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Analysis of dengue model with fractal-fractional Caputo–Fabrizio operator

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Abstract

In this work, we study the dengue dynamics with fractal-factional Caputo–Fabrizio operator. We employ real statistical data of dengue infection cases of East Java, Indonesia, from 2018 and parameterize the dengue model. The estimated basic reduction number for this dataset is $\mathcal{R}_0 \approx 2.2020$. We briefly show the stability results of the model for the case when the basic reproduction number is $\mathcal{R}_0 < 1$. We apply the fractal-fractional operator in the framework of Caputo–Fabrizio to the model and present its numerical solution by using a novel approach. The parameter values estimated for the model are used to compare with fractal-fractional operator, and we suggest that the fractal-fractional operator provides the best fitting for real cases of dengue infection when varying the values of both operators' orders. We suggest some more graphical illustration for the model variables with various orders of fractal and fractional.

Keywords: Fractal-fractional; Caputo–Fabrizio operator; Dengue model; Real cases; Parameter estimates

Dengue infection is a vector-borne infection that is caused through a female mosquito from genus Flavivirus, family Flaviviridae. DENV1 to DENV4 are different serotypes associated with the dengue virus. A person infected with one DENV serotype has a lifetime of immunity from that serotype, but is not guaranteed immunity against other serotypes. Thus, it is possible for a person to be infected four times, once with each serotype [1]. Dengue fever is transmitted through the bite of female Aedes Aegypti and Aedes albopictus. Aedes Aegypti mosquitoes will attack healthy humans and people with dengue fever so that the virus enters the human body and reproduces itself; it will be in the blood for one week. If the human immune system is weak, it will be easy for humans to become dengue sufferers.

Dengue fever is a growing public health concern worldwide. As mobility and population density increase, the number of sufferers and the area of its spread are increasing. In recent years, dengue fever has become endemic in more than 100 countries including Africa, America, the Eastern Mediterranean, Southeast Asia, and the Western Pacific. The worst areas affected are in the United States, the Western Pacific, and Southeast Asia [2, 3]. These areas had the highest number of dengue cases, with a total of 1.2 million cases in 2008 and more than 3.34 million cases in 2016 reported [4].

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In Indonesia, dengue was first discovered in the city of Surabaya in 1968, where as many as 58 people were infected and 24 of them died, with a mortality rate of 41.3%. Since then, the disease has spread widely throughout Indonesia [5]. In 2015, there were 126,675 dengue fever sufferers across 34 provinces in Indonesia, and 1229 of them died. This number is higher than the previous year, as many as 907 patients died and there were 100,347 sufferers due to dengue fever. This increasing figure could be caused by climate change and low awareness to maintain environmental cleanliness [6].

Many researchers have utilized mathematical modeling to capture complexities of dengue infection transmission [7–14]. All of the models are stated in the form of ordinary integer-order derivative to describe the dynamics of the dengue infection. The limitation of these models is that they do not present the information about memory and learning mechanisms. In addition, it is difficult to explore the dynamics between two different points using the models based on integer-order derivatives. In order to address these concerns, various fractional-order derivative models were proposed in the literature (see [15, 16] and the references therein). Recently, some important results regarding the fractional operators were considered in [17–24]. In these works, the authors utilized the concepts of the fractional operators and obtained results for their proposed models.

Recently, some new results regarding the fractional operators in fractal-fractional the sense were considered in [16] and utilized in [25, 26] by the real statistical applications to banking data. The authors showed in their works that the fractal-fractional operators are effective for data fitting of real data. The fractal-fractional operators used in [25, 26] were in Caputo and Atangana–Baleanu sense. Due to the recent application of fractal-fractional operators, we aim in this work to apply the fractal-fractional operator in Caputo–Fabrizio sense and consider its application to real statistical data of Indonesia for dengue fever. Some more published work on the fractal-fractional and fractional modeling and its applications to science and engineering problems can be seen in [27–33]. For example, the authors considered a fractal-fractional model of an oscillator and presented the results in [27]. Avian influenza model with fractal-fractional derivative has been studied in [28]. The Valles model and its multiple attractors have been obtained using fractional derivative in [29]. A new chaotic model using fractal-fractional differentiation has been considered in [30]. A malaria model with fractal-fractional derivative was studied in [31]. Chaotic attractors with simple illustrations have been considered in [32]. The fractal-fractional applications to complex systems have been studied in [33].

The present work explores the dengue dynamics with hospitalization class in the framework of the fractal-fractional operator where the fractional operator is considered in the sense of Caputo–Fabrizio. Using this new idea, we formulate a dengue model and investigate its parameters using the real data of East Java, Indonesia, for the year 2018. We give a comparison of the fractal-fractional orders versus the real data of dengue cases and show some interesting results. We structure the rest of the paper as follows: The dengue model formulation and the related mathematical results are shown in Sect. 2. The basic concepts of fractal-fractional operators, the model construction using fractal-fractional operators, and associated results are shown in Sect. 3. The numerical solution of the fractal-fractional dengue model is presented in Sect. 4. Parameter estimates and the numerical results of the model using the fractal-fractional operator are discoursed in Sect. 5. Section 6 summarizes the work.

1 Dengue model transmission

In this present section, we describe a host–vector model for dengue transmission. The host–vector model is divided into three mosquito populations, susceptible (S_m), exposed (E_m), and infectious (I_m), and five human (host) populations, susceptible (S_h), exposed (E_h), infectious (I_h), hospitalized and/or notified infectious (P_h), and recovered (R_h). Thus, the total human population, denoted by N_h, is given as $N_h = S_h + E_h + I_h + P_h + R_h$, where the total dynamics of mosquitos, denoted by N_m, is $N_m = S_m + E_m + I_m$. The system of differential equations that describes the host–vector model is written as

$$
\begin{cases}
\frac{dS_m}{dt} = \Pi_m - \tau\alpha_h S_m \frac{I_h}{N_h} - \mu_m S_m, \\
\frac{dE_m}{dt} = \tau\alpha_h S_m \frac{I_h}{N_h} - (\beta_m + \mu_m)E_m, \\
\frac{dI_m}{dt} = \beta_m E_m - \mu_m I_m, \\
\frac{dS_h}{dt} = \Pi_h - \tau\alpha_v I_m \frac{S_h}{N_h} - \nu_h S_h, \\
\frac{dE_h}{dt} = \tau\alpha_v I_m \frac{S_h}{N_h} - (\beta_h + \nu_h)E_h, \\
\frac{dI_h}{dt} = \beta_h E_h - (\psi_h + \omega_h + \nu_h)I_h, \\
\frac{dP_h}{dt} = \psi_h I_h - (\tau_h + \sigma_h + \nu_h)P_h, \\
\frac{dR_h}{dt} = \omega_h I_h + \sigma_h P_h - \nu_h R_h,
\end{cases}
\tag{1}
$$

subject to the initial conditions

$$
\begin{cases}
S_m(0) = S_{m0} \geq 0, \\
E_m(0) = E_{m0} \geq 0, \\
I_m(0) = I_{m0} \geq 0, \\
S_h(0) = S_{h0} \geq 0, \\
E_h(0) = E_{h0} \geq 0, \\
I_h(0) = I_{h0} \geq 0, \\
P_h(0) = P_{h0} \geq 0, \\
R_h(0) = R_{h0} \geq 0.
\end{cases}
\tag{2}
$$

In model (1) above, the populations of vectors and humans are denoted by Π_m and Π_h. The natural death rate of mosquitoes and humans are given by μ_m and ν_h. The parameter τ measures the biting rate per mosquito per person. The transmission probability from an infected human to susceptible mosquitos, and the transmission probability from infected mosquitos to susceptible humans are shown respectively by α_h and α_v. The intrinsic incubation period of mosquitos is given by β_m, and that of the humans is shown by β_h. The human infected cases that are notified and get hospitalization increase at a rate ψ_h. The infected human recover naturally at a rate ω_h while the hospitalized individuals get recovery at a rate σ_h. The hospitalized individuals die due to disease at a rate τ_h. The complete details of the parameter descriptions of the dengue model (1) are presented briefly in Table 1.

1.1 Positivity and boundedness of the solution

In order to show that the dengue model given by (1) is epidemiologically meaningful, we have to prove that the associated state variables of the model stay nonnegative. It can also

Table 1 Biological meaning of parameters for dengue model (1)

Parameter	Description
Π_m	Recruitment rate of mosquito
τ	Average biting rate per mosquito per person
α_h	Transmission probability from infected human to susceptible mosquito
μ_m	Natural death rate of mosquito
β_m	Extrinsic incubation of mosquito
Π_h	Recruitment rate of human
α_v	Transmission probability from infected mosquito to susceptible human
β_h	Extrinsic incubation of human
ψ_h	Rate of hospitalization and/or notification of infected human
ω_h	Natural recovery rate of infected human
σ_h	Recovery rate of hospitalized and/or notified infected human
τ_h	Disease-related death rate of human
ν_h	Natural death rate of human

be explained that the solution of the dengue model with nonnegative initial conditions will remain nonnegative for every time greater than zero. We have the following lemma.

