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Oblique explicit wave solutions of the fractional biological population (BP) and equal width (EW) models

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Abstract

This research uses the extended exp-($-\varphi(\vartheta)$)-expansion and the Jacobi elliptical function methods to obtain a fashionable explicit format for solutions to the fragmented biological population and the same width models that depict popular logistics because of deaths or births. In mathematical terminology, the linear, hyperbolic, and trigonometric equation solutions that have been found describe several innovative aspects from the two models. Sketching these solutions in different types is used to give them more details. The accuracy and performance of the method adopted show their ability to be applied to various nonlinear developmental equations.

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1 Introduction

Previously, a system of nonlinear evolutionary equations has been used to formulate a fraction of a population in specific fields [1–5]. In numerous distinct branches of science such as mathematics, chemistry, biology, ecology, chaos syncing, mechanics engineering, physics and anomalous spreads, and so on [6–8], many researchers have investigated analytical, semiautomatic, and numerical solutions of fractional models. Such phenomena have been modeled by the fractional mathematical models based on experimental results to demonstrate their nonlocal properties, where this form of property [9–13] cannot be expressed by nonlinear partial differential equations with an integer order.

Based on the ability to form multiple complex phenomena in diverse fields such as biology, plasma physics, hydrodynamics, fluid mechanics, optics, and so forth, several precise and computational schemes such as [14–23] have been developed. In the treatment to these problems, electronic and technical developments are known to be of essential value across derivative structures. These systems have been recently considered to be basic tools in the development of various waveform travel formulas such as [24–30]. These are complex phenomena. In the nonlinear partial differential equation (NLPDE), several scientists have struggled to extract and formulate several complicated anomalies in an integer order [31, 32]. Therefore, fractional is assumed to be an effective solution for this question when a nonlocal property is found that is not NLPDE dependent with [33, 34] an integer order. This makes many fractional models and definitions of derivatives represented and formulated as in [35–47].

In this research, we study two primary models in biological science. These models are named with the fractional BP model, and fractional EW equation is given by

- Fractional BP model [48–51]:

  This paradigm illustrates community dynamics and is given by

$$D_t^{\kappa}\mathcal{H} = D_{xx}^{2\kappa}\mathcal{H}^2 + D_{yy}^{2\kappa}\mathcal{H}^2 + \upsilon\big(\mathcal{H}^2 - s\big), \tag{1}$$

  where $\mathcal{H}$ is the function of the population density, while $\upsilon(\mathcal{H}^2 - s)$ represents the population logistics according to deaths and births. Additionally, ν, s, κ $(0 < \kappa \leq 1)$ are arbitrary constants.

  The population model describes the number of organisms of the same species (human, animal, and plant) living simultaneously in a particular geographic area with interbreeding capacity.

- Fractional EW equation [52–55]:

  It is an alternative form of nonlinear dispersive waves first introduced by Morrison et al. and formulated as follows:

$$D_t^{\kappa}\Lambda + 2h\Lambda D_x^{\kappa}\Lambda - rD_{xxt}^{3\kappa}\Lambda = 0, \tag{2}$$

  where h, r, κ $(0 < \kappa \leq 1)$ are arbitrary constants. Without deformation, waves may spread in a wave medium that is non-dispersive. Electromagnetic waves with unbounded free space are both non-dispersive and non-dissipative, thus can spread over astronomical distances. Sound waves in air are also virtually non-dispersive in the ultrasound range. If not, if high-frequency notes (e.g. piccolo) and low-frequency notes (e.g. basis) spread at different speeds, they could reach the ears at different times. However, the majority of the waves in material media are scattering, and initially established wave forms will change so that wave power can spread or disperse more spatially.

Implementation of the following conformable derivative definitions (for further definition and properties of the conformable fractional derivatives, see the Appendix) on Eqs. (1) and (2) with the following respective order $\mathcal{H}(x,y,t) = \mathcal{H}(\vartheta)$, $\vartheta = \varrho\frac{x^{\kappa}}{\kappa} + i\varrho\frac{y^{\kappa}}{\kappa} + \frac{ct^{\kappa}}{\kappa}$, $\Lambda(x,t) = \Lambda(\vartheta)$, $\vartheta = \frac{x^{\kappa}}{\kappa} + \frac{ct^{\kappa}}{\kappa}$, where ϱ, c, κ are arbitrary constants, transforms the fractional PDE into integer order ODE which are given by

$$c\mathcal{H}' - \upsilon\mathcal{H}^2 + \upsilon s = 0, \tag{3}$$

$$c\Lambda + h\Lambda^2 - rc\Lambda'' = 0. \tag{4}$$

The remaining sections in our research paper are ordered as follows: Sect. 2 manipulates the extended exp-$(-\varphi(\vartheta))$-expansion method [56–60] and the Jacobi elliptical function method [61–63] to procure novel solitary solutions of both suggested models. Section 5 gives the conclusion of this research.

2 Applications

This section applies the extended exp-($-\varphi(\vartheta)$)-expansion and the Jacobi elliptical function techniques on the considered fractional models.

Balancing the terms in Eqs. (3), (4) to get the balance value of each of them leads to respectively $n+1=m$ & $n+2=m$, where n, m are arbitrary constants. Supposing the value of $n=1$ and according to the general solution that is suggested by the extended exp-($-\varphi(\vartheta)$)-expansion method and the Jacobi elliptical function method, the general solution of Eqs. (3) is given respectively by

$$\mathcal{H}(\vartheta)=\frac{\sum_{i=0}^{m}a_i e^{-i\varphi(\vartheta)}}{\sum_{j=0}^{n}b_j e^{-j\varphi(\vartheta)}}=\frac{a_1 e^{-\varphi(\vartheta)}+a_2 e^{-2\varphi(\vartheta)}+a_0}{b_1 e^{-\varphi(\vartheta)}+b_0}, \tag{5}$$

$$\mathcal{H}(\vartheta)=\sum_{i=1}^{n}a_i\phi(\vartheta)^i+\sum_{i=1}^{n}b_i\phi(\vartheta)^{-i}+a_0=a_1\phi(\vartheta)+a_0+\frac{b_1}{\phi(\vartheta)}, \tag{6}$$

while the general solution of Eqs. (4) is given by

$$\Lambda(\vartheta)=\frac{\sum_{i=0}^{m}a_i e^{-i\varphi(\vartheta)}}{\sum_{j=0}^{n}b_j e^{-j\varphi(\vartheta)}}=\frac{a_1 e^{-\varphi(\vartheta)}+a_2 e^{-2\varphi(\vartheta)}+a_3 e^{-3\varphi(\vartheta)}+a_0}{b_1 e^{-\varphi(\vartheta)}+b_0}, \tag{7}$$

$$\begin{aligned}\Lambda(\vartheta)&=\sum_{i=1}^{n}a_i\phi(\vartheta)^i+\sum_{i=1}^{n}b_i\phi(\vartheta)^{-i}+a_0\\&=a_2\phi(\vartheta)^2+a_1\phi(\vartheta)+a_0+\frac{b_2}{\phi(\vartheta)^2}+\frac{b_1}{\phi(\vartheta)},\end{aligned} \tag{8}$$

where a_i, b_j $(i,j=0,1,2,\ldots)$. Additionally, $\varphi(\vartheta)$ is the solution function of $[\varphi'(\vartheta)=\chi+\gamma e^{\varphi(\vartheta)}+\frac{1}{e^{\varphi(\vartheta)}}$ & $\phi'(\vartheta)=\sqrt{p\phi(\vartheta)^2+q\phi(\vartheta)^4+\rho}]$, where χ,γ,r,p,q are arbitrary constants.

