RESEARCH   **Open Access**

Novel extended dissipativity criteria for generalized neural networks with interval discrete and distributed time-varying delays

Sunisa Luemsai[1], Thongchai Botmart[1] and Wajaree Weera[2*]

*Correspondence:
wajaree.we@up.ac.th
[2]Department of Mathematics,
University of Pha Yao, 56000 Pha
Yao, Thailand
Full list of author information is
available at the end of the article

Abstract

The problem of asymptotic stability and extended dissipativity analysis for the generalized neural networks with interval discrete and distributed time-varying delays is investigated. Based on a suitable Lyapunov–Krasovskii functional (LKF), an improved Wirtinger single integral inequality, a novel triple integral inequality, and convex combination technique, the new asymptotic stability and extended dissipativity criteria are achieved for the generalized neural networks with interval discrete and distributed time-varying delays. By the above methods, the less conservative asymptotic stability criteria are obtained for a special case of the generalized neural networks. By using the Matlab LMI toolbox, the derived new asymptotic stability and extended dissipativity criteria are expressed in terms of linear matrix inequalities (LMIs) that cover H_∞, L_2–L_∞, passivity, and dissipativity performance by setting parameters in the general performance index. Finally, we show numerical examples which are less conservative than other examples in the literature. Moreover, we present numerical examples for asymptotic stability and extended dissipativity performance of the generalized neural networks, including a special case of the generalized neural networks.

Keywords: Generalized neural networks; Extended dissipativity analysis; Asymptotic stability; Interval discrete and distributed time-varying delays

1 Introduction

In the numerous science and engineering fields, neural networks (NNs) have been studied extensively in recent years due to the wide range of their applications such as in signal processing, fault diagnosis, pattern recognition, associative memory, reproducing moving pictures, optimization problems, and industrial automation [1–5]. Theoretical stability analysis of the equilibrium is initially performed to make the above applications possible. To obtain the model for theoretical analysis, the important factors that affect the system are examined, and one important factor is the time delay. It is well known that the time delay always occurs in real world situations, and it causes oscillation, instability, and poor performance of the system. Furthermore, the time delay in neural networks is caused by the finite speed of information processing and the communication time of neurons. There-

Springer

fore, many researchers are interested in investigating the stability or performance of the neural networks with time delay. The problem of stability or performance analysis for neural networks with constant, discrete, and distributed time-varying delays has received much attention [6–10].

In addition, neural networks can be classified into two types, local-field neural networks (LFNNs) and static neural networks (SNNs), based on the different neuron states. Many studies have separated the stability or performance of LFNNs and SNNs. For example, Zeng et al. [11] investigated the stability and dissipativity problem of static neural networks with interval time-varying delay. In [6], the stability for local-field neural networks with time-varying interval was investigated and stability criteria were improved by using some new techniques. Moreover, in 2011 for the first time, Zhang and Han [12] combined them into a unified system model, called generalized neural networks (GNNs), which covers both SNNs and LFNNs models, using certain assumptions. And later the GNNs with time delay model have been extensively used for stability or performance analyses [13–18].

On the other hand, the performance of neural networks has been analyzed by a variety of techniques, which often have input and output relationships, and they play an important role in science and engineering applications. For example, Du et al. [19] studied the problem of robust reliable H_∞ control for neural networks with mixed time delays based on the LMI technique and the Lyapunov stability theory. In [20], the problem of finite-time non-fragile passivity control for neural networks with time-varying delay is investigated based on a new Lyapunov–Krasovskii function with tripple and quadruple integral terms and utilizing Wirtinger-type inequality technique. Passivity performance analysis for neural networks is examined in [21–24]. In [25], the issue of L_2–L_∞ state estimation design for delayed neural networks (NNs) is considered via quadratic-type generalized free-matrix-based integral inequality. The problem of dissipative analysis for aircraft flight control systems and uncertain discrete-time neural networks is addressed in [26, 27]. It is well known that the concept of dissipativity was first studied by Willems [28]. Also, many researchers have studied the dissipativity theory, since it does not only cover H_∞ and passivity performance, but is also advisable to be used in a convenient control structure in engineering applications such as chemical process control [29] and power converters [30]. Dissipativity theory is widely studied in neural networks because it provides a fundamental framework for the analysis and design problems of control systems, and it can keep the system internally stable. Recently, many researchers have studied the dissipativity for stochastic fuzzy neural networks, static neural networks, stochastic Markovian switching CVNNs, and so on [11, 31, 32]. However, the analysis of dissipativity does not relate to L_2–L_∞ performance. To fill this gap, Zhang et al. [33] created a new general performance index, called an extended dissipativity performance index, which links all of these performance indexes. Therefore, many problems have been studied to analyze extended dissipativity for neural networks with time delays, and the results have been reported in [34–36]. Moreover, the extended dissipative analysis was studied for the GNNs with interval time-varying delay signals [37]. It is interesting to study the extended dissipativity performance for GNNs with interval discrete and distributed time-varying delays, which has not been studied, yet.

The problem of asymptotic stability and extended dissipativity analysis for the generalized neural networks with interval discrete and distributed time-varying delays is investigated in this paper. The main contributions of this research are as follows:

● We construct more general systems named the generalized neural networks that cover both SNNs and LFNNs. Moreover, the interval discrete and distributed time-varying delays are not necessarily differentiable functions, the lower bound of the delays is not restricted to be 0, the activation functions are different, and the output contains terms of the state vector with interval discrete time-varying delay and the disturbance.