Lemma 1 *Consider the initial data $G(0) \geq 0$, where*

$$G(t) = \big(S_m(t), E_m(t), I_m(t), S_h(t), E_h(t), I_h(t), P_h(t), R_h(t)\big).$$

Then the solutions of the model given by (1) *are nonnegative for every time $t > 0$. Further,*

$$\lim_{t\to\infty} N_m(t) \leq \frac{\Pi_m}{\mu_m} \quad and \quad \lim_{t\to\infty} N_h(t) \leq \frac{\Pi_h}{\nu_h},$$

with $N_m(t) = S_m(t) + E_m(t) + I_m(t)$ and $N_h(t) = S_h(t) + E_h(t) + I_h(t) + P_h(t) + R_h(t)$.

Proof Consider $t_1 = \sup\{t > 0 : G(t) > 0\}$. So, $t_1 > 0$. The first equation of the dengue model (1) leads to the following:

$$\frac{dS_m}{dt} = \Pi_m - \tau\alpha_h S_m \frac{I_h}{N_h} - \mu_m S_m, \tag{3}$$

with $\lambda_v = \tau\alpha_h \frac{I_h}{N_h}$, then the above equation (3) becomes

$$\frac{dS_m}{dt} = \Pi_m - \lambda_v S_m - \mu_m S_m. \tag{4}$$

The equation given by (4) can be expressed further as follows:

$$\frac{d}{dt}\left\{ S_m(t) \exp\left(\mu_m t + \int_0^t \lambda_v(\phi)\,d\phi \right) \right\} = \lambda_v \exp\left(\mu_m t + \int_0^t \lambda_v(\phi)\,d\phi \right). \tag{5}$$

Hence,

$$S_m(t_1) \exp\left(\mu_m t_1 + \int_0^{t_1} \lambda_v(\phi)\,d\phi \right) - S_m(0)$$

$$= \lambda_v \exp\left(\mu_m y + \int_0^y \lambda_v(\psi)\,d\psi \right) dy, \tag{6}$$

so that

$$S_m(t_1) = S_m(0) \exp\left\{ -\left(\mu_m t_1 + \int_0^{t_1} \lambda_v(\phi) \, d\phi \right) \right\} + \exp\left\{ -\left(\mu_m t_1 + \int_0^{t_1} \lambda_v(\phi) \, d\phi \right) \right\}$$
$$\times \int_0^{t_1} \lambda_v \exp\left(\mu_m y + \int_0^y \lambda_v(\psi) \, d\psi \right) dy > 0. \tag{7}$$

Similar steps can be followed for the rest of the equations of the dengue model (1) that is, $G(t) > 0$ for every $t > 0$. In order to show the second part of the result, note that $0 < S_m(0) \le N_m(t)$, $0 < E_m(0) \le N_m(t)$, $0 < I_m(0) \le N_m(t)$, $0 < S_h(0) \le N_h(t)$, $0 \le E_h(0) \le N_h(t)$, $0 \le I_h(0) \le N_h(t)$, $0 \le P_h(0) \le N_h(t)$, $0 \le R_h(0) \le N_h(t)$. Now, summing the mosquito and human compartments leads to the following:

$$\frac{dN_m}{dt} = \Pi_m - \mu_m N_v$$

and

$$\frac{dN_h}{dt} = \Pi_h - v_h N_h.$$

Thus,

$$\lim_{t \to \infty} N_m(t) \le \frac{\Pi_m}{\mu_m} \quad \text{and} \quad \lim_{t \to \infty} N_h(t) \le \frac{\Pi_h}{v_h},$$

which is the required claim. $\qquad \square$

Next, we show the invariant regions for the given dengue model (1). Consider the feasible region $\Theta = \Theta_v \times \Theta_h \subset \mathbb{R}_+^3 \times \mathbb{R}_+^5$, with

$$\Theta_v = \left\{ \left(S_m(t), E_m(t), I_m(t) \right) \in \mathbb{R}_+^3 : N_m(t) \le \frac{\Pi_m}{\mu_m} \right\}$$

and

$$\Theta_h = \left\{ \left(S_h(t), E_h(t), I_h(t), P_h(t), R_h(t) \right) \in \mathbb{R}_+^5 : N_h(t) \le \frac{\Pi_h}{v_h} \right\}.$$

We have the following results for this feasible region.

Lemma 2 *The region given by $\Theta = \Theta_v \times \Theta_h \subset \mathbb{R}_+^3 \times \mathbb{R}_+^5$ is positively invariant for the dengue model (1) with the nonnegative initial conditions in (2).*

Proof The summation of the mosquito and human populations of the dengue model (1) leads to

$$\frac{dN_m}{dt} = \Pi_m - \mu_m N_m$$

and

$$\frac{dN_h}{dt} = \Pi_h - v_h N_h.$$

Hence, $\frac{dN_m(t)}{dt} \leq 0$, if $N_m(0) \geq \frac{\Pi_m}{\mu_m}$ and $\frac{dN_h(t)}{dt} \leq 0$, if $N_h(0) \geq \frac{\Pi}{\nu_h}$. So, $N_m(t) \leq N_m(0)e^{-\mu_m t} + \frac{\Pi_m}{\mu_m}(1 - e^{-\mu_m t})$ and $N_h(t) \leq N_h(0)e^{-\nu_h t} + \frac{\Pi_h}{\nu_h}(1 - e^{-\nu_h t})$. Thus, the region given by Θ is positively invariant. Also, if $N_m(0) > \frac{\Pi_m}{\mu_m}$ and $N_h(0) > \frac{\Pi_h}{\nu_h}$, then either the solutions enter Θ in finite time, or $N_m(t)$ tends to $\frac{\Pi_m}{\mu_m}$ and $N_h(t)$ tends to $\frac{\Pi_h}{\nu_h}$ asymptotically. So, the regions given by Θ attract all the solutions in $\mathbb{R}_+^8$.   □

2 Preliminaries

This section explores the related results regarding fractal-fractional calculus, the model construction in fractal-fractional Caputo–Fabrizio operator and model stability results. We consider first the basic of fractal-fractional (FF) calculus by following the literature in [34, 35].

Definition 1 If a function $g(t)$ is continuous and fractally differentiable over the given interval (a_1, a_2) with order θ_2, then the definition of the FF derivative of $g(t)$ with order θ_1 in Riemann–Liouville sense with exponentially decaying kernel is given by:

$$^{FFE}D_{0,t}^{\theta_1,\theta_2}\big(g(t)\big) = \frac{M(\theta_1)}{1-\theta_1} \frac{d}{dt^{\theta_2}} \int_0^t \exp\left(-\frac{\theta_1}{1-\theta_1}(t-s)\right)g(s)\,ds, \tag{8}$$

with $\theta_1 > 0$, $\theta_2 \leq m \in \mathbb{N}$ and $M(0) = M(1) = 1$.

Definition 2 For a function $g(t)$ which is continuous and fractally differentiable over the given interval (a_1, a_2), the definition of FF integral of $g(t)$ with order θ_1 and exponentially decaying kernel is given by:

$$^{FFE}J_{0,t}^{\theta_1}\big(g(t)\big) = \frac{\theta_1\theta_2}{M(\theta_1)} \int_0^t s^{\theta_1-1}g(s)\,ds + \frac{\theta_2(1-\theta_1)t^{\theta_2-1}g(t)}{M(\theta_1)}. \tag{9}$$