2.1 Fractional BP model

2.1.1 Extended exp-($-\varphi(\vartheta)$)-expansion method

Employing Eqs. (5) and (3) in the framework of the extended exp-($-\varphi(\vartheta)$)-expansion method to solve Eq. (3) yields the following.

Family I

$$\left[a_0\to\frac{b_0\chi\sqrt{s}}{\sqrt{\chi^2-4\gamma}},a_1\to\frac{2b_0\sqrt{s}}{\sqrt{\chi^2-4\gamma}}+\frac{b_1\chi\sqrt{s}}{\sqrt{\chi^2-4\gamma}},a_2\to\frac{2b_1\sqrt{s}}{\sqrt{\chi^2-4\gamma}},c\to-\frac{2\sqrt{s}\nu}{\sqrt{\chi^2-4\gamma}}\right].$$

Consequently, the explicit wave solutions of Eq. (1) are given by:

In case of $[\chi^2-4\gamma>0$ & $\gamma\neq 0]$,

$$\mathcal{H}_1(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma+\chi\sqrt{\chi^2-4\gamma}\tanh(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi+\sqrt{\chi^2-4\gamma}\tanh(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}, \tag{9}$$

$$\mathcal{H}_2(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma+\chi\sqrt{\chi^2-4\gamma}\coth(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi+\sqrt{\chi^2-4\gamma}\coth(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}. \tag{10}$$

In case of $[\chi^2-4\gamma>0\ \&\ \gamma=0\ \&\ \chi\neq 0]$,

$$\mathcal{H}_3(x,y,t)=\frac{\chi\sqrt{s}\coth(\frac{\chi(\eta\kappa-\frac{2\sqrt{s}\nu t^\kappa}{\sqrt{\chi^2}}+\varrho(x^\kappa+iy^\kappa))}{2\kappa})}{\sqrt{\chi^2}}. \tag{11}$$

In case of $[\chi^2-4\gamma=0\ \&\ \gamma\neq 0\ \&\ \chi\neq 0]$,

$$\mathcal{H}_4(x,y,t)=-\frac{2\kappa\chi\sqrt{s}}{2\chi\sqrt{s}\nu t^\kappa-\sqrt{\chi^2-4\gamma}(\kappa(\eta\chi+2)+\chi\varrho(x^\kappa+iy^\kappa))}. \tag{12}$$

In case of $[\chi^2-4\gamma<0\ \&\ \gamma\neq 0]$,

$$\mathcal{H}_5(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma-\chi\sqrt{4\gamma-\chi^2}\tan(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}\nu t^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi-\sqrt{4\gamma-\chi^2}\tan(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}\nu t^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}, \tag{13}$$

$$\mathcal{H}_6(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma-\chi\sqrt{4\gamma-\chi^2}\cot(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}\nu t^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi-\sqrt{4\gamma-\chi^2}\cot(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}\nu t^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}. \tag{14}$$

Family II

$$\left[a_0\to\frac{b_1\chi^2\sqrt{s}}{2\sqrt{\chi^2-4\gamma}},a_1\to\frac{2b_1\chi\sqrt{s}}{\sqrt{\chi^2-4\gamma}},a_2\to\frac{2b_1\sqrt{s}}{\sqrt{\chi^2-4\gamma}},b_0\to\frac{b_1\chi}{2},c\to-\frac{2\sqrt{s}\nu}{\sqrt{\chi^2-4\gamma}}\right].$$

Consequently, the explicit wave solutions of Eq. (1) are given by:

In case of $[\chi^2-4\gamma>0\ \&\ \gamma\neq 0]$,

$$\mathcal{H}_7(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma+\chi\sqrt{\chi^2-4\gamma}\tanh(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi+\sqrt{\chi^2-4\gamma}\tanh(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}, \tag{15}$$

$$\mathcal{H}_8(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma+\chi\sqrt{\chi^2-4\gamma}\coth(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi+\sqrt{\chi^2-4\gamma}\coth(\frac{-2\sqrt{s}\nu t^\kappa+\sqrt{\chi^2-4\gamma}(\eta\kappa+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}. \tag{16}$$

In case of $[\chi^2-4\gamma>0\ \&\ \gamma=0\ \&\ \chi\neq 0]$,

$$\mathcal{H}_9(x,y,t)=\frac{\chi\sqrt{s}\coth(\frac{\chi(\eta\kappa-\frac{2\sqrt{s}\nu t^\kappa}{\sqrt{\chi^2}}+\varrho(x^\kappa+iy^\kappa))}{2\kappa})}{\sqrt{\chi^2}}. \tag{17}$$

In case of $[\chi^2-4\gamma=0\ \&\ \gamma\neq 0\ \&\ \chi\neq 0]$,

$$\mathcal{H}_{10}(x,y,t)=-\frac{2\kappa\chi\sqrt{s}}{2\chi\sqrt{s}\nu t^\kappa-\sqrt{\chi^2-4\gamma}(\kappa(\eta\chi+2)+\chi\varrho(x^\kappa+iy^\kappa))}. \tag{18}$$

In case of $[\chi^2-4\gamma<0\ \&\ \gamma\neq 0]$,

$$\mathcal{H}_{11}(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma-\chi\sqrt{4\gamma-\chi^2}\tan(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}vt^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi-\sqrt{4\gamma-\chi^2}\tan(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}vt^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))},\tag{19}$$

$$\mathcal{H}_{12}(x,y,t)=\frac{\sqrt{s}(\chi^2-4\gamma-\chi\sqrt{4\gamma-\chi^2}\cot(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}vt^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}{\sqrt{\chi^2-4\gamma}(\chi-\sqrt{4\gamma-\chi^2}\cot(\frac{\sqrt{4\gamma-\chi^2}(\eta\kappa-\frac{2\sqrt{s}vt^\kappa}{\sqrt{\chi^2-4\gamma}}+\varrho x^\kappa+i\varrho y^\kappa)}{2\kappa}))}.\tag{20}$$

2.1.2 Jacobi elliptical function method

Substituting Eq. (6) and its derivative into Eq. (3) leads to a system of equations. Solving them yields

$$[a_0\to-\sqrt{s},a_1\to 0,b_1\to 0]$$

and

$$[a_0\to\sqrt{s},a_1\to 0,b_1\to 0].$$

This result shows the failure of the Jacobi elliptical function method which is considered as a proof of the fact that there exists no unified computational method that can be used on all nonlinear evolution equation.