● We create a suitable Lyapunov–Krasovskii functional (LKF) for application in asymptotic stability and extended dissipativity analysis of the GNNs with new inequalities.

● For the first time, we use a novel triple integral inequality and an improved Wirtinger single integral inequality with convex combination technique for estimation to obtain the upper bound of the interval discrete time-varying delay that is better than in other references.

● We obtain new asymptotic stability and extended dissipativity criteria that cover H_∞, L_2-L_∞, passivity, and dissipativity performance by setting parameters in the general performance index.

This paper is structured in five sections as follows. In Sect. 2, the generalized neural networks model is formulated, and some definitions, lemmas, and assumptions are introduced. In Sect. 3, we show the asymptotic stability and extended dissipativity criteria for the generalized neural networks and a special case of the generalized neural networks. Numerical examples are shown in Sect. 4 to demonstrate the effectiveness of asymptotic stability and extended dissipativity performance for the generalized neural networks, including a special case of the generalized neural networks. Finally, conclusions are addressed in Sect. 5.

2 Network model and preliminaries

Notations   Throughout this paper, $\mathbb{R}$ and $\mathbb{R}^+$ represent the set of real numbers and the set of nonnegative real numbers, respectively; $\mathbb{R}^n$ and $\mathbb{R}^{n\times r}$ denote the n-dimensional Euclidean space and the set of $n \times r$ real matrices, respectively; I is the identity matrix with appropriate dimensions; $\mathcal{C}([-\varrho, 0], \mathbb{R}^n)$ represents the space of all continuous vector-valued functions mapping $[-\varrho, 0]$ into $\mathbb{R}^n$, where $\varrho \in \mathbb{R}^+$; $\mathcal{L}_2[0, \infty)$ denotes the space of functions $\phi : \mathbb{R}^+ \to \mathbb{R}^n$ with the norm $\|\phi\|_{\mathcal{L}_2} = [\int_0^\infty |\phi(\theta)|^2 \, d\theta]^{\frac{1}{2}}$; P^T is the transpose of the matrix P; $P = P^T$ denotes that the matrix P is a symmetric matrix; $P > (\geq)0$ means that the symmetric matrix P is positive (semi-positive) definite; $P < (\leq)0$ denotes that the symmetric matrix P is negative (semi-negative) definite; $\text{Sym}\{P\}$ represents $P + P^T$; e_i represents the unit column vector having 1 on its ith row and zeros elsewhere.

Consider the following generalized neural networks model with both interval discrete and distributed time-varying delays:

$$\dot{w}(t) = -Cw(t) + B_0 f\big(Ww(t)\big) + B_1 g\big(Ww\big(t - \delta(t)\big)\big)$$
$$+ B_2 \int_{t-\sigma_2(t)}^{t-\sigma_1(t)} h\big(Ww(s)\big) \, ds + B_3 u(t), \tag{1}$$
$$z(t) = D_1 w(t) + D_2 w\big(t - \delta(t)\big) + D_3 u(t),$$
$$w(t) = \phi(t), \quad \forall t \in [-\varrho, 0],$$

where $w(t) = [w_1(t), w_2(t), \ldots, w_n(t)]^T \in \mathbb{R}^n$ is the neuron state vector; $z(t) \in \mathbb{R}^n$ is the output vector; $u(t) \in \mathbb{R}^n$ is the deterministic disturbance input which belongs to $\mathcal{L}_2[0, \infty)$; $f(\cdot), g(\cdot), h(\cdot) \in \mathbb{R}^n$ are the neuron activation functions; $C = \text{diag}\{c_1, c_2, \ldots, c_n\}$ is a positive

diagonal matrix; B_0, B_1, B_2, and W are connection weight matrices; B_3, D_1, D_2, and D_3 are given real matrices; $\phi(t) \in \mathcal{C}[[-\varrho, 0], \mathbb{R}^n]$ is the initial function. The variables $\delta(t)$ and $\sigma_i(t)$ $(i = 1, 2)$ denote the interval discrete and distributed time-varying delays that satisfy $0 \leq \delta_1 \leq \delta(t) \leq \delta_2$, $0 \leq \sigma_1 \leq \sigma_1(t) \leq \sigma_2(t) \leq \sigma_2$ where δ_1, δ_2, σ_1, σ_2, $\varrho = \max\{\delta_2, \sigma_2\}$ are known real constants.

The neuron activation functions $f(\cdot)$, $g(\cdot)$, and $h(\cdot)$ satisfy the following conditions:

(A1) The neuron activation function f is continuous and there exist constants F_i^- and F_i^+ such that

$$F_i^- \leq \frac{f_i(\alpha_1) - f_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq F_i^+$$

for all $\alpha_1 \neq \alpha_2$; we also let $\tilde{F}_i = \max\{|F_i^-|, |F_i^+|\}$, where $f(\cdot) = [f_1(\cdot), f_2(\cdot), \ldots, f_n(\cdot)]^T$ and for any $i \in \{1, 2, \ldots, n\}$, $f_i(0) = 0$.