2.1 Fractal-fractional dengue model

In the present subsection, we apply the fractal-fractional operator in the sense of Caputo–Fabrizio to the dengue model (1) described above. And we have

$$\begin{cases} {}^{FF}D_{0,t}^{\theta_1,\theta_2}(S_m(t)) = \Pi_m - \tau\alpha_h S_m \frac{I_h}{N_h} - \mu_m S_m, & S_m(0) = S_{m0} \geq 0, \\ {}^{FF}D_{0,t}^{\theta_1,\theta_2}(E_m(t)) = \tau\alpha_h S_m \frac{I_h}{N_h} - (\beta_m + \mu_m)E_m, & E_m(0) = E_{m0} \geq 0, \\ {}^{FF}D_{0,t}^{\theta_1,\theta_2}(I_m(t)) = \beta_m E_m - \mu_m I_m, & I_m(0) = I_{m0} \geq 0, \\ {}^{FF}D_{0,t}^{\theta_1,\theta_2}(S_h(t)) = \Pi_h - \tau\alpha_v I_m \frac{S_h}{N_h} - \nu_h S_h, & S_h(0) = S_{h0} \geq 0, \\ {}^{FF}D_{0,t}^{\theta_1,\theta_2}(E_h(t)) = \tau\alpha_v I_m \frac{S_h}{N_h} - (\beta_h + \nu_h)E_h, & E_h(0) = E_{h0} \geq 0, \\ {}^{FF}D_{0,t}^{\theta_1,\theta_2}(I_h(t)) = \beta_h E_h - (\psi_h + \omega_h + \nu_h)I_h, & I_h(0) = I_{h0} \geq 0, \\ {}^{FF}D_{0,t}^{\theta_1,\theta_2}(P_h(t)) = \psi_h I_h - (\tau_h + \sigma_h + \nu_h)P_h, & P_h(0) = P_{h0} \geq 0, \\ {}^{FF}D_{0,t}^{\theta_1,\theta_2}(R_h(t)) = \omega_h I_h + \sigma_h P_h - \nu_h R_h, & R_h(0) = R_{h0} \geq 0, \end{cases} \tag{10}$$

where θ_1 and θ_2 respectively represent the fractional and fractal order. The model (10) described above for the dengue infection using the fractal-fractional Caputo–Fabrizio operator is used further to obtain its numerical solution by using a novel numerical procedure in the following section.

2.2 Stability analysis of the disease-free case

This section explores the stability results for the dengue model given at the disease-free equilibrium (DFE) at E_0. We set the right-hand side of the dengue model (10) equal to zero and obtain the following expressions:

$$E_0 = \left(S_m^0, 0, 0, S_h^0, 0, 0, 0, 0\right) = \left(\frac{\Pi_m}{\mu_m}, 0, 0, \frac{\Pi_h}{\nu_h}, 0, 0, 0, 0\right).$$

The stability of DFE at E_0 can be analyzed by using the next generation matrix of the dengue model (10). Considering the infected compartments in the dengue model (10), namely E_m, I_m, E_h, I_h, P_h, and following the instructions given in [36], the matrices F and V are obtained as follows:

$$F = \begin{pmatrix} 0 & 0 & 0 & \frac{\tau \alpha_h \nu_h \Pi_m}{\Pi_h \mu_m} & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & \tau \alpha_\nu & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}, \quad \text{and} \quad V = \begin{pmatrix} \pi_1 & 0 & 0 & 0 & 0 \\ -\beta_m & \mu_m & 0 & 0 & 0 \\ 0 & 0 & \pi_2 & 0 & 0 \\ 0 & 0 & -\beta_h & \pi_3 & 0 \\ 0 & 0 & 0 & -\psi_h & \pi_4 \end{pmatrix}.$$

where $\pi_1 = (\beta_m + \mu_m)$, $\pi_2 = (\beta_h + \nu_h)$, $\pi_3 = (\psi_h + \omega_h + \nu_h)$, and $\pi_4 = (\tau_h + \sigma_h + \nu_h)$. The required basic reproduction number of the given model is obtained through the spectral radius of the matrix $\mathcal{R}_0 = \rho(FV^{-1})$, which is given by the following expression:

$$\mathcal{R}_0 = \sqrt{\frac{\tau^2 \alpha_h \beta_h \mu_h \alpha_\nu \beta_m \Pi_m}{\pi_1 \pi_2 \pi_3 \Pi_h \mu_m^2}}.$$

In the following theorem, we show that the dengue model given by (10) is locally asymptotically stable at E_0. We give the following result:

Theorem 1 *The dengue model given by* (10) *at* E_0 *is locally asymptotically stable whenever* $\mathcal{R}_0 < 1$.

Proof In order to prove the given theorem, we need to obtain the Jacobian matrix by evaluating the model (10) at E_0, and we have

$$J(E_0) = \begin{pmatrix} -\mu_m & 0 & 0 & 0 & 0 & -\frac{\tau \alpha_h \Pi_\nu \nu_h}{\Pi_h \mu_m} & 0 & 0 \\ 0 & -\pi_1 & 0 & 0 & 0 & \frac{\tau \alpha_h \Pi_\nu \nu_h}{\Pi_h \mu_m} & 0 & 0 \\ 0 & \beta_\nu & -\mu_m & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -\tau \alpha_\nu & -\nu_h & 0 & 0 & 0 & 0 \\ 0 & 0 & \tau \alpha_\nu & 0 & -\pi_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \gamma_h & -\pi_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \psi_h & -\pi_4 & 0 \\ 0 & 0 & 0 & 0 & 0 & \omega_h & \sigma_h & -\nu_h \end{pmatrix}.$$

It can be seen from the above matrix $J(E_0)$ that the eigenvalues $-\mu_m$, $-\nu_h$, $-\nu_h$, and $-\pi_4$ are obviously negative, while the remaining four eigenvalues with negative real parts can be obtained through the following equation:

$$\lambda^4 + \varpi_1 \lambda^3 + \varpi_2 \lambda^2 + \varpi_3 \lambda + \varpi_4 = 0,$$

where

$$\varpi_1 = \pi_1 + \pi_2 + \pi_3 + \mu_m,$$

$$\varpi_2 = \pi_3 \mu_m + \pi_2(\pi_3 + \mu_m) + \pi_1(\pi_2 + \pi_3 + \mu_m),$$

$$\varpi_3 = \pi_2 \pi_3 \mu_m + \pi_1\big(\pi_3 \mu_m + \pi_2(\pi_3 + \mu_m)\big),$$

$$\varpi_4 = \pi_1 \pi_2 \pi_3 \mu_m\big(1 - \mathcal{R}_0^2\big).$$

The coefficients given by ϖ_i for $i = 1, 2, \ldots, 4$ are obviously positive for ϖ_i, $i = 1, 2, 3$ while ϖ_4 can be positive or negative based on the value of $\mathcal{R}_0$. For the DFE case, the value of the basic reproduction number should be less than 1, so the last coefficient is positive when $\mathcal{R}_0 < 1$. So, for all the coefficients ϖ_i for $i = 1, 2, \ldots, 4$ to be positive, they should satisfy the Rough–Hurtwiz criterion, which is easy to be satisfied, for the conditions supplied, $\varpi_1 \varpi_2 \varpi_3 > \varpi_3^2 + \varpi_1^2 \varpi_4$, where $\varpi_i > 0$ for all $i = 1, 2, \ldots, 4$. Thus, the condition of Rough–Hurtwiz criterion ensures the local asymptotic stability of the dengue model given by (10) at E_0. □

2.3 Endemic equilibria and their stability

This subsection presents the endemic equilibria of the dengue model (10) denoted by $E_1 = (S_m^*, E_m^*, I_m^*, S_h^*, E_h^*, I_h^*, P_h^*, R_h^*)$ and given by

$$\begin{cases} S_m^* = \frac{\Pi_m}{\lambda_v^* + \mu_m}, \\ E_m^* = \frac{\lambda_v^* \Pi_v}{\pi_1(\lambda_v + \mu_m)}, \\ I_m^* = \frac{\beta_m \lambda_v^* \Pi_m}{\pi_1 \mu_m(\lambda_v^* + \mu_m)}, \\ S_h^* = \frac{\Pi_h}{\lambda_h^* + \nu_h}, \\ E_h^* = \frac{\Pi_h \lambda_h^*}{\pi_2(\lambda_h^* + \nu_h)}, \\ I_h^* = \frac{\Pi_h \beta_h \lambda_h^*}{\pi_2 \pi_3(\lambda_h^* + \nu_h)}, \\ P_h^* = \frac{\psi_h \Pi_h \beta_h \lambda_h^*}{\pi_2 \pi_3 \pi_4(\lambda_h^* + \nu_h)}, \\ R_h^* = \frac{\Pi_h \beta_h \lambda_h^*(\pi_4 \omega_h + \psi_h \sigma_h)}{\pi_2 \pi_3 \pi_4 \nu_h(\lambda_h^* + \nu_h)}, \end{cases} \tag{11}$$

where

$$\lambda_v^* = \frac{\tau \alpha_h I_h^*}{N_h^*}, \qquad \lambda_h^* = \frac{\tau I_m^* \alpha_v}{N_h^*}. \tag{12}$$

