(\frac{1}{x_1} - \frac{1}{x_2}\right)F(x_1, x_2) + 0.5\sigma_1^2\big(x_1^2 - x_2^2\big)\right]
$$

$$
+ \mathrm{sgn}(y_1 - y_2)\left[\left(\frac{1}{y_1} - \frac{1}{y_2}\right)G(y_1, y_2) + 0.5\sigma_2^2\big(y_1^2 - y_2^2\big)\right]
$$

$$
\le -\big[(a_{11} - a_{12})^- - b_{11}^+ - \alpha_{11}^+\|\phi_1\| - \alpha_{11}^+\big]|x_1 - x_2|\, dt
$$

$$
- \big[(a_{22} - a_{21})^- - b_{22}^+ - \alpha_{21}^+\|\phi_2\| - \alpha_{21}^+\big]|y_1 - y_2|\, dt. \tag{4.7}
$$

In view of assumptions of Theorem 4.1, integrating both sides of (4.7) leads to

$$
V(t) + \int_0^t \big[(a_{11} - a_{12})^- - b_{11}^+ - \alpha_{11}^+\|\phi_1\| - \alpha_{11}^+\big]\big|x_1(s) - x_2(s)\big|\, ds
$$

$$
+ \int_0^t \big[(a_{22} - a_{21})^- - b_{22}^+ - \alpha_{21}^+\|\phi_2\| - \alpha_{21}^+\big]\big|y_1(s) - y_2(s)\big|\, ds
$$

$$
\le V(0) < \infty.
$$

Thus, $|x_1(t) - x_2(t)|, |y_1(t) - y_2(t)| \in L^1[0, \infty)$ and the desired assertion follows from Lemmas 4.3 and 4.4 immediately. $\qquad\square$

5 Numerical examples

Consider the following example:

$$
\begin{cases}
dx_1(t) = [-(12 + \sin t) + (2 + \sin t)e^{-x_1(t)} + 2 - \sin t - (2 - \sin t)e^{-x_2(t)} \\
\qquad + (2 - \sin^2 t)x_1(t-1)e^{-(2+\sin^2 t)x_1(t-1.5)} \\
\qquad + (2 - \cos^2 t)x_1(t-1)e^{-(2+\cos^2 t)x_1(t-1.2)}]\,dt + 0.2x_1(t)\,dB_1(t), \\
dx_2(t) = [-(12 - \cos^2 t) + (2 - \cos t)e^{-x_2(t)} + 2 - \cos^2 t - (2 - \cos^2 t)e^{-x_1(t)} \\
\qquad + (2 - \cos^2 t)x_2(t-1.5)e^{-(3+\sin^2 t)x_2(t-1.2)} \\
\qquad + (2 + \cos^2 t)x_2(t-0.5)e^{-(3+\cos^2 t)x_2(t-1.2)}]\,dt + 0.1x_2(t)\,dB_2(t),
\end{cases}
\tag{5.1}
$$

where

$$a_{11}(t) = 12 + \sin t, \qquad b_{11}(t) = 2 + \sin t, \qquad a_{12}(t) = 2 - \sin t, \qquad b_{12}(t) = 2 - \sin t,$$

$$\alpha_{11}(t) = 2 - \sin^2 t, \qquad \tau_{11}(t) = 1, \qquad \beta_{11}(t) = 2 + \sin^2 t, \qquad \gamma_{11}(t) = 1.5,$$

$$\alpha_{12}(t) = 2 - \cos^2 t, \qquad \tau_{12}(t) = 0.5, \qquad \beta_{12}(t) = 2 + \cos^2 t, \qquad \gamma_{12}(t) = \cos^2 \frac{\pi t}{2},$$

$$\sigma_1 = 0.2,$$

$$a_{22}(t) = 12 - \cos^2 t, \qquad b_{22}(t) = 2 - \cos t, \qquad a_{21}(t) = 2 - \cos^2 t,$$

$$b_{21}(t) = 2 - \cos^2 t,$$

$$\alpha_{21}(t) = 2 - \cos^2 t, \qquad \tau_{21}(t) = 1.5, \qquad \beta_{21}(t) = 3 + \sin^2 t, \qquad \gamma_{21}(t) = 1.2,$$