(A2) The neuron activation function g is continuous and there exist constants G_i^- and G_i^+ such that

$$G_i^- \leq \frac{g_i(\alpha_1) - g_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq G_i^+$$

for all $\alpha_1 \neq \alpha_2$; and we let $\tilde{G}_i = \max\{|G_i^-|, |G_i^+|\}$, where $g(\cdot) = [g_1(\cdot), g_2(\cdot), \ldots, g_n(\cdot)]^T$ and for any $i \in \{1, 2, \ldots, n\}$, $g_i(0) = 0$.

(A3) The neuron activation function h is continuous and there exist constants H_i^- and H_i^+ such that

$$H_i^- \leq \frac{h_i(\alpha_1) - h_i(\alpha_2)}{\alpha_1 - \alpha_2} \leq H_i^+$$

for all $\alpha_1 \neq \alpha_2$; and we let $\tilde{H}_i = \max\{|H_i^-|, |H_i^+|\}$, where $h(\cdot) = [h_1(\cdot), h_2(\cdot), \ldots, h_n(\cdot)]^T$ and for any $i \in \{1, 2, \ldots, n\}$, $h_i(0) = 0$.

Remark 1 The NNs model (1) provides a general form of delay NNs model, which covers both LFNNs and SNNs. It can be simply reduced to each of them by setting B_0, B_1, B_2, and W. That is, if we set $W = I$, the NNs model (1) leads to the following model, namely *LFNNs*:

$$\dot{w}(t) = -Cw(t) + B_0 f\big(w(t)\big) + B_1 g\big(w\big(t - \delta(t)\big)\big) + B_2 \int_{t-\sigma_2(t)}^{t-\sigma_1(t)} h\big(w(s)\big)\, ds$$

$$+ B_3 u(t),$$

$$z(t) = D_1 w(t) + D_2 w\big(t - \delta(t)\big) + D_3 u(t).$$

In the same way, if we set $B_0 = B_1 = B_2 = I$, the NNs model (1) is changed to the following model, namely *SNNs*:

$$\dot{w}(t) = -Cw(t) + f\big(Ww(t)\big) + g\big(Ww\big(t - \delta(t)\big)\big) + \int_{t-\sigma_2(t)}^{t-\sigma_1(t)} h\big(Ww(s)\big)\, ds$$

$$+ B_3 u(t),$$

$$z(t) = D_1 w(t) + D_2 w\big(t - \delta(t)\big) + D_3 u(t).$$

Before moving to the main results, we introduce the definitions, lemmas, and assumptions which are necessary to state the new criteria.

Assumption (H1) ([34]) For given real symmetric matrices $\Gamma_1 \leq 0$, $\Gamma_3, \Gamma_4 \geq 0$, and a real matrix Γ_2, the following conditions are satisfied:

(1) $\|D_3\| \cdot \|\Gamma_4\| = 0$;

(2) $(\|\Gamma_1\| + \|\Gamma_2\|) \cdot \|\Gamma_4\| = 0$;

(3) $D_3^T \Gamma_1 D_3 + D_3^T \Gamma_2 + \Gamma_2^T D_3 + \Gamma_3 > 0$.

Definition 2.1 ([34]) For given matrices Γ_1, Γ_2, Γ_3, and Γ_4 satisfying Assumption (H1), system (1) is said to be extended dissipative, if, under the zero initial condition, there exists a scalar λ such that the following inequality holds for any $t_f \geq 0$ and all $u(t) \in \mathcal{L}_2[0, \infty)$:

$$\int_0^{t_f} J(s)\,ds \geq \sup_{0 \leq t \leq t_f} z^T(t)\Gamma_4 z(t) + \lambda, \tag{2}$$

where

$$J(s) = z^T(s)\Gamma_1 z(s) + 2z^T(s)\Gamma_2 u(s) + u^T(s)\Gamma_3 u(s). \tag{3}$$

Remark 2 The inequality (2) implies that the new performance measure is more general by allowing to set the weighting matrices Γ_i, $i = 1, 2, 3, 4$, i.e.,

- If $\Gamma_1 = 0$, $\Gamma_2 = 0$, $\Gamma_3 = \gamma^2 I$, $\Gamma_4 = I$, and $\lambda = 0$ then the inequality (2) describes the $L_2 - L_\infty$ performance;
- If $\Gamma_1 = -I$, $\Gamma_2 = 0$, $\Gamma_3 = \gamma^2 I$, $\Gamma_4 = 0$, and $\lambda = 0$ then the inequality (2) determines the H_∞ performance;
- If $\Gamma_1 = 0$, $\Gamma_2 = I$, $\Gamma_3 = \gamma I$, $\Gamma_4 = 0$, and $\lambda = 0$ then the inequality (2) reduces to the passivity performance;
- If $\Gamma_1 = \mathcal{Q}$, $\Gamma_2 = \mathcal{S}$, $\Gamma_3 = \mathcal{R} - \gamma I$, $\Gamma_4 = 0$, and $\lambda = 0$ then the inequality (2) degenerates to the $(\mathcal{Q}, \mathcal{S}, \mathcal{R}) - \gamma$-dissipativity performance.