Using the expressions given by (11) in (12), this leads to the following:

$$g_1 \lambda_h^{*2} + g_2 \lambda_h^* + g_3 = 0,$$

where

$$g_1 = \pi_1 \Pi_h \mu_m\big(\pi_4\big(\nu_h(\beta_h + \pi_3) + \omega_h \beta_h\big) + \psi_h \beta_h(\nu_h + \sigma_h)\big)$$

$$\times \big(\pi_4\big(\tau \alpha_h \beta_h \nu_h + \mu_m\big(\nu_h(\beta_h + \pi_3) + \omega_h \beta_h\big)\big) + \psi_h \beta_h \mu_m(\nu_h + \sigma_h)\big),$$

$$g_2 = \pi_2 \pi_3 \pi_4 \nu_h\big(\pi_4\big(2\pi_1 \Pi_h \mu_m^2\big(\nu_h(\beta_h + \pi_3) + \omega_h \beta_h\big) + \tau \alpha_h \beta_h \nu_h(\pi_1 \Pi_h \mu_m - \tau \alpha_v \beta_m \Pi_m)\big)\big)$$

$$+ 2\pi_2\pi_3\pi_4\nu_h\psi_h\pi_1\Pi_h\beta_h\mu_m^2(\nu_h + \sigma_h),$$

$$g_3 = \pi_1\pi_2^2\pi_3^2\pi_4^2\Pi_h\nu_h^2\mu_m^2\left(1 - \mathcal{R}_0^2\right).$$

Here, $g_1 > 0$, g_3 depends on the sign of $\mathcal{R}_0$, and is positive when $\mathcal{R}_0 < 1$ and negative when $\mathcal{R}_0 > 1$. We establish the following result:

Theorem 2 *The dengue model given by* (10) *has the following properties*:

(i) *If $g_3 < 0$ and $\mathcal{R}_0 > 1$, then there exists a unique endemic equilibrium*;

(ii) *If $g_2 < 0$ and $g_3 = 0$, then we have a unique endemic equilibrium*;

(iii) *If $g_3 > 0$, $g_2 < 0$, and their discriminant is positive, then two endemic equilibria exist*; *and*

(iv) *there are no equilibria otherwise*.

It can be seen from the first point (i) of Theorem 2 that for the case of $\mathcal{R}_0 > 1$, we clearly have a unique positive endemic equilibrium. Theorem 2(iii) shows the possible existence of the backward bifurcation when $\mathcal{R}_0 < 1$.

3 Novel solution procedure for fractal-fractional model

For the numerical solution of the fractal-fractional dengue model (10), we present a novel procedure here that is based on the Adams–Bashforth technique. In order to have numerical scheme for the fractal-fractional model (10), we first express the model (10) in the following form:

$$
\begin{cases}
{}^{CF}D_{0,t}^{\theta_1}(S_m(t)) = \theta_2 t^{\theta_2-1}f_1(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t), \\
{}^{CF}D_{0,t}^{\theta_1}(E_m(t)) = \theta_2 t^{\theta_2-1}f_2(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t), \\
{}^{CF}D_{0,t}^{\theta_1}(I_m(t)) = \theta_2 t^{\theta_2-1}f_3(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t), \\
{}^{CF}D_{0,t}^{\theta_1}(S_h(t)) = \theta_2 t^{\theta_2-1}f_4(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t), \\
{}^{CF}D_{0,t}^{\theta_1}(E_h(t)) = \theta_2 t^{\theta_2-1}f_5(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t), \\
{}^{CF}D_{0,t}^{\theta_1}(I_h(t)) = \theta_2 t^{\theta_2-1}f_6(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t), \\
{}^{CF}D_{0,t}^{\theta_1}(R_h(t)) = \theta_2 t^{\theta_2-1}f_7(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t), \\
{}^{CF}D_{0,t}^{\theta_1}(I_m(t)) = \theta_2 t^{\theta_2-1}f_8(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t).
\end{cases}
\tag{13}
$$

We apply the CF integral on equation (13), which leads to the following:

$$S_m(t) = S_m(0) + \frac{\theta_2}{\Gamma(\theta_1)}\int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1}f_1(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\,d\lambda,$$

$$E_m(t) = E_m(0) + \frac{\theta_2}{\Gamma(\theta_1)}\int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1}f_2(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\,d\lambda,$$

$$I_m(t) = I_m(0) + \frac{\theta_2}{\Gamma(\theta_1)}\int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1}f_3(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\,d\lambda,$$

$$S_h(t) = S_h(0) + \frac{\theta_2}{\Gamma(\theta_1)}\int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1}f_4(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\,d\lambda,$$

$$E_h(t) = E_h(0) + \frac{\theta_2}{\Gamma(\theta_1)}\int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1}f_5(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\,d\lambda, \tag{14}$$

$$I_h(t) = I_h(0) + \frac{\theta_2}{\Gamma(\theta_1)} \int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1} f_6(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$P_h(t) = P_h(0) + \frac{\theta_2}{\Gamma(\theta_1)} \int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1} f_7(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$R_h(t) = R_h(0) + \frac{\theta_2}{\Gamma(\theta_1)} \int_0^t \lambda^{\theta_2-1}(t-\lambda)^{\theta_1-1}$$
$$\times f_8(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

where

$$f_1(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \Pi_m - \tau\alpha_h S_m \frac{I_h}{N_h} - \mu_m S_m,$$

$$f_2(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \tau\alpha_h S_m \frac{I_h}{N_h} - (\beta_m + \mu_m)E_m,$$

$$f_3(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \beta_m E_m - \mu_m I_m,$$

$$f_4(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \Pi_h - \tau\alpha_v I_m \frac{S_h}{N_h} - \nu_h S_h,$$

$$f_5(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \tau\alpha_v I_m \frac{S_h}{N_h} - (\beta_h + \nu_h)E_h,$$

$$f_6(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \beta_h E_h - (\psi_h + \omega_h + \nu_h)I_h,$$

$$f_7(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \psi_h I_h - (\tau_h + \sigma_h + \nu_h)P_h,$$

$$f_8(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) = \omega_h I_h + \sigma_h P_h - \nu_h R_h.$$

We are presenting now a novel approach for the above model and use the approach at t_{n+1}. We have the following:

$$S_m(t) = S_m^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_1(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$
$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_1(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$E_m(t) = E_m^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_2(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$
$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_2(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$I_m(t) = I_m^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_3(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$
$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_3(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$S_h(t) = S_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_4(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$
$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_4(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda, \tag{15}$$

$$E_h(t) = E_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_5(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_5(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$I_h(t) = I_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_6(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_6(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$P_h(t) = P_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_7(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_7(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$R_h(t) = R_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_8(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, t)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^t \lambda^{\theta_2-1} f_8(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda.$$

At t_{n+1}, we have the following:

$$S_m^{n+1} = S_m^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_1\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_1(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$E_m^{n+1} = E_m^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_2\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_2(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$I_m^{n+1} = I_m^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_3\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_3(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$S_h^{n+1} = S_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_4\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_4(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$E_h^{n+1} = E_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_5\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_5(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$I_h^{n+1} = I_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_6\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_6(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$P_h^{n+1} = P_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_7\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$(16)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_7(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$R_h^{n+1} = R_h^0 + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_8\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_0^{t_{n+1}} \lambda^{\theta_2-1} f_8(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda.$$

The following is obtained by taking the difference between consecutive terms:

$$S_m^{n+1} = S_m^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_1\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_1\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_1(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$E_m^{n+1} = E_m^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_2\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_2\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_2(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$I_m^{n+1} = I_m^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_3\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_3\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_3(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$S_h^{n+1} = S_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_4\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_4\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_4(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$E_h^{n+1} = E_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_5\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_5\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_5(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda)\, d\lambda,$$

$$I_h^{n+1} = I_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_6\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_6\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$(17)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_6(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$P_h^{n+1} = P_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_7\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_7\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_7(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda,$$

$$R_h^{n+1} = R_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_8\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_8\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \int_{t_n}^{t_{n+1}} \lambda^{\theta_2-1} f_8(S_m, E_m, I_m, S_h, E_h, I_h, P_h, R_h, \lambda) \, d\lambda.$$