$$\alpha_{22}(t) = 2 + \cos^2 t, \qquad \tau_{22}(t) = 0.5, \qquad \beta_{22}(t) = 3 + \cos^2 t, \qquad \gamma_{21}(t) = 1.2,$$

$$\sigma_2 = 0.1.$$

Obviously, $\tau = \max\{\tau_{ij}, \gamma_{ij}, i,j = 1,2\} = 1.5$, then the initial value of system (4.2) takes $x_i(t) = \phi_i(t) = \sin^2 t$, $i = 1, 2$, $t \in [-1.5, 0]$. After simple calculation, we have

$$(a_{11} - a_{12})^- = 10, \qquad b_{11}^+ = 3, \qquad \alpha_{11}^+ = 2, \qquad \|\phi_1\| = 1,$$

$$(a_{11} - a_{12})^- - b_{11}^+ - \alpha_{11}^+\|\phi_1\| - \alpha_{11}^+ = 3 > 0,$$

$$(a_{22} - a_{21})^- = 8, \qquad b_{22}^+ = 3, \qquad \alpha_{21}^+ = 2, \qquad \|\phi_2\| = 1,$$

$$(a_{22} - a_{21})^- - b_{22}^+ - \alpha_{21}^+\|\phi_2\| - \alpha_{21}^+ = 1 > 0.$$

Then the conditions of Theorem 4.1 hold and system (5.1) is globally asymptotically stable. The numerical solutions with proper initial values are shown in Fig. 1.

6 Conclusions

In this paper, we study a patch structure stochastic Nicholson's blowflies system with mixed delay and obtain some very simple conditions for guaranteeing the existence and global asymptotic stability of the considered system. It is interesting that the delays of system (1.3) are not the same, which is different from the corresponding ones of the past work. The methods in this paper can be extended to study other types of differential dynamic

Figure 1 State trajectories of system (5.1) for $x(t)$
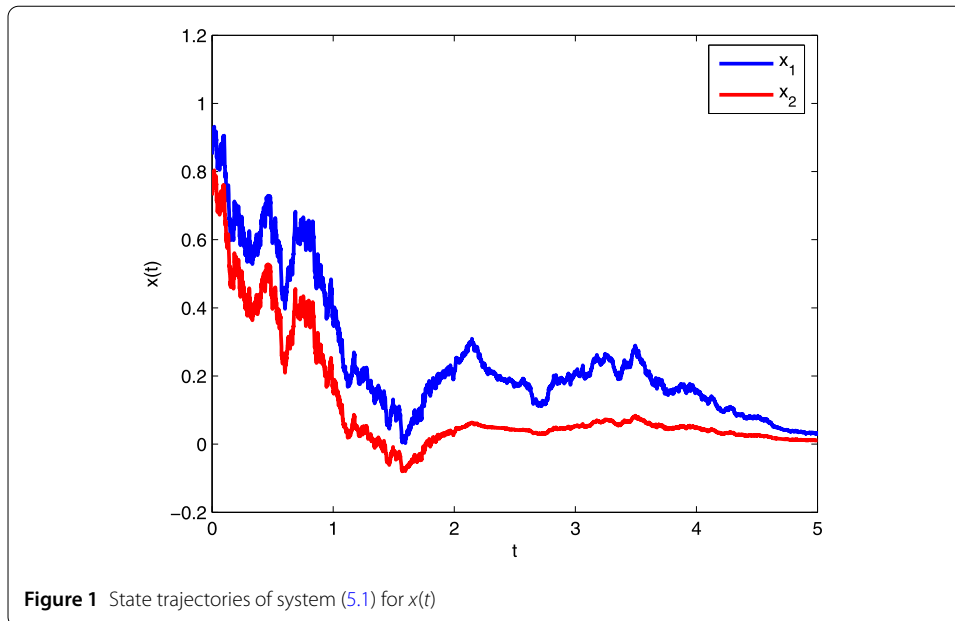
systems such as stochastic differential impulsive equations, stochastic fractional differential equations, etc. We hope other researchers can use the method provided in this article to do more in-depth research on various types of stochastic differential dynamic systems.

Availability of data and materials
Data sharing not applicable to this article as no datasets were generated or analyzed during the current study.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors contributed equally to the writing of this paper. All authors read and approved the final manuscript.

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