Lemma 2.2 ([38]) *For a given symmetric positive definite matrix $P \in \mathbb{R}^{n \times n}$, scalars t, a, and b satisfying $b \geq a \geq 0$, and a vector function $w : [t - b, t] \to \mathbb{R}^n$ such that the integrals involved are well defined, the following inequality holds*:

$$\frac{1}{2}(b^2 - a^2) \int_{-b}^{-a} \int_{t+\theta}^{t} w^T(s)Pw(s)\,ds\,d\theta \geq \int_{-b}^{-a} \int_{t+\theta}^{t} w^T(s)\,ds\,d\theta$$
$$\times P \int_{-b}^{-a} \int_{t+\theta}^{t} w(s)\,ds\,d\theta.$$

Lemma 2.3 ([39]) *For any constant matrices $P \in \mathbb{R}^{n \times n}$, $X \in \mathbb{R}^{2n \times 2n}$, and $Y \in \mathbb{R}^{2n \times n}$ with $\begin{bmatrix} X & Y \\ * & P \end{bmatrix} \geq 0$, and such that the following inequality is well defined, it holds that*

$$-\int_{-b}^{-a} \int_{\theta}^{0} \int_{t+\beta}^{t} \dot{w}^T(s)P\dot{w}(s)\,ds\,d\beta\,d\theta$$
$$\leq \Omega_1^T(t)\left[(b^2 - a^2)\,\mathrm{Sym}\{Y\Theta\} + \frac{b^3 - a^3}{6}X\right]\Omega_1(t),$$

where $\Theta = [I, -I]$ and $\Omega_1 = [w^T(t), \int_{-b}^{-a} \int_{t+\theta}^{t} \frac{2}{b^2 - a^2} w^T(s)\,ds\,d\theta]^T$.

Lemma 2.4 ([40]) *For a given matrix $P > 0$, the following inequality holds for all continuous differentiable function $w(t)$ in $[a, b] \in \mathbb{R}^n$:*

$$-\int_a^b \dot{w}^T(s)P\dot{w}(s)\, ds \leq -\frac{1}{b-a}\big(\Omega_2^T P\Omega_2 + 3\Omega_3^T P\Omega_3 + 5\Omega_4^T P\Omega_4 + 7\Omega_5^T P\Omega_5\big),$$

where

$$\Omega_2 = w(b) - w(a),$$

$$\Omega_3 = w(b) + w(a) - \frac{2}{b-a}\int_a^b w(s)\, ds,$$

$$\Omega_4 = w(b) - w(a) + \frac{6}{b-a}\int_a^b w(s)\, ds - \frac{12}{(b-a)^2}\int_a^b\int_u^b w(s)\, ds\, du,$$

$$\Omega_5 = w(b) + w(a) - \frac{12}{b-a}\int_a^b w(s)\, ds + \frac{60}{(b-a)^2}\int_a^b\int_u^b w(s)\, ds\, du$$

$$\qquad - \frac{120}{(b-a)^3}\int_a^b\int_u^b\int_s^b w(r)\, dr\, ds\, du.$$

Lemma 2.5 ([41]) *Suppose that $w(t) \in \mathbb{R}^n$ and $\eta \in \mathbb{R}$. Then for any positive definite matrix P, the following inequality holds*:

$$-\frac{\eta_2^3}{6}\int_{-\eta_2}^0\int_\beta^0\int_{t+\lambda}^t w^T(s)Pw(s)\, ds\, d\lambda\, d\beta$$

$$\leq -\int_{-\eta_2}^0\int_\beta^0\int_{t+\lambda}^t w^T(s)\, ds\, d\lambda\, d\beta\, P\int_{-\eta_2}^0\int_\beta^0\int_{t+\lambda}^t w(s)\, ds\, d\lambda\, d\beta.$$

3　Main results

In what follows, for the simplification, some notations are introduced as:

$$G_1 = e_1 - e_3, \qquad G_2 = e_1 + e_3 - 2e_8, \qquad G_3 = e_1 - e_3 + 6e_8 - 12e_{14},$$

$$G_4 = e_1 + e_3 - 12e_8 + 60e_{14} - 120e_{20}, \qquad G_5 = e_1 - e_4,$$

$$G_6 = e_1 + e_4 - 2e_9, \qquad G_7 = e_1 - e_4 + 6e_9 - 12e_{15},$$

$$G_8 = e_1 + e_4 - 12e_9 + 60e_{15} - 120e_{21}, \qquad G_9 = e_5 - e_4,$$

$$G_{10} = e_5 + e_4 - 2e_{11}, \qquad G_{11} = e_5 - e_4 + 6e_{11} - 12e_{16},$$

$$G_{12} = e_5 + e_4 - 12e_{11} + 60e_{16} - 120e_{22}, \qquad G_{13} = e_3 - e_5,$$

$$G_{14} = e_3 + e_5 - 2e_{10}, \qquad G_{15} = e_3 - e_5 + 6e_{10} - 12e_{17},$$

$$G_{16} = e_3 + e_5 - 12e_{10} + 60e_{17} - 120e_{23}, \qquad G_{17} = \begin{bmatrix} e_1 & 2e_{12} + 2e_{13} \end{bmatrix},$$

$$G_{18} = e_1 W^T F_p^T - e_6, \qquad G_{19} = e_6 - F_m We_1, \qquad G_{20} = e_5 W^T G_p^T - e_7,$$