2.2 Fractional EW model

2.2.1 Extended exp-$(-\varphi(\vartheta))$-expansion method

Replacing Eq. (7) into Eq. (4) and collecting all terms with the same power for $[e^{-\varepsilon\varphi(\vartheta)},\varepsilon=0,1,2,\dots]$ lead to a system of algebraic equation. Solving this system yields the following.

Family I

$$\Bigg[a_0\to\frac{a_3b_0(-\sqrt{(\chi^2-4\gamma)^2}+\chi^2+8\gamma)}{12b_1},$$
$$a_1\to\frac{1}{12}a_3\left(\frac{12b_0\chi}{b_1}-\sqrt{\left(\chi^2-4\gamma\right)^2}+\chi^2+8\gamma\right),a_2\to\frac{a_3(b_1\chi+b_0)}{b_1},$$
$$r\to\frac{1}{\sqrt{\chi^4-8\chi^2\gamma+16\gamma^2}},h\to\frac{6b_1c}{a_3\sqrt{(\chi^2-4\gamma)^2}}\Bigg].$$

Consequently, the explicit wave solutions of Eq. (2) are given by:

In case of $[\chi^2-4\gamma>0\ \&\ \gamma\neq 0]$,

$$\Lambda_1(x,y,t)=\frac{1}{12b_1(\sqrt{\chi^2-4\gamma}\tanh(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa})+\chi)^2}$$
$$\times\left[a_3\operatorname{sech}^2\left(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}\right)\right.$$
$$\times\left(-(\sqrt{\left(\chi^2-4\gamma\right)^2}-\chi^2+4\gamma)\right.$$

$$\begin{aligned}
&\times \left(\chi\sqrt{\chi^2-4\gamma}\sinh\left(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\right)\right.\\
&\left.+\left(\chi^2-2\gamma\right)\cosh\left(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\right)\right)\\
&\left.-2\gamma\left(\sqrt{\left(\chi^2-4\gamma\right)^2}+5\chi^2-20\gamma\right)\right)\bigg], \qquad (21)
\end{aligned}$$

$$\begin{aligned}
\Lambda_2(x,y,t) &= \frac{1}{12b_1(\sqrt{\chi^2-4\gamma}\coth(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa})+\chi)^2}\\
&\times \left[a_3\operatorname{csch}^2\left(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}\right)\right.\\
&\times \left(2\gamma\left(\sqrt{\left(\chi^2-4\gamma\right)^2}+5\chi^2-20\gamma\right)\right.\\
&-\left(\sqrt{\left(\chi^2-4\gamma\right)^2}-\chi^2+4\gamma\right)\left(\chi\sqrt{\chi^2-4\gamma}\right.\\
&\times\sinh\left(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\right)\\
&\left.\left.\left.+\left(\chi^2-2\gamma\right)\cosh\left(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\right)\right)\right)\right]. \qquad (22)
\end{aligned}$$

In case of $[\chi^2-4\gamma>0 \;\&\; \gamma=0 \;\&\; \chi\neq 0]$,

$$\Lambda_3(x,y,t)=\frac{a_3(3\chi^2\operatorname{csch}^2(\frac{\chi(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa})-\sqrt{\chi^4}+\chi^2)}{12b_1}. \qquad (23)$$

In case of $[\chi^2-4\gamma=0 \;\&\; \gamma\neq 0 \;\&\; \chi\neq 0]$,

$$\Lambda_4(x,y,t)=\frac{a_3(\chi^2(\frac{12\kappa^2}{(c\chi t^\kappa+\eta\kappa\chi+2\kappa+\chi x^\kappa)^2}-2)-\sqrt{(\chi^2-4\gamma)^2}+8\gamma)}{12b_1}. \qquad (24)$$

In case of $[\chi^2-4\gamma=0 \;\&\; \gamma=0 \;\&\; \chi=0]$,

$$\Lambda_5(x,y,t)=\frac{a_3\kappa^2}{b_1(ct^\kappa+\eta\kappa+x^\kappa)^2}. \qquad (25)$$

In case of $[\chi^2-4\gamma<0 \;\&\; \gamma\neq 0]$,

$$\begin{aligned}
\Lambda_6(x,y,t) &= \frac{1}{12b_1(\chi-\sqrt{4\gamma-\chi^2}\tan(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}))^2}\\
&\times\left[a_3\sec^2\left(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}\right)\right.\\
&\times\left(-\left(\sqrt{\left(\chi^2-4\gamma\right)^2}-\chi^2+4\gamma\right)\right.\\
&\times\left(\left(\chi^2-2\gamma\right)\cos\left(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\right)\right.
\end{aligned}$$

$$
\begin{aligned}
&-\chi\sqrt{4\gamma-\chi^2}\sin\left(\frac{\sqrt{4\gamma-\chi^2}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{\kappa}\right)\bigg)\\
&-2\gamma\left(\sqrt{\left(\chi^2-4\gamma\right)^2}+5\chi^2-20\gamma\right)\bigg)\bigg], \qquad (26)
\end{aligned}
$$

$$
\begin{aligned}
\Lambda_7(x,y,t) &= \frac{1}{12b_1(\chi-\sqrt{4\gamma-\chi^2}\cot(\frac{\sqrt{4\gamma-\chi^2}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{2\kappa}))^2}\\
&\times\left[a_3\csc^2\left(\frac{\sqrt{4\gamma-\chi^2}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{2\kappa}\right)\right.\\
&\times\left(\left(\sqrt{\left(\chi^2-4\gamma\right)^2}-\chi^2+4\gamma\right)\right.\\
&\times\left(\chi\sqrt{4\gamma-\chi^2}\sin\left(\frac{\sqrt{4\gamma-\chi^2}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{\kappa}\right)\right.\\
&+\left(\chi^2-2\gamma\right)\cos\left(\frac{\sqrt{4\gamma-\chi^2}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{\kappa}\right)\bigg)\\
&-2\gamma\left(\sqrt{\left(\chi^2-4\gamma\right)^2}+5\chi^2-20\gamma\right)\bigg)\bigg]. \qquad (27)
\end{aligned}
$$