It follows from the Lagrange polynomial interpolation and integrating the following expressions that

$$S_m^{n+1} = S_m^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_1\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_1\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \left[\frac{3h}{2} t_n^{\theta_2-1} f_1\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right) \right.$$

$$\left. - \frac{h}{2} t_{n-1}^{\theta_2-1} f_1\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right) \right],$$

$$E_m^{n+1} = E_m^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_2\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_2\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \left[\frac{3h}{2} t_n^{\theta_2-1} f_2\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right) \right.$$

$$\left. - \frac{h}{2} t_{n-1}^{\theta_2-1} f_2\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right) \right],$$

$$I_m^{n+1} = I_m^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_3\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_3\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right)$$

$$+ \frac{\theta_1 \theta_2}{M(\theta_1)} \left[\frac{3h}{2} t_n^{\theta_2-1} f_3\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right) \right.$$

$$\left. - \frac{h}{2} t_{n-1}^{\theta_2-1} f_3\left(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\right) \right],$$

$$S_h^{n+1} = S_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_4\left(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\right)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_4\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \bigg[\frac{3h}{2} t_n^{\theta_2-1} f_4\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{h}{2} t_{n-1}^{\theta_2-1} f_4\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big) \bigg],$$

$$\tag{18}$$

$$E_h^{n+1} = E_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_5\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_5\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \bigg[\frac{3h}{2} t_n^{\theta_2-1} f_5\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{h}{2} t_{n-1}^{\theta_2-1} f_5\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big) \bigg],$$

$$I_h^{n+1} = I_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_6\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_6\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \bigg[\frac{3h}{2} t_n^{\theta_2-1} f_6\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{h}{2} t_{n-1}^{\theta_2-1} f_6\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big) \bigg],$$

$$P_h^{n+1} = P_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_7\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_7\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \bigg[\frac{3h}{2} t_n^{\theta_2-1} f_7\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{h}{2} t_{n-1}^{\theta_2-1} f_7\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big) \bigg],$$

$$R_h^{n+1} = R_h^n + \frac{\theta_2 t^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_8\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{\theta_2 t_{n-1}^{\theta_2-1}(1-\theta_1)}{M(\theta_1)} f_8\big(S_v^{n-1}, E_v^{n-1}, I_v^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big)$$

$$+ \frac{\theta_1\theta_2}{M(\theta_1)} \bigg[\frac{3h}{2} t_n^{\theta_2-1} f_8\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \frac{h}{2} t_{n-1}^{\theta_2-1} f_8\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big) \bigg].$$

Further simplifications of (18) lead to the following:

$$S_m^{n+1} = S_m^n + \theta_2 t_n^{\theta_2-1} \bigg(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)} \bigg) f_1\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \theta_2 t_{n-1}^{\theta_2-1} \bigg(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)} \bigg)$$

$$\times f_1\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big),$$

$$E_m^{n+1} = E_m^n + \theta_2 t_n^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)}\right) f_2\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \theta_2 t_{n-1}^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)}\right)$$

$$\times f_2\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big),$$

$$I_m^{n+1} = I_m^n + \theta_2 t_n^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)}\right) f_3\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \theta_2 t_{n-1}^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)}\right)$$

$$\times f_3\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big),$$

$$S_h^{n+1} = S_h^n + \theta_2 t_n^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)}\right) f_4\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \theta_2 t_{n-1}^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)}\right)$$

$$\times f_4\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big),$$

$$E_h^{n+1} = E_h^n + \theta_2 t_n^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)}\right) f_5\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big) \tag{19}$$

$$- \theta_2 t_{n-1}^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)}\right)$$

$$\times f_5\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big),$$

$$I_h^{n+1} = I_h^n + \theta_2 t_n^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)}\right) f_6\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \theta_2 t_{n-1}^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)}\right)$$

$$\times f_6\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big),$$

$$P_h^{n+1} = P_h^n + \theta_2 t_n^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)}\right) f_7\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \theta_2 t_{n-1}^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)}\right)$$

$$\times f_7\big(S_v^{n-1}, E_v^{n-1}, I_v^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big),$$

$$R_h^{n+1} = R_h^n + \theta_2 t_n^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{3\theta_1 h}{2M(\theta_1)}\right) f_8\big(S_m^n, E_m^n, I_m^n, S_h^n, E_h^n, I_h^n, P_h^n, R_h^n, t_n\big)$$

$$- \theta_2 t_{n-1}^{\theta_2-1}\left(\frac{1-\theta_1}{M(\theta_1)} + \frac{\theta_1 h}{2M(\theta_1)}\right)$$

$$\times f_8\big(S_m^{n-1}, E_m^{n-1}, I_m^{n-1}, S_h^{n-1}, E_h^{n-1}, I_h^{n-1}, P_h^{n-1}, R_h^{n-1}, t_{n-1}\big).$$

4 Numerical results

In order to graphically illustrate the numerical results for the fractal-fractional model in the sense of Caputo–Fabrizio operator, we first estimate the model parameters and the initial conditions for the model. We present the following subsection for parameter estimates.

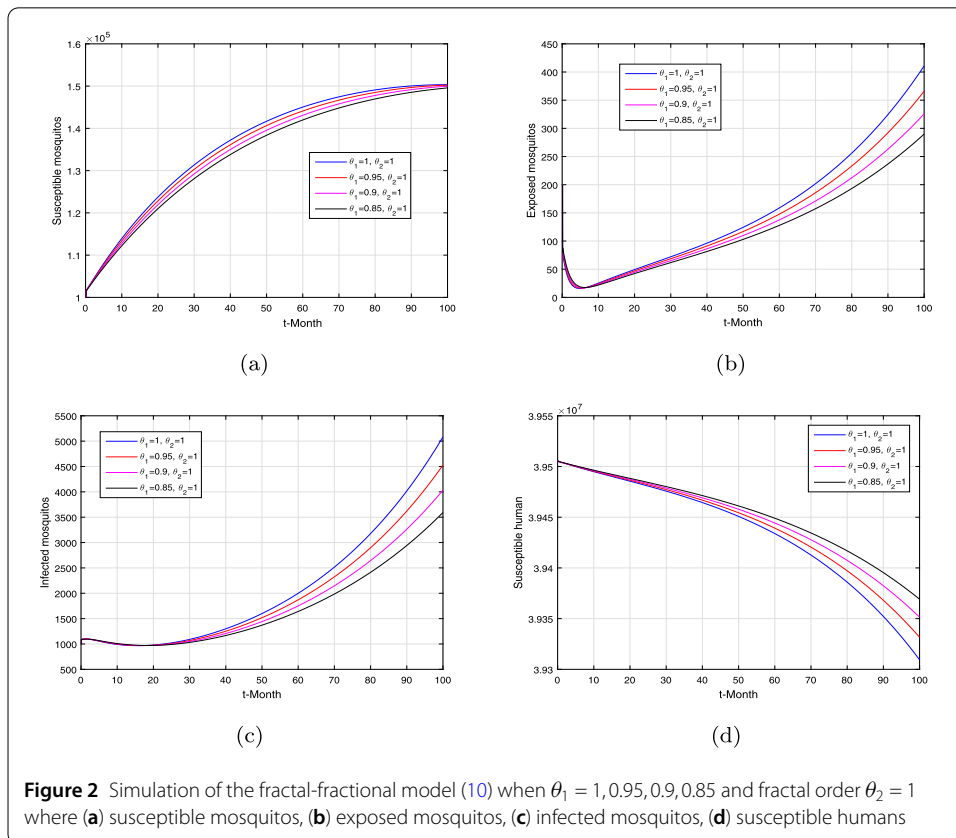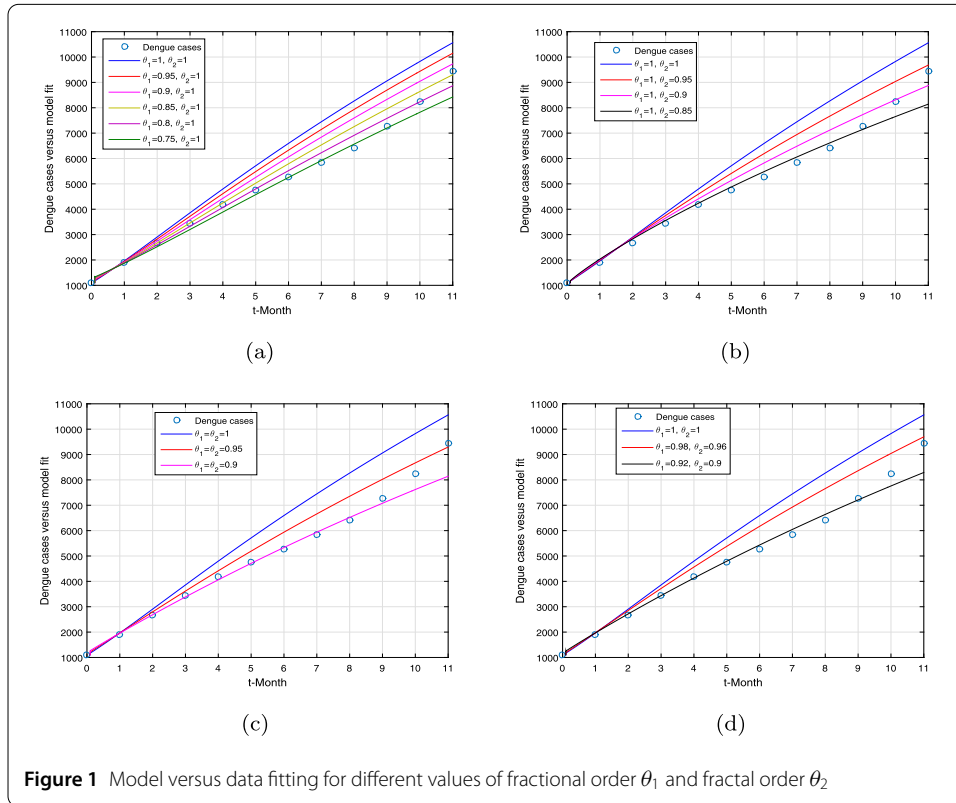### 4.1 Parameter estimation of dengue model