$$G_{21} = e_7 - G_m We_5, \qquad G_{22} = e_1 W^T H_p^T - e_{24}, \qquad G_{23} = e_{24} - H_m We_1,$$

$$\bar{Y} = \begin{bmatrix} Y_1 + Y_1^T & -Y_1 + Y_2^T \\ * & -Y_2 - Y_2^T \end{bmatrix}, \qquad \bar{X} = \begin{bmatrix} X_1 + X_1^T & X_2 + X_3^T \\ * & X_4 + X_4^T \end{bmatrix},$$

$$F_p = \text{diag}\{F_1^+, F_2^+, \ldots, F_n^+\}, \qquad F_m = \text{diag}\{F_1^-, F_2^-, \ldots, F_n^-\},$$

$$G_p = \text{diag}\{G_1^+, G_2^+, \ldots, G_n^+\}, \qquad G_m = \text{diag}\{G_1^-, G_2^-, \ldots, G_n^-\},$$

$$H_p = \text{diag}\{H_1^+, H_2^+, \ldots, H_n^+\}, \qquad H_m = \text{diag}\{H_1^-, H_2^-, \ldots, H_n^-\},$$

$$Z_1 = \text{diag}\{z_{11}, z_{12}, \ldots, z_{1n}\} \geq 0, \qquad Z_2 = \text{diag}\{z_{21}, z_{22}, \ldots, z_{2n}\} \geq 0,$$

$$Z_3 = \text{diag}\{z_{31}, z_{32}, \ldots, z_{3n}\} \geq 0,$$

$$\begin{aligned}
\varsigma^T(t) = \Bigg[& w^T(t), \dot{w}^T(t), w^T(t - \delta_1), w^T(t - \delta_2), w^T(t - \delta(t)), f^T(Ww(t)), \\
& g^T(Ww(t - \delta(t))), \frac{1}{\delta_1} \int_{t-\delta_1}^{t} w^T(s)\, ds, \frac{1}{\delta_2} \int_{t-\delta_2}^{t} w^T(s)\, ds, \\
& \frac{1}{\delta(t) - \delta_1} \int_{t-\delta(t)}^{t-\delta_1} w^T(s)\, ds, \frac{1}{\delta_2 - \delta(t)} \int_{t-\delta_2}^{t-\delta(t)} w^T(s)\, ds, \\
& \frac{1}{\delta_2^2 - \delta_1^2} \int_{-\delta(t)}^{-\delta_1} \int_{t+\theta}^{t} w^T(s)\, ds\, d\theta, \frac{1}{\delta_2^2 - \delta_1^2} \int_{-\delta_2}^{-\delta(t)} \int_{t+\theta}^{t} w^T(s)\, ds\, d\theta, \\
& \frac{1}{\delta_1^2} \int_{t-\delta_1}^{t} \int_{\theta}^{t} w^T(s)\, ds\, d\theta, \frac{1}{\delta_2^2} \int_{t-\delta_2}^{t} \int_{\theta}^{t} w^T(s)\, ds\, d\theta, \\
& \frac{1}{(\delta_2 - \delta(t))^2} \int_{t-\delta_2}^{t-\delta(t)} \int_{\theta}^{t-\delta(t)} w^T(s)\, ds\, d\theta, \\
& \frac{1}{(\delta(t) - \delta_1)^2} \int_{t-\delta(t)}^{t-\delta_1} \int_{\theta}^{t-\delta_1} w^T(s)\, ds\, d\theta, \int_{-\delta_2}^{-\delta(t)} \int_{\beta}^{0} \int_{t+\lambda}^{t} w^T(s)\, ds\, d\lambda\, d\beta, \\
& \int_{-\delta(t)}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} w^T(s)\, ds\, d\lambda\, d\beta, \frac{1}{\delta_1^3} \int_{t-\delta_1}^{t} \int_{u}^{t} \int_{s}^{t} w^T(r)\, dr\, ds\, du, \\
& \frac{1}{\delta_2^3} \int_{t-\delta_2}^{t} \int_{u}^{t} \int_{s}^{t} w^T(r)\, dr\, ds\, du, \\
& \frac{1}{(\delta_2 - \delta(t))^3} \int_{t-\delta_2}^{t-\delta(t)} \int_{u}^{t-\delta(t)} \int_{s}^{t-\delta(t)} w^T(r)\, dr\, ds\, du, \\
& \frac{1}{(\delta(t) - \delta_1)^3} \int_{t-\delta(t)}^{t-\delta_1} \int_{u}^{t-\delta_1} \int_{s}^{t-\delta_1} w^T(r)\, dr\, ds\, du \Bigg],
\end{aligned}$$

$$\bar{\varsigma}^T(t) = \left[\varsigma^T(t), u^T(t) \right],$$

$$\eta^T(t) = \left[\varsigma^T(t), h^T(Ww(t)), \int_{t-\sigma_2(t)}^{t-\sigma_1(t)} h^T(Ww(s))\, ds \right],$$

$$\bar{\eta}^T(t) = \left[\eta^T(t), u^T(t) \right].$$

3.1 Stability analysis for generalized neural networks

In this section, new asymptotic stability criteria for the generalized neural networks (1), and their special case, are obtained based on a suitable Lyapunov–Krasovskii functional (LKF), an improved Wirtinger single integral inequality, a novel triple integral inequality, and convex combination technique.