Family II

$$
\begin{aligned}
&\bigg[b_1\to\frac{2b_0}{\chi}, a_0\to\frac{1}{24}a_3\chi\left(\sqrt{\left(\chi^2-4\gamma\right)^2}+\chi^2+8\gamma\right),\\
&a_1\to\frac{1}{12}a_3\left(\sqrt{\left(\chi^2-4\gamma\right)^2}+7\chi^2+8\gamma\right),\\
&a_2\to\frac{3a_3\chi}{2}, r\to-\frac{1}{\sqrt{\chi^4-8\chi^2\gamma+16\gamma^2}}, h\to-\frac{12b_0c}{a_3\chi\sqrt{(\chi^2-4\gamma)^2}}\bigg].
\end{aligned}
$$

Consequently, the explicit wave solutions of Eq. (2) are given by:

In case of $[\chi^2-4\gamma>0\ \&\ \gamma\neq 0]$,

$$
\begin{aligned}
\Lambda_8(x,y,t) &= \frac{1}{24b_0(\sqrt{\chi^2-4\gamma}\tanh(\frac{\sqrt{\chi^2-4\gamma}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{2\kappa})+\chi)^2}\\
&\times\left[a_3\chi\,\mathrm{sech}^2\left(\frac{\sqrt{\chi^2-4\gamma}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{2\kappa}\right)\right.\\
&\times\left(\left(\sqrt{\left(\chi^2-4\gamma\right)^2}+\chi^2-4\gamma\right)\right.\\
&\times\left(\chi\sqrt{\chi^2-4\gamma}\sinh\left(\frac{\sqrt{\chi^2-4\gamma}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{\kappa}\right)\right.\\
&+\left(\chi^2-2\gamma\right)\cosh\left(\frac{\sqrt{\chi^2-4\gamma}(ct^{\kappa}+\eta\kappa+x^{\kappa})}{\kappa}\right)\bigg)\\
&+2\gamma\left(\sqrt{\left(\chi^2-4\gamma\right)^2}-5\chi^2+20\gamma\right)\bigg)\bigg], \qquad (28)
\end{aligned}
$$

$$\begin{aligned}\Lambda_9(x,y,t) &= \frac{1}{24b_0(\sqrt{\chi^2-4\gamma}\coth(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa})+\chi)^2} \\ &\quad\times\biggl[a_3\chi\operatorname{csch}^2\biggl(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}\biggr) \\ &\quad\times\Bigl(\Bigl(\sqrt{\bigl(\chi^2-4\gamma\bigr)^2}+\chi^2-4\gamma\Bigr) \\ &\quad\times\biggl(\chi\sqrt{\chi^2-4\gamma}\sinh\biggl(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\biggr) \\ &\quad+\bigl(\chi^2-2\gamma\bigr)\cosh\biggl(\frac{\sqrt{\chi^2-4\gamma}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\biggr)\biggr) \\ &\quad-2\gamma\Bigl(\sqrt{\bigl(\chi^2-4\gamma\bigr)^2}-5\chi^2+20\gamma\Bigr)\Bigr)\biggr]. \end{aligned} \tag{29}$$

In case of $[\chi^2-4\gamma>0\ \&\ \gamma=0\ \&\ \chi\neq 0]$,

$$\Lambda_{10}(x,y,t)=\frac{a_3\chi(3\chi^2\operatorname{csch}^2(\frac{\chi(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa})+\sqrt{\chi^4}+\chi^2)}{24b_0}. \tag{30}$$

In case of $[\chi^2-4\gamma=0\ \&\ \gamma\neq 0\ \&\ \chi\neq 0]$,

$$\Lambda_{11}(x,y,t)=\frac{a_3\chi(\chi^2(\frac{12\kappa^2}{(c\chi t^\kappa+\eta\kappa\chi+2\kappa+\chi x^\kappa)^2}-2)+\sqrt{(\chi^2-4\gamma)^2}+8\gamma)}{24b_0}. \tag{31}$$

In case of $[\chi^2-4\gamma<0\ \&\ \gamma\neq 0]$,

$$\begin{aligned}\Lambda_{12}(x,y,t) &= \frac{1}{24b_0(\chi-\sqrt{4\gamma-\chi^2}\tan(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}))^2} \\ &\quad\times\biggl[a_3\chi\sec^2\biggl(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}\biggr) \\ &\quad\times\Bigl(\Bigl(\sqrt{\bigl(\chi^2-4\gamma\bigr)^2}+\chi^2-4\gamma\Bigr) \\ &\quad\times\biggl(\bigl(\chi^2-2\gamma\bigr)\cos\biggl(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\biggr) \\ &\quad-\chi\sqrt{4\gamma-\chi^2}\sin\biggl(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\biggr)\biggr) \\ &\quad\times\gamma\Bigl(\sqrt{\bigl(\chi^2-4\gamma\bigr)^2}-5\chi^2+20\gamma\Bigr)\Bigr)\biggr], \end{aligned} \tag{32}$$

$$\begin{aligned}\Lambda_{13}(x,y,t) &= \frac{-1}{24b_0(\chi-\sqrt{4\gamma-\chi^2}\cot(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}))^2} \\ &\quad\times\biggl[a_3\chi\csc^2\biggl(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{2\kappa}\biggr) \\ &\quad\times\Bigl(\Bigl(\sqrt{\bigl(\chi^2-4\gamma\bigr)^2}+\chi^2-4\gamma\Bigr) \end{aligned}$$

Table 1 Some solutions of auxiliary equation of Eq. 8

ρ	p	q	$\phi(\vartheta)$
1	$-(1+m^2)$	m^2	$sn(\vartheta)$
$1-m^2$	$2m^2-1$	$-m^2$	$cn(\vartheta)$
m^2-1	$2-m^2$	-1	$dn(\vartheta)$
$\frac{1}{4}$	$\frac{1-2m^2}{2}$	$\frac{1}{4}$	$ns(\vartheta)\pm cs(\vartheta)$ or $\frac{sn(\vartheta)}{1\pm cn(\vartheta)}$
$\frac{1-m^2}{4}$	$\frac{1+m^2}{2}$	$\frac{1-m^2}{4}$	$nc(\vartheta)\pm sc(\vartheta)$ or $\frac{cn(\vartheta)}{1\pm sn(\vartheta)}$
1	$2-m^2$	$1-m^2$	$sc(\vartheta)$
$1-m^2$	$2-m^2$	1	$cs(\vartheta)$

$$\times\left(\chi\sqrt{4\gamma-\chi^2}\sin\left(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\right)\right.$$
$$\left.+\left(\chi^2-2\gamma\right)\cos\left(\frac{\sqrt{4\gamma-\chi^2}(ct^\kappa+\eta\kappa+x^\kappa)}{\kappa}\right)\right)$$
$$\left.-2\gamma\left(\sqrt{\left(\chi^2-4\gamma\right)^2}-5\chi^2+20\gamma\right)\right)\Bigg]. \tag{33}$$