In the present section, we estimate the parameters of the model (10) for the case when $\theta_1 = \theta_2 = 1$ of dengue fever cases in East Java, Indonesia, for the year 2018. Based on data obtained from the East Java provincial health office, it was reported that the incidence rate of Dengue Hemorrhagic Fever (DHF) in East Java in 2016 was 64.8 per 100,000 people, an increase compared to 2015 which was 54.18 per 100,000 people, while the incidence rate of DHF in 2017 was 20 per 100,000 people [37]. Although in 2017 the number of cases of DHF decreased compared to the previous year, the awareness of the surge in cases in the next year needs to be improved. In 2017, the total number of DHF in East Java reached 7854 people, while in 2018 it reached 9452 people [38]. From this, it appears that there is an increase in the number of DHF cases in 2018 compared to 2017. Hence, the cumulative monthly reported DHF cases from January to December 2018 are used to parameterize the model (10). In order to get a good fit to the real data, we estimate the parameters using the least square curve fitting technique except for the recruitment rate of human Π_h and the natural death rate of human ν_h. The parameter ν_h is calculated as the inverse of the average lifespan in East Java so that $\nu_h = 1/70.97$ per year, where 70.97 year is the average lifespan population in East Java province [39]. The parameter Π_h is computed as follows. Since the total population of East Java province as of 2018 was 39,507,370, we have that $\Pi_h/\nu_h = 39{,}507{,}370$ is the maximum human population without the disease, therefore $\Pi_h = 556{,}677.0466$ per year. The other parameters are obtained using the least-squares curve fitting method. The fitted and estimated parameter values of the model (10) are listed in Table 2. The result of fitting the model (10) to the actual data of dengue incidence is displayed in Fig. 1. Using the parameter values in Table 2, the basic reproduction number in East Java is $\mathcal{R}_0 \approx 2.2020$. The parameters listed in Table 2 are used further to obtain the graphical results for the fractal-fractional model (10).
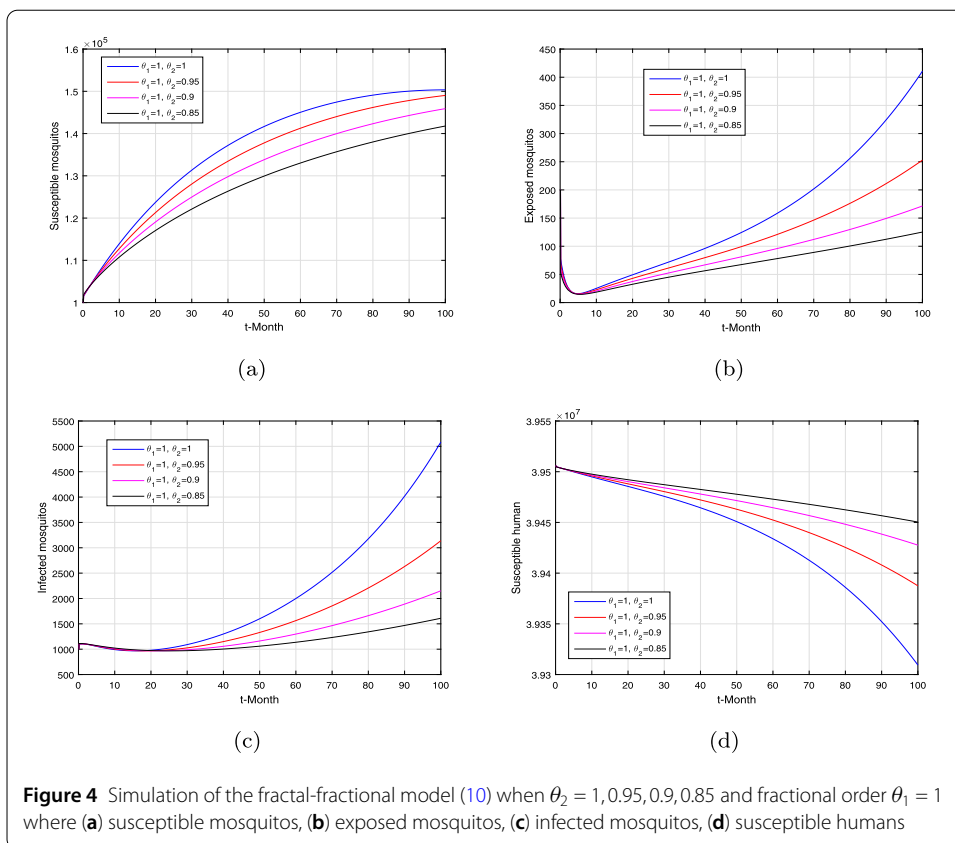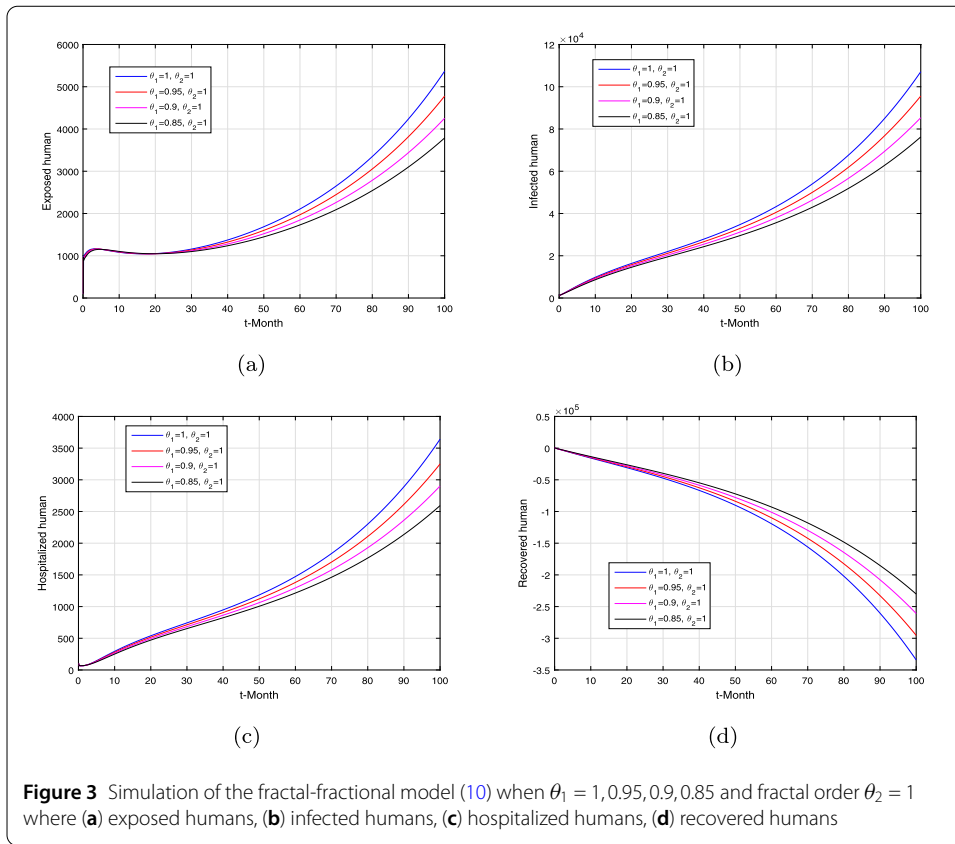
4.2 Simulation results

In this subsection, we apply the novel approach considered above for the fractal-fractional model of dengue in the sense of Caputo–Fabrizio operator to illustrate its numerical results graphically. Throughout these simulation results, the subgraphs in Figs. 2, 4, 6, 8, which represent respectively the dynamics of susceptible, exposed, and infected mosquitos and the susceptible humans, while the subgraphs in Figs. 3, 5, 7, 9 show respectively the components of the exposed, infected, hospitalized, and recovered humans.