Theorem 3.1 *For given scalars δ_1, δ_2, σ_1, σ_2, β_1, and β_2, if there exist symmetric positive definite matrices $P, U_1, U_2, T_1, T_2, T_3, L, S_1, S_2, Q \in \mathbb{R}^{n \times n}$, positive definite matrices $N_1, N_2 \in$*

$\mathbb{R}^{n\times n}$, *positive diagonal matrices* $Z_1, Z_2, Z_3 \in \mathbb{R}^{n\times n}$, *any matrices* $X_1, X_2, X_3, X_4, Y_1, Y_2 \in \mathbb{R}^{n\times n}$, *and positive scalars* c_1, c_2 *such that the following LMIs hold*:

$$\Xi + 2\Xi_i + 2c_1 I < 0, \quad i = 1, 2, \tag{4}$$

$$\Xi + 2\Xi_j - 2c_2 I < 0, \quad j = 3, 4, \tag{5}$$

$$c_1 - c_2 > 0, \tag{6}$$

$$\begin{bmatrix} X_1 + X_1^T & X_2 + X_3^T & Y_1 \\ * & X_4 + X_4^T & Y_2 \\ * & * & S_1 \end{bmatrix} \geq 0, \tag{7}$$

where

$$\begin{aligned}
\Xi = {}& 2e_1 P e_2^T + e_1 U_1 e_1^T - e_3 U_1 e_3^T + e_1 U_2 e_1^T - e_4 U_2 e_4^T + \delta_1^2 e_2 T_1 e_2^T - G_1 T_1 G_1^T \\
& - 3 G_2 T_1 G_2^T - 5 G_3 T_1 G_3^T - 7 G_4 T_1 G_4^T + \delta_2^2 e_2 T_2 e_2^T - G_5 T_2 G_5^T - 3 G_6 T_2 G_6^T \\
& - 5 G_7 T_2 G_7^T - 7 G_8 T_2 G_8^T + (\delta_2 - \delta_1)^2 e_2 T_3 e_2^T - G_9 T_3 G_9^T - 3 G_{10} T_3 G_{10}^T \\
& - 5 G_{11} T_3 G_{11}^T - 7 G_{12} T_3 G_{12}^T - G_{13} T_3 G_{13}^T - 3 G_{14} T_3 G_{14}^T - 5 G_{15} T_3 G_{15}^T \\
& - 7 G_{16} T_3 G_{16}^T + \frac{(\delta_2^2 - \delta_1^2)^2}{4} e_1 L e_1^T - (\delta_2^2 - \delta_1^2)^2 e_{13} L e_{13}^T - (\delta_2^2 - \delta_1^2)^2 e_{12} L e_{12}^T \\
& + \frac{(\delta_2^3 - \delta_1^3)^2}{36} e_2 S_1 e_2^T + G_{17}\left((\delta_2^2 - \delta_1^2)\bar{Y} + \frac{\delta_2^3 - \delta_1^3}{6}\bar{X} \right) G_{17}^T \\
& + \frac{(\delta_2^3 - \delta_1^3)^2}{36} e_1 S_2 e_1^T - e_{18} S_2 e_{18}^T - e_{19} S_2 e_{19}^T + 2 G_{18} Z_1 G_{19}^T + 2 G_{20} Z_2 G_{21}^T \\
& + (\sigma_2 - \sigma_1)^2 e_{24} Q e_{24}^T - e_{25} Q e_{25}^T + 2 G_{22} Z_3 G_{23}^T \\
& - 2 e_1 \beta_1 N_1^T e_2^T - 2 e_1 \beta_1 N_1^T C e_1^T + 2 e_1 \beta_1 N_1^T B_0 e_6^T + 2 e_1 \beta_1 N_1^T B_1 e_7^T \\
& + 2 e_1 \beta_1 N_1^T B_2 e_{25}^T - 2 e_2 \beta_2 N_2^T e_2^T - 2 e_2 \beta_2 N_2^T C e_1^T \\
& + 2 e_2 \beta_2 N_2^T B_0 e_6^T + 2 e_2 \beta_2 N_2^T B_1 e_7^T + 2 e_2 \beta_2 N_2^T B_2 e_{25}^T,
\end{aligned}$$

$$\Xi_1 = -(\delta_2^2 - \delta_1^2)^2 e_{13} L e_{13}^T,$$

$$\Xi_2 = -(\delta_2^2 - \delta_1^2)^2 e_{12} L e_{12}^T,$$

$$\Xi_3 = -e_{18} S_2 e_{18}^T,$$

$$\Xi_4 = -e_{19} S_2 e_{19}^T,$$

then, the system (1) *with* $u(t) = 0$ *is asymptotically stable.*

Proof We consider the following Lyapunov–Krasovskii functional candidate for the system (1):

$$V(w(t), t) = \sum_{i=1}^{10} V_i(w(t), t), \tag{8}$$

where

$$V_1\big(w(t),t\big) = w^T(t)Pw(t),$$

$$V_2\big(w(t),t\big) = \int_{t-\delta_1}^t w^T(s)U_1 w(s)\, ds,$$

$$V_3\big(w(t),t\big) = \int_{t-\delta_2}^t w^T(s)U_2 w(s)\, ds,$$

$$V_4\big(w(t),t\big) = \delta_1 \int_{-\delta_1}^0 \int_{t+s}^t \dot{w}^T(\tau)T_1 \dot{w}(\tau)\, d\tau\, ds,$$