2.2.2 Jacobi elliptical function method

Substituting Eq. (8) and its derivative into Eq. (4) leads to a system of equations. Solving them yields

$$\left[a_0\to\frac{1}{2}\left(\frac{cp}{h\sqrt{p^2-3q\rho}}-\frac{c}{h}\right),a_1\to 0,a_2\to\frac{6cq}{h\sqrt{16p^2-48q\rho}},\right.$$
$$\left.b_1\to 0,b_2\to 0,r\to\frac{1}{\sqrt{16p^2-48q\rho}}\right].$$

Using these values and Table 1 leads to formulating the explicit wave solutions of Eq. (2) in the following formats:

$$\Lambda_{14}(x,y,t)|_{\rho\to 1,p\to -2,q\to 1}=-\frac{3c\,\mathrm{sech}^2(\frac{ct^\kappa+x^\kappa}{\kappa})}{2h}, \tag{34}$$

$$\Lambda_{15}(x,y,t)|_{\rho\to\frac{1}{4},p\to-\frac{1}{2},q\to\frac{1}{4}}=\frac{3c((\coth(\frac{ct^\kappa+x^\kappa}{\kappa})\pm\mathrm{csch}(\frac{ct^\kappa+x^\kappa}{\kappa}))^2-1)}{2h}, \tag{35}$$

$$\Lambda_{16}(x,y,t)|_{\rho\to 0,p\to 1,q\to 1}=\frac{3c\,\mathrm{csch}^2(\frac{ct^\kappa+x^\kappa}{\kappa})}{2h}, \tag{36}$$

$$\Lambda_{16}(x,y,t)|_{\rho\to\frac{1}{4},p\to\frac{1}{2},q\to\frac{1}{4}}=\frac{c(3(\csc(\frac{ct^\kappa+x^\kappa}{\kappa})\pm\cot(\frac{ct^\kappa+x^\kappa}{\kappa}))^2+1)}{2h}, \tag{37}$$

$$\Lambda_{18}(x,y,t)|_{\rho\to\frac{1}{4},p\to\frac{1}{2},q\to\frac{1}{4}}=\frac{c(3(\sec(\frac{ct^\kappa+x^\kappa}{\kappa})\pm\tan(\frac{ct^\kappa+x^\kappa}{\kappa}))^2+1)}{2h}, \tag{38}$$

$$\Lambda_{19}(x,y,t)|_{\rho\to 1,p\to 2,q\to 1}=\frac{c(3(\sec(\frac{ct^\kappa+x^\kappa}{\kappa})\pm\tan(\frac{ct^\kappa+x^\kappa}{\kappa}))^2+1)}{2h}, \tag{39}$$

$$\Lambda_{20}(x,y,t)|_{\rho\to 1,p\to 2,q\to 1}=\frac{c(3\cot^2(\frac{ct^\kappa+x^\kappa}{\kappa})+1)}{2h}. \tag{40}$$

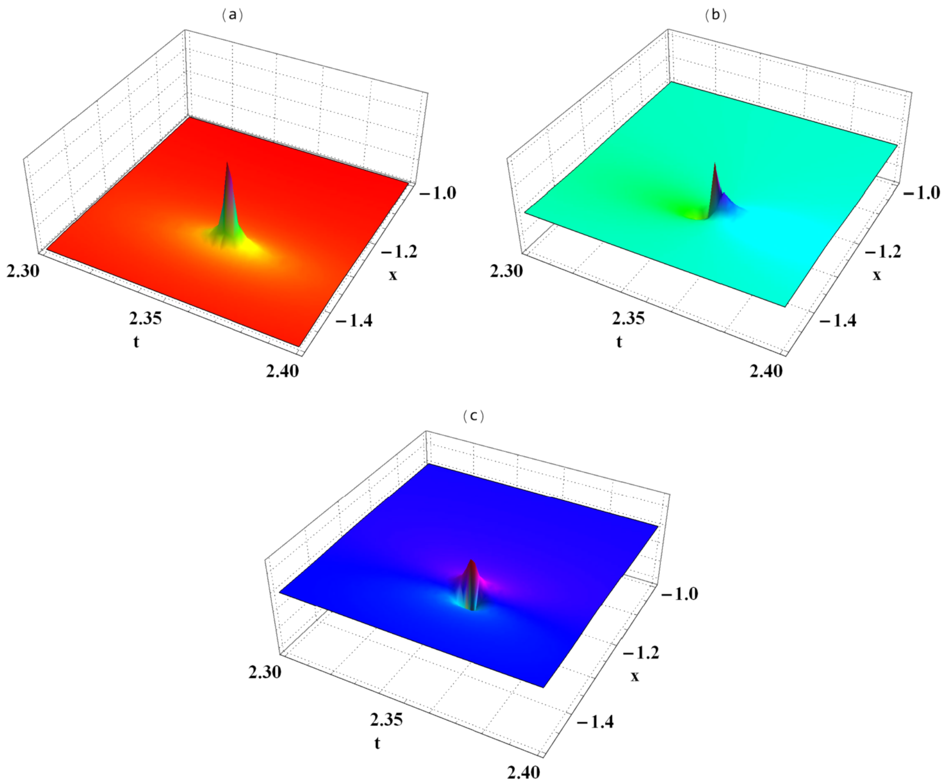

Figure 1 Cone wave solution for the absolute, real, and imaginary parts of (9) in three dimensions

3 Figures representation

This section explains the shown figures in our paper. All these plots depend on individual values of the indicated parameters in the obtained solution. Now, we discuss the representation of the shown figures with their settings as follows:

- Fig. 1 represents a cone wave solution of Eq. (9) when $\eta = 1$, $\kappa = 0.5$, $\lambda = 3$, $\mu = 2$, $s = 4$, $\nu = 6$, $y = 7$, $\varrho = 5$ in a three-dimensional sketch.
- Fig. 2 shows a contour plot of Eq. (9) when $\eta = 1$, $\kappa = 0.5$, $\lambda = 3$, $\mu = 0$, $s = 4$, $\nu = 6$, $y = 7$, $\varrho = 5$ that explains the surface is symmetric and peaks in the center.
- Fig. 3 represents a dark wave solution of Eq. (21) when $(a_3 = 3)$; $b_1 = 2$, $c = 5$, $\eta = 1$, $\kappa = 0.5$, $\lambda = 3$, $\mu = 2$, $s = 4$ in three distinct types of plots.
- Fig. 4 shows dark wave solution of Eq. (23) when $a_3 = 3$, $b_1 = 2$, $c = 5$, $\eta = 1$, $\kappa = 0.5$, $\lambda = 3$, $\mu = 2$, $s = 4$ in three different plots.
- Fig. 5 shows dark wave solution of Eq. (34) when $a_3 = 3$, $b_1 = 2$, $c = 5$, $\eta = 1$, $\kappa = 0.5$, $\lambda = 3$, $\mu = 0$, $s = 4$ in three various kind of plots.
- Fig. 6 shows bright wave solution of Eq. (35) when $c = 3$, $h = 2$, $\kappa = 0.5$ in three different forms of sketches.