Table 2 Fitted and estimated values for the parameters of the model (10)

Parameter	Units	Baselines value	References
Π_m	month^{-1}	3839.9	Fitted
τ	month^{-1}	0.9961	Fitted
α_h	–	0.6341	Fitted
μ_m	month^{-1}	1/42	[12, 40]
β_m	month^{-1}	0.586	Fitted
Π_h	month^{-1}	46390	Estimated
α_v	–	0.989	Fitted
β_h	month^{-1}	0.8850	Fitted
ψ_h	month^{-1}	0.0184	Fitted
ω_h	month^{-1}	0.00154	Fitted
σ_h	month^{-1}	0.440	Fitted
τ_h	month^{-1}	0.0769	Fitted
ν_h	month^{-1}	$\frac{1}{(70.97 \times 12)}$	Estimated

Figure 1 Model versus data fitting for different values of fractional order θ_1 and fractal order θ_2
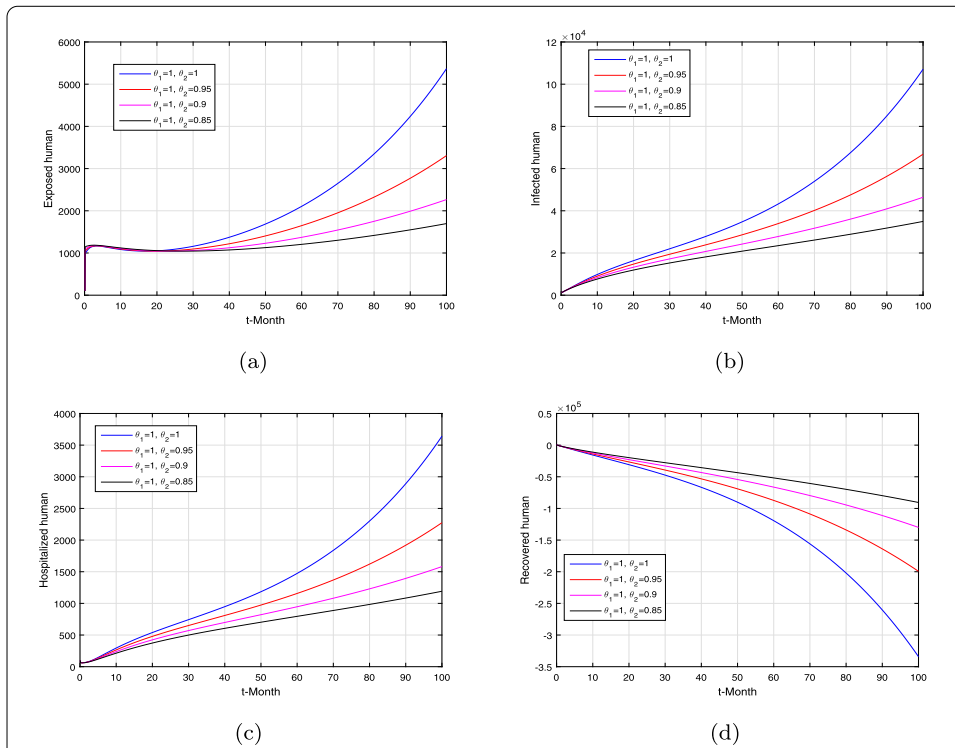

Figure 2 Simulation of the fractal-fractional model (10) when $\theta_1 = 1, 0.95, 0.9, 0.85$ and fractal order $\theta_2 = 1$ where (**a**) susceptible mosquitos, (**b**) exposed mosquitos, (**c**) infected mosquitos, (**d**) susceptible humans

Figure 3 Simulation of the fractal-fractional model (10) when $\theta_1 = 1, 0.95, 0.9, 0.85$ and fractal order $\theta_2 = 1$ where (**a**) exposed humans, (**b**) infected humans, (**c**) hospitalized humans, (**d**) recovered humans

Figure 4 Simulation of the fractal-fractional model (10) when $\theta_2 = 1, 0.95, 0.9, 0.85$ and fractional order $\theta_1 = 1$ where (**a**) susceptible mosquitos, (**b**) exposed mosquitos, (**c**) infected mosquitos, (**d**) susceptible humans

Figure 5 Simulation of the fractal-fractional model (10) when $\theta_2 = 1, 0.95, 0.9, 0.85$ and fractional order $\theta_1 = 1$ where (**a**) exposed humans, (**b**) infected humans, (**c**) hospitalized humans, (**d**) recovered humans
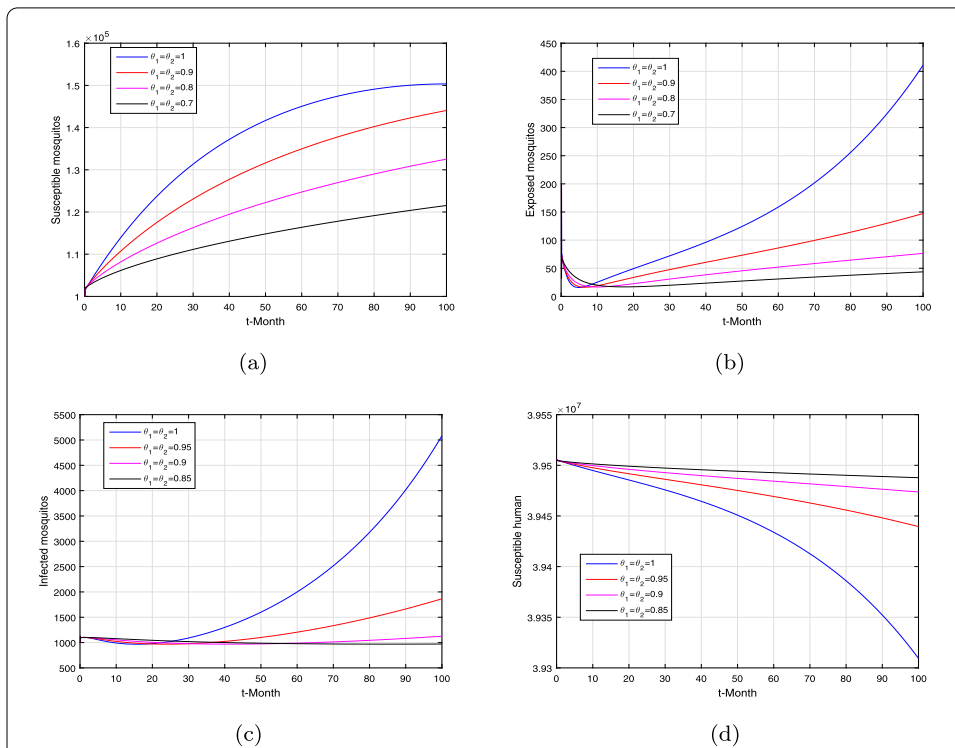
Figure 6 Simulation of the fractal-fractional model (10) when $\theta_1 = \theta_2 = 1, 0.9, 0.8, 0.7$ where (**a**) susceptible mosquitos, (**b**) exposed mosquitos, (**c**) infected mosquitos, (**d**) susceptible humans
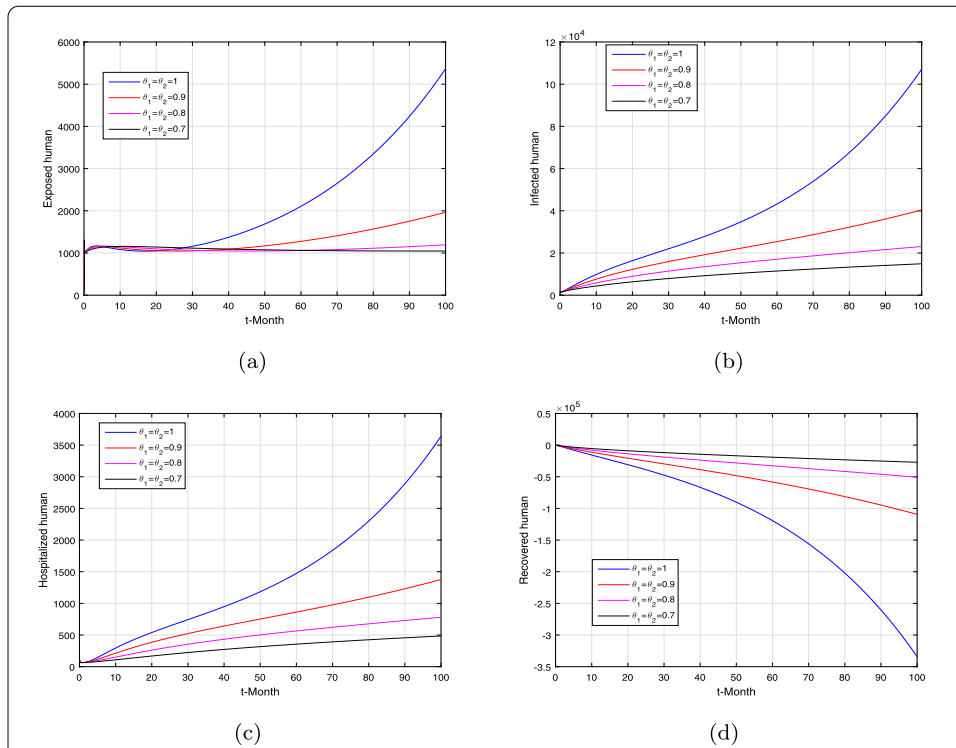
Figure 7 Simulation of the fractal-fractional model (10) when $\theta_1 = \theta_2 = 1, 0.9, 0.8, 0.7$ where (**a**) exposed humans, (**b**) infected humans, (**c**) hospitalized humans, (**d**) recovered humans
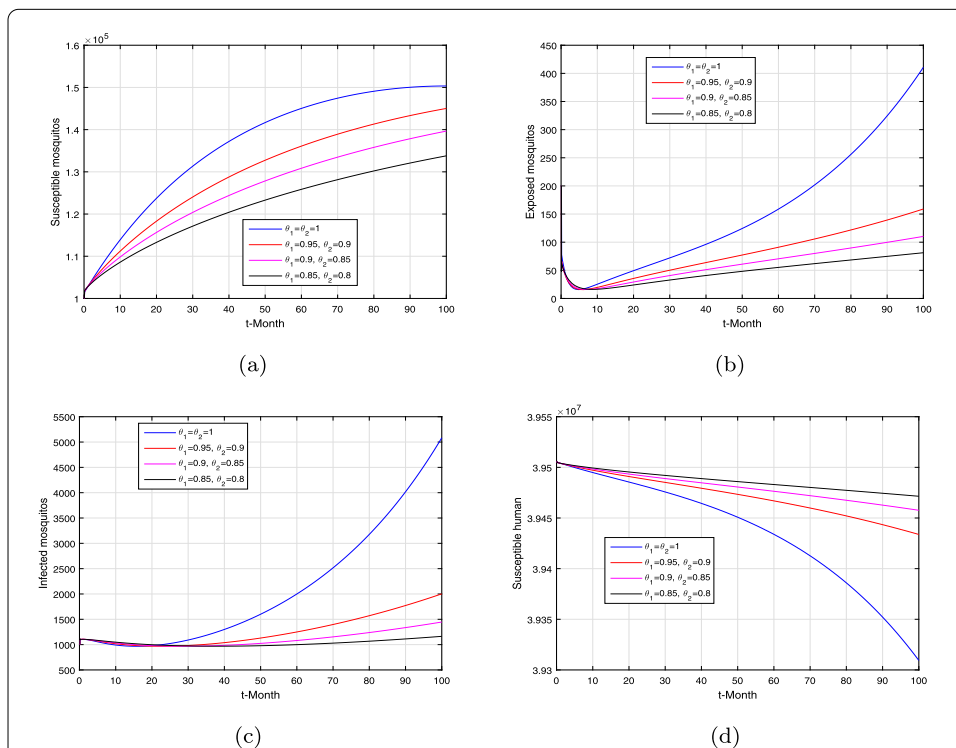