$$V_5\big(w(t),t\big) = \delta_2 \int_{-\delta_2}^0 \int_{t+s}^t \dot{w}^T(\tau)T_2 \dot{w}(\tau)\, d\tau\, ds,$$

$$V_6\big(w(t),t\big) = (\delta_2 - \delta_1) \int_{-\delta_2}^{-\delta_1} \int_{t+s}^t \dot{w}^T(\tau)T_3 \dot{w}(\tau)\, d\tau\, ds,$$

$$V_7\big(w(t),t\big) = \frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^0 \int_{t+\lambda}^t w^T(s)Lw(s)\, ds\, d\lambda\, d\beta,$$

$$V_8\big(w(t),t\big) = \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^0 \int_{\lambda}^0 \int_{t+\varphi}^t \dot{w}^T(s)S_1 \dot{w}(s)\, ds\, d\varphi\, d\lambda\, d\beta,$$

$$V_9\big(w(t),t\big) = \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^0 \int_{\lambda}^0 \int_{t+\varphi}^t w^T(s)S_2 w(s)\, ds\, d\varphi\, d\lambda\, d\beta,$$

$$V_{10}\big(w(t),t\big) = (\sigma_2 - \sigma_1) \int_{-\sigma_2}^{-\sigma_1} \int_{t+s}^t h^T\big(Ww(r)\big)Qh\big(Ww(r)\big)\, dr\, ds.$$

Time derivatives of $V_i(w(t),t)$, $i = 1, 2, \ldots, 10$, along the trajectories of (1) are as follows:

$$\dot{V}_1\big(w(t),t\big) = w^T(t)P\dot{w}(t) + \dot{w}^T(t)Pw(t), \tag{9}$$

$$\dot{V}_2\big(w(t),t\big) = w^T(t)U_1 w(t) - w^T(t - \delta_1)U_1 w(t - \delta_1), \tag{10}$$

$$\dot{V}_3\big(w(t),t\big) = w^T(t)U_2 w(t) - w^T(t - \delta_2)U_2 w(t - \delta_2), \tag{11}$$

$$\dot{V}_4\big(w(t),t\big) = \delta_1 \int_{-\delta_1}^0 \big[\dot{w}^T(t)T_1 \dot{w}(t) - \dot{w}^T(t + s)T_1 \dot{w}(t + s)\big]\, ds$$

$$= \delta_1^2 \dot{w}^T(t)T_1 \dot{w}(t) - \delta_1 \int_{t-\delta_1}^t \dot{w}^T(\alpha)T_1 \dot{w}(\alpha)\, d\alpha, \tag{12}$$

$$\dot{V}_5\big(w(t),t\big) = \delta_2 \int_{-\delta_2}^0 \big[\dot{w}^T(t)T_2 \dot{w}(t) - \dot{w}^T(t + s)T_2 \dot{w}(t + s)\big]\, ds$$

$$= \delta_2^2 \dot{w}^T(t)T_2 \dot{w}(t) - \delta_2 \int_{t-\delta_2}^t \dot{w}^T(\alpha)T_2 \dot{w}(\alpha)\, d\alpha, \tag{13}$$

$$\dot{V}_6\big(w(t),t\big) = (\delta_2 - \delta_1) \int_{-\delta_2}^{-\delta_1} \big[\dot{w}^T(t)T_3 \dot{w}(t) - \dot{w}^T(t + s)T_3 \dot{w}(t + s)\big]\, ds$$

$$= (\delta_2 - \delta_1)^2 \dot{w}^T(t)T_3 \dot{w}(t) - (\delta_2 - \delta_1) \int_{t-\delta_2}^{t-\delta_1} \dot{w}^T(\alpha)T_3 \dot{w}(\alpha)\, d\alpha, \tag{14}$$

$$\dot{V}_7\big(w(t),t\big) = \frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \big[w^T(t)Lw(t) - w^T(t+\lambda)Lw(t+\lambda)\big]\, d\lambda\, d\beta$$

$$= \frac{(\delta_2^2 - \delta_1^2)^2}{4} w^T(t)Lw(t)$$

$$- \frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{t+\beta}^{t} w^T(s)Lw(s)\, ds\, d\beta, \tag{15}$$

$$\dot{V}_8\big(w(t),t\big) = \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{\lambda}^{0} \big[\dot{w}^T(t)S_1\dot{w}(t)$$

$$- \dot{w}^T(t+\varphi)S_1\dot{w}(t+\varphi)\big]\, d\varphi\, d\lambda\, d\beta$$

$$= \frac{(\delta_2^3 - \delta_1^3)^2}{36} \dot{w}^T(t)S_1\dot{w}(t)$$

$$- \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} \dot{w}^T(s)S_1\dot{w}(s)\, ds\, d\lambda\, d\beta, \tag{16}$$

$$\dot{V}_9\big(w(t),t\big) = \frac{(\delta_2^3 - \delta_1^3)^2}{36} w^T(t)S_2w(t)$$