4 Results and discussion

This segment displays our approaches and their latest functions. Check out how close we are and how the ideas we have found can be contrasted with the previously published papers. The key elements of our debate are the methodological approach employed and the approaches produced.

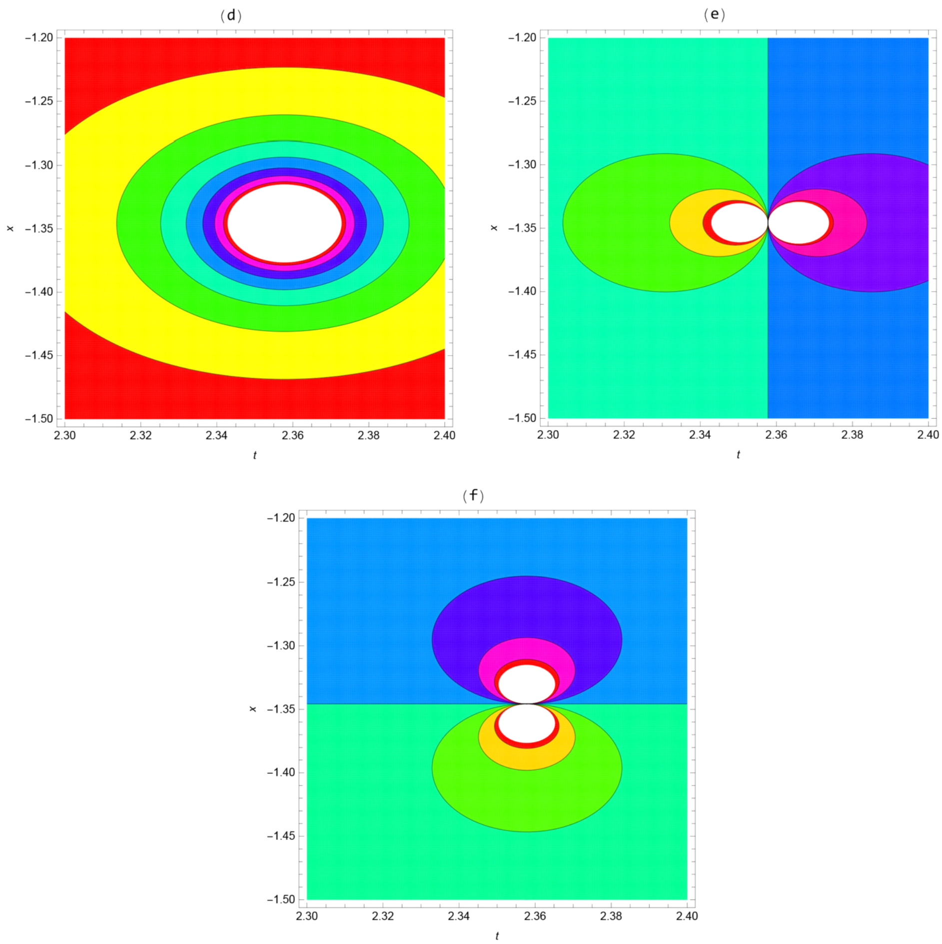

Figure 2 Cone wave solution for the absolute, real, and imaginary parts of (9) in the contour plot

1. *The computational schemes used*:

   We used two computational methods (the extended exp-($-\varphi(\vartheta)$)-expansion method and the Jacobi elliptical function method) for the fractional BP model and the EW model for constructing the exact traveling and solitary wave solutions. A compliant fractional operator was employed to transform fractional aspects of the equation to a nonlinear ordinary differential equation. This fractional operator enables the classification schemes to be extended to the transformed shape. All systems have the desired approach.

$$\mathcal{H}(\vartheta) = \Lambda(\vartheta) = \begin{cases} \frac{\sum_{i=0}^{m} a_i e^{-i\phi(\vartheta)}}{\sum_{i=0}^{n} b_i e^{-i\phi(\vartheta)}}, \\ \quad \phi'(\vartheta) \to \lambda + \mu e^{\phi(\vartheta)} + \frac{1}{e^{\phi(\vartheta)}}, \\ \sum_{i=1}^{n} a_i \phi(\vartheta)^i + \sum_{i=1}^{n} b_i \phi(\vartheta)^{-i} + a_0, \\ \quad \phi'(\vartheta) \to \sqrt{p\phi(\vartheta)^2 + q\phi(\vartheta)^4 + \rho}, \end{cases} \tag{41}$$

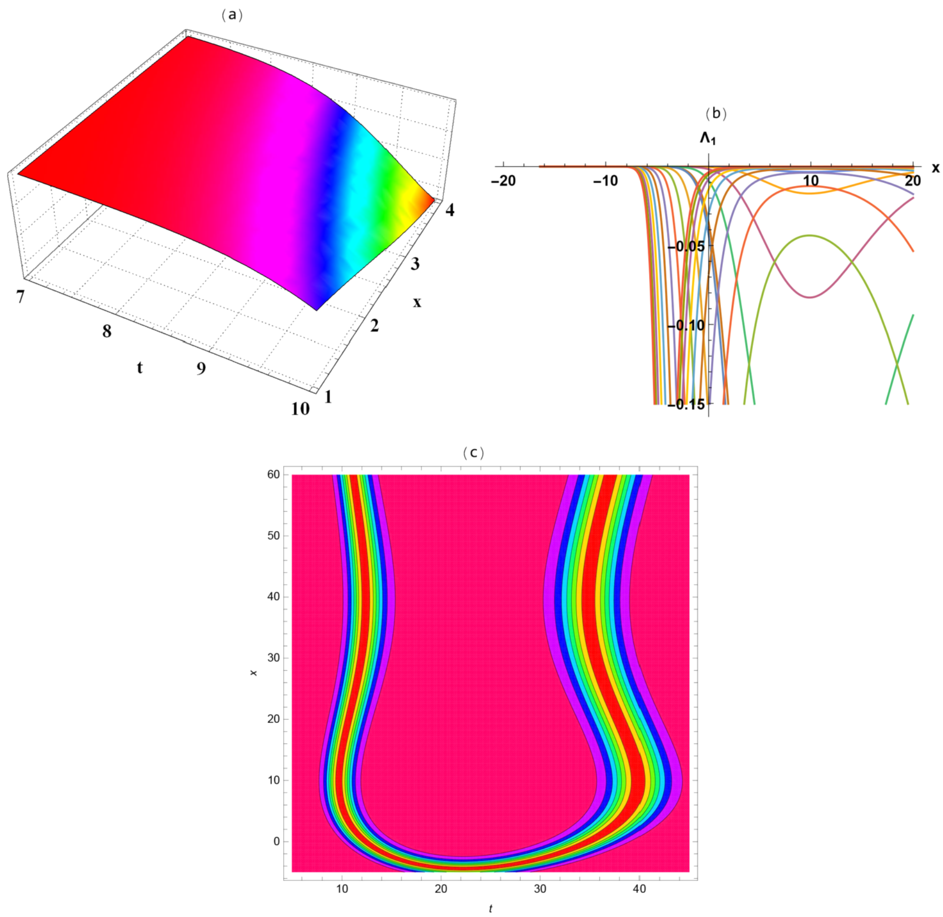

Figure 3 Dark wave solution of (21) in three (**a**), two-dimensional (**b**), and contour plots (**c**)