Figure 8 Simulation of the fractal-fractional model (10) when $\theta_1 = 1, 0.95, 0.9, 0.85$ and fractal order $\theta_2 = 1, 0.9, 0.85, 0.8$ where (**a**) susceptible mosquitos, (**b**) exposed mosquitos, (**c**) infected mosquitos, (**d**) susceptible humans

Figure 9 Simulation of the fractal-fractional model (10) when $\theta_1 = 1, 0.95, 0.9, 0.85$ and fractal order $\theta_2 = 1, 0.9, 0.85, 0.8$ where (**a**) exposed humans, (**b**) infected humans, (**c**) hospitalized humans, (**d**) recovered humans
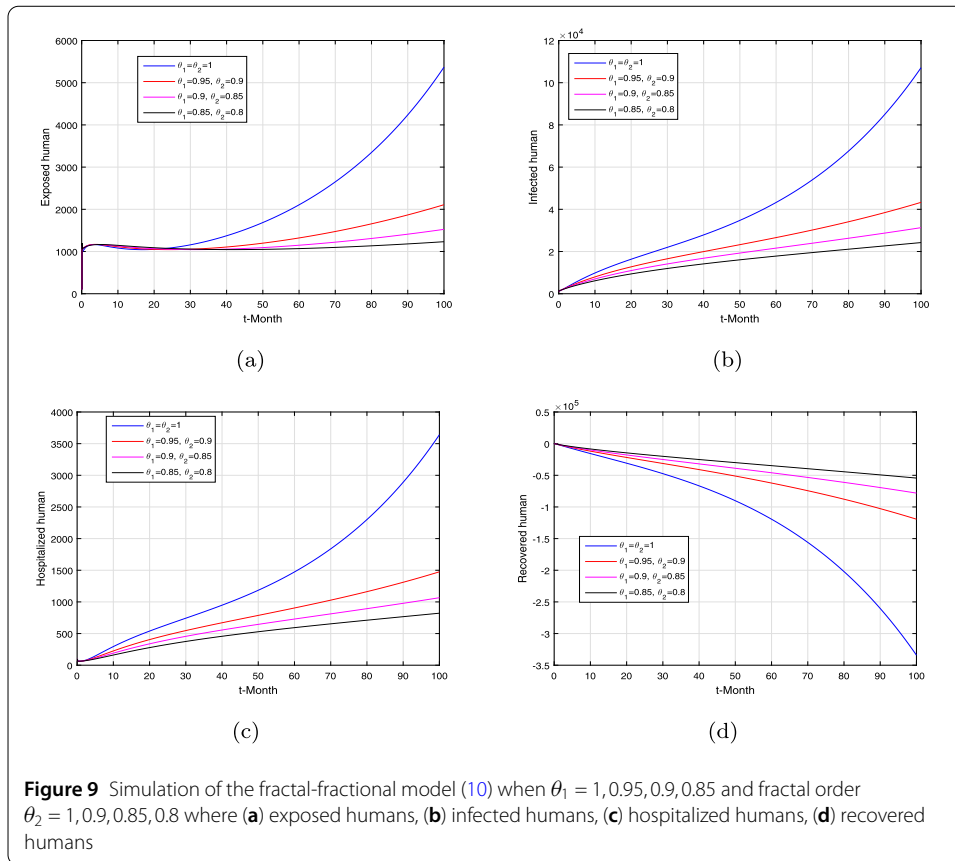
We use the estimated and fitted parameters in Table 2 for the simulation results and the units are taken in months. We present the graphical results using the fractal and fractional order parameters θ_1 and θ_2 in different scenarios. Initially, we choose θ_1 as varying, fix the fractal order θ_2 and obtain the graphical results shown in Figs. 2 and 3. We obtain Figs. 4 and 5 by choosing the fractal order as varying and fixing the fractional order. Figures 6 and 7 are obtained by setting equal values to the fractional and fractal orders, θ_1 and θ_2. Figures 7 and 9 are presented for different orders of fractal and fractional parameters simultaneously. We observe from these numerical results that by varying the fractal and fractional orders, the dynamics of infected compartments in both humans and mosquitos decrease much faster than that when using integer order equations, either by varying only fractal order or only fractional orders. Thus, it is concluded that the fractal-fractional operator provides a better understanding for the epidemic disease model with real data.

5 Conclusions

We obtained a dengue fever model in the framework of fractal-fractional operator. We obtained the basic reproduction number for the proposed dengue model equal to $\mathcal{R}_0 \approx 2.2020$ for the real cases of East Java, Indonesia, for 2018. We presented the model stability results and found that the model is locally asymptotically stable at the disease-free case when $\mathcal{R}_0 < 1$. Then, the application of fractal-fractional operator in the sense of Caputo–Fabrizio was applied to the model, and we obtained a generalized model. The latter model was then used to present a novel numerical procedure and solution. The numerical re-

sults for the fractal and fractional orders have been compared with the real cases of dengue fever, and we have found that, by varying the values of the fractal and fractional orders arbitrary, the best fitting has been obtained. Moreover, we used the fractal and fractional order parameter values and presented numerous graphical results for the model. We fixed the fractal order and varied the fractional order, showing the graphical results. We also fixed the fractional order and varied the fractal order to obtain the numerical results. Similarly, we varied both fractal and fractional orders equally and unequally and obtained many graphical results. Based on these numerical results, we came to a conclusion that varying both fractal and fractional orders provides the best results for the minimization of infected compartments in mosquitoes and humans and for the increase of the noninfected compartments of humans and mosquitos by decreasing the fractal and fractional orders. From the present analysis of the dengue infection model, we suggest to the readers that the application of fractal-fractional operators to a real life problem provides better results than using the ordinary order.

Authors' contributions
The main idea of this paper was proposed by F and MAK. The mathematical results are obtained by CA, F and MAK. The numerical experiment was performed by MAK. F, CA, MAK, EA revised the results and improved the paper. All authors read and approved the final manuscript.

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