where $[n, m]$ are arbitrary constants to be determined by using the homogeneous balance rule, while $[\lambda, \mu, p, q, \rho]$ are arbitrary constants to be determined by solving the obtained algebraic system of equations. Moreover, applying two different schemes on one model shows its ability to be used for other various schemes where there is no unified method that is applied for all nonlinear evolution equations, and that is well demonstrated in our paper.

2. *The obtained solutions*:

   This part gives a comparison between our obtained solutions and those obtained in previously published papers. In [64], Mostafa M. A. Khater, Raghda A. M. Attia, and Dianchen Lu applied the modified Khater method to a fractional biological population model, fractional equal width model, and fractional modified equal width equation. They got many distinct types of solutions for these fractional biological models.

   - Eq. (27, [64]) is equal to Eq. (17) when $[\chi = 2, \lambda = \nu, n\kappa = 0, \varrho = \mu\sqrt{-\alpha\sigma}]$.
   - All other obtained solutions of the fractional BP model are different from those obtained in [64].
   - All our obtained solutions of the fractional EW model are new and different from those obtained in [64].

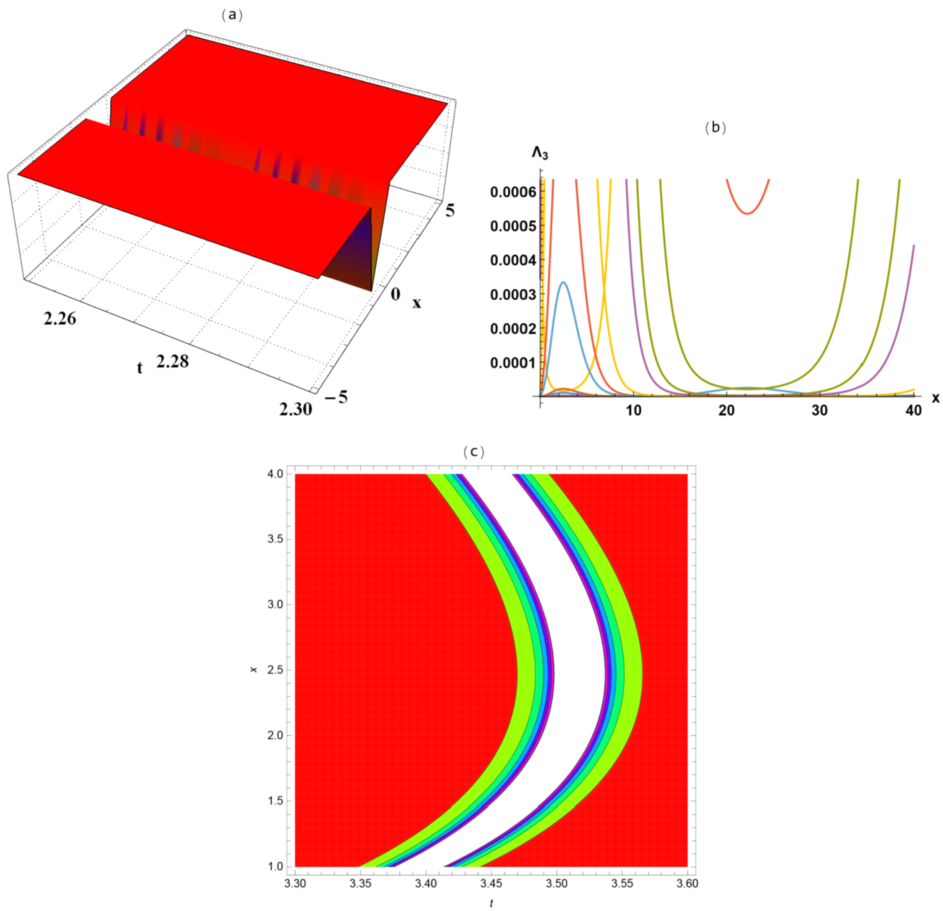

Figure 4 Dark wave solution of (23) in three (**a**), two-dimensional (**b**), and contour plots (**c**)

5 Conclusion

This paper examined two nonlinear BP and EW fractional models via exp-($-\varphi(\vartheta)$)-expansion and the Jacobi elliptical function method. The conformable fractional derivative has been employed to convert the nonlinear partial differential equations to an ordinary differential equation with an integer order. Many distinct wave solutions have been obtained and have been represented in three-, two-dimensional, and contour plots. These solutions were explained by different illustrations which clarify the new features of the fractional models in question. Our solutions have been explained in terms of precision and innovation. The novelty of our solutions is shown by comparing our solution with that obtained in previous research papers. The powerful and effective application of the used method is examined and tested to show its ability to be applied to other nonlinear evolution equations.

Appendix

Given function $f : [0,\infty) \to R$. Then the (conformable fractional derivative) of f of order α is defined by

$$T_\alpha(f)(t) = \lim_{\varepsilon\to 0}\frac{f(t+\varepsilon t^{1-\alpha}) - f(t)}{\varepsilon}. \tag{42}$$

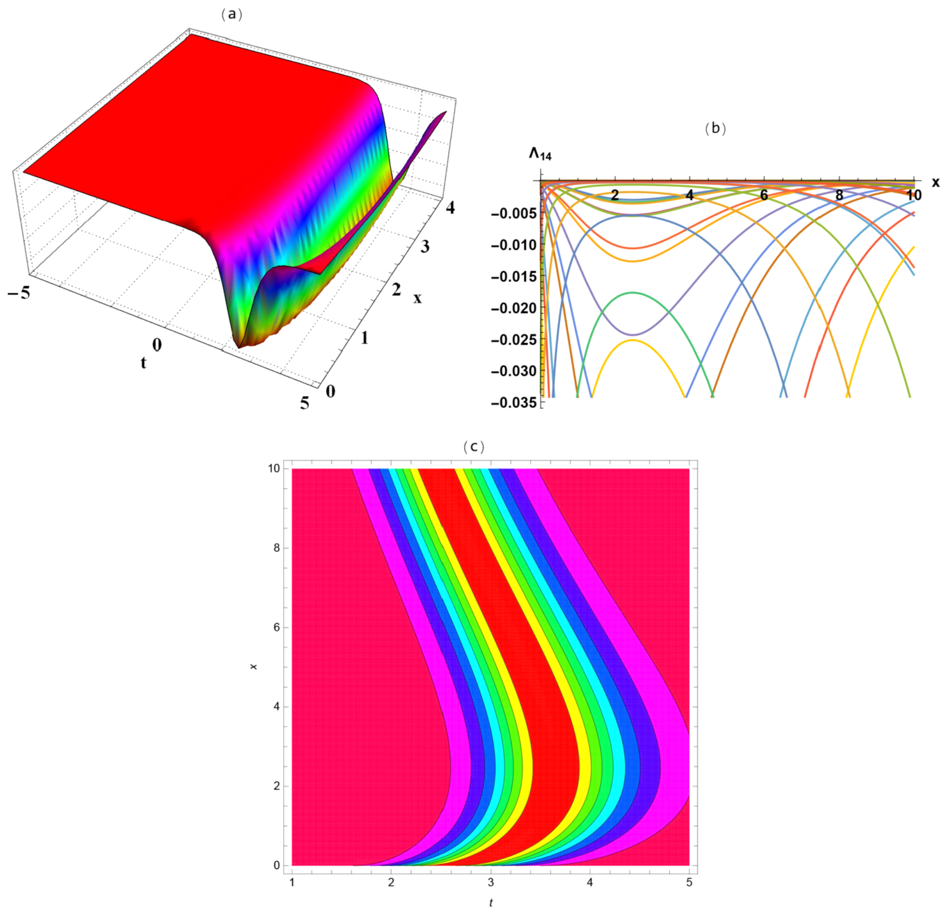

Figure 5 Dark wave solution of (34) in three (**a**), two-dimensional (**b**), and contour plots (**c**)

For all $t > 0$, $\alpha \in (0,1)$ if f is α-differentiable in some $(0,\alpha)$. $\alpha > 0$ and $\lim_{t\to 0} f^{\alpha}(t)$ exists, then define $f^{\alpha}(0) = \lim_{t\to 0} f^{\alpha}(t)$.

We sometimes write $f^{\alpha}(t)$ for $T_{\alpha}(f)(t)$ to denote the conformable fractional derivatives of f of order α. In addition, if the conformable fractional derivative of f of order α exists, then we say f is α-differentiable. We should take into consideration that $T_{\alpha}(t^p) = pt^{p-\alpha}$. Further, this definition coincides with the same of traditional definition of Riemann–Liouville and of Caputo on polynomials (up to a constant multiple).

The conformable fractional properties:

- $T_{\alpha}(e^{ct}) = ct^{1-\alpha}e^{ct}$.
- $T_{\alpha}(\sin(at)) = at^{1-\alpha}\cos(at)$.
- $T_{\alpha}\sin(at) = -at^{1-\alpha}\sin(at)$.
- $T_{\alpha}(\tan(at)) = at^{1-\alpha}\sec^2(at)$.
- $T_{\alpha}(\cot(at)) = -at^{1-\alpha}\csc^2(at)$.
- $T_{\alpha}(\sec(at)) = at^{1-\alpha}\sec(at)\tan(at)$.
- $T_{\alpha}(\csc(at)) = -at^{1-\alpha}\csc(at)\cot(at)$.
- $T_{\alpha}(\frac{t^{\alpha}}{\alpha}) = 1$.

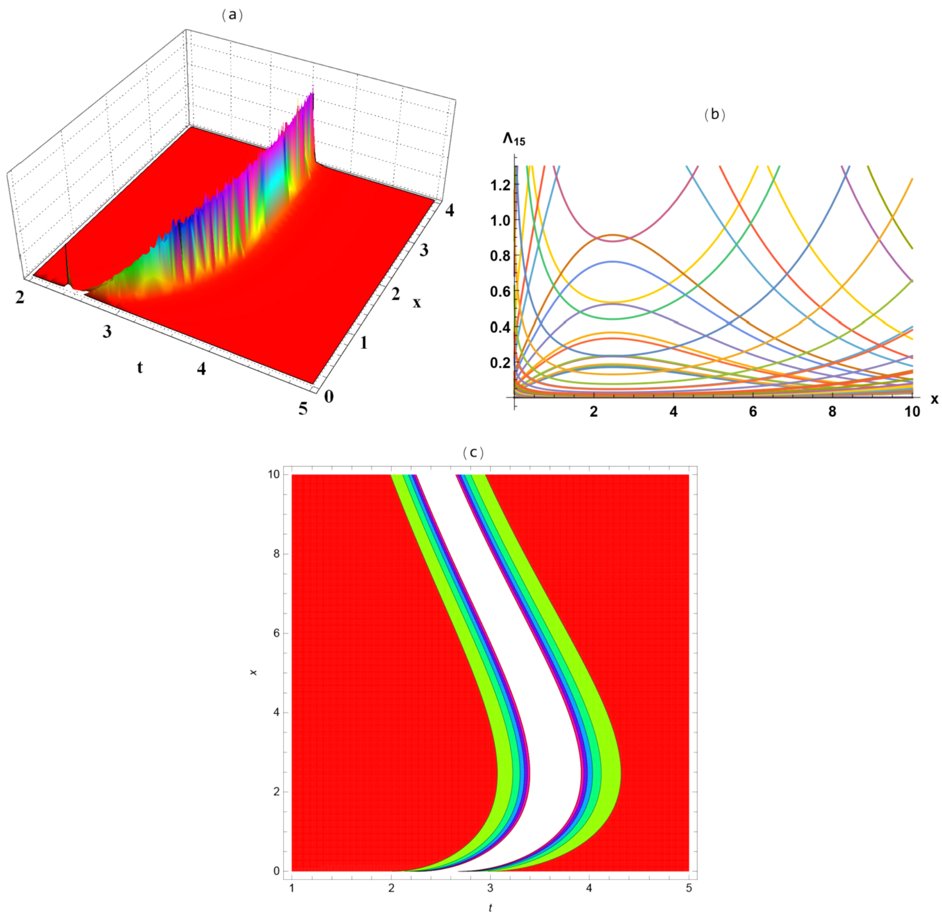

Figure 6 Bright wave solution of (35) in three (**a**), two-dimensional (**b**), and contour plots (**c**)

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Availability of data and materials

Not applicable.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

All authors conceived of the study, participated in its design and coordination, drafted the manuscript, participated in the sequence alignment, and read and approved the final manuscript.

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