$$- \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} w^T(s)S_2w(s)\, ds\, d\lambda\, d\beta, \tag{17}$$

$$\dot{V}_{10}\big(w(t),t\big) = (\sigma_2 - \sigma_1)^2 h^T\big(Ww(t)\big)Qh\big(Ww(t)\big)$$

$$- (\sigma_2 - \sigma_1) \int_{t-\sigma_2}^{t-\sigma_1} h^T\big(Ww(r)\big)Qh\big(Ww(r)\big)\, dr$$

$$\leq (\sigma_2 - \sigma_1)^2 h^T\big(Ww(t)\big)Qh\big(Ww(t)\big)$$

$$- \big(\sigma_2(t) - \sigma_1(t)\big) \int_{t-\sigma_2(t)}^{t-\sigma_1(t)} h^T\big(Ww(r)\big)Qh\big(Ww(r)\big)\, dr$$

$$\leq (\sigma_2 - \sigma_1)^2 \eta^T(t)e_{24}Qe_{24}^T\eta(t) - \eta^T(t)e_{25}Qe_{25}^T\eta(t). \tag{18}$$

Utilizing Lemma 2.4, the following relations are easily obtained:

$$-\delta_1 \int_{t-\delta_1}^{t} \dot{w}^T(\alpha)T_1\dot{w}(\alpha)\, d\alpha$$

$$\leq -\eta^T(t)(e_1 - e_3)T_1(e_1 - e_3)^T\eta(t)$$

$$- 3\eta^T(t)(e_1 + e_3 - 2e_8)T_1(e_1 + e_3 - 2e_8)^T\eta(t)$$

$$- 5\eta^T(t)(e_1 - e_3 + 6e_8 - 12e_{14})T_1(e_1 - e_3 + 6e_8 - 12e_{14})^T\eta(t)$$

$$- 7\eta^T(t)(e_1 + e_3 - 12e_8 + 60e_{14} - 120e_{20})$$

$$\times T_1(e_1 + e_3 - 12e_8 + 60e_{14} - 120e_{20})^T\eta(t), \tag{19}$$

$$-\delta_2 \int_{t-\delta_2}^{t} \dot{w}^T(\alpha)T_2\dot{w}(\alpha)\, d\alpha$$

$$\leq -\eta^T(t)(e_1 - e_4)T_2(e_1 - e_4)^T\eta(t)$$

$$- 3\eta^T(t)(e_1 + e_4 - 2e_9)T_2(e_1 + e_4 - 2e_9)^T\eta(t)$$

$$- 5\eta^T(t)(e_1 - e_4 + 6e_9 - 12e_{15})T_2(e_1 - e_4 + 6e_9 - 12e_{15})^T\eta(t)$$

$$- 7\eta^T(t)(e_1 + e_4 - 12e_9 + 60e_{15} - 120e_{21})$$

$$\times T_2(e_1 + e_4 - 12e_9 + 60e_{15} - 120e_{21})^T \eta(t), \tag{20}$$

$$-(\delta_2 - \delta_1) \int_{t-\delta_2}^{t-\delta_1} \dot{w}^T(\alpha) T_3 \dot{w}(\alpha)\, d\alpha$$

$$= -(\delta_2 - \delta_1) \int_{t-\delta_2}^{t-\delta(t)} \dot{w}^T(\alpha) T_3 \dot{w}(\alpha)\, d\alpha - (\delta_2 - \delta_1) \int_{t-\delta(t)}^{t-\delta_1} \dot{w}^T(\alpha) T_3 \dot{w}(\alpha)\, d\alpha$$

$$\leq -\eta^T(t)(e_5 - e_4) T_3 (e_5 - e_4)^T \eta(t)$$

$$- 3\eta^T(t)(e_5 + e_4 - 2e_{11}) T_3 (e_5 + e_4 - 2e_{11})^T \eta(t)$$

$$- 5\eta^T(t)(e_5 - e_4 + 6e_{11} - 12e_{16}) T_3 (e_5 - e_4 + 6e_{11} - 12e_{16})^T \eta(t)$$

$$- 7\eta^T(t)(e_5 + e_4 - 12e_{11} + 60e_{16} - 120e_{22})$$

$$\times T_3(e_5 + e_4 - 12e_{11} + 60e_{16} - 120e_{22})^T \eta(t)$$

$$- \eta^T(t)(e_3 - e_5) T_3 (e_3 - e_5)^T \eta(t)$$

$$- 3\eta^T(t)(e_3 + e_5 - 2e_{10}) T_3 (e_3 + e_5 - 2e_{10})^T \eta(t)$$

$$- 5\eta^T(t)(e_3 - e_5 + 6e_{10} - 12e_{17}) T_3 (e_3 - e_5 + 6e_{10} - 12e_{17})^T \eta(t)$$

$$- 7\eta^T(t)(e_3 + e_5 - 12e_{10} + 60e_{17} - 120e_{23})$$

$$\times T_3(e_3 + e_5 - 12e_{10} + 60e_{17} - 120e_{23})^T \eta(t). \tag{21}$$

On the other hand, we have the following inequality from Lemma 2.2:

$$-\frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{t+\beta}^{t} w^T(s) L w(s)\, ds\, d\beta$$

$$\leq -(\delta_2^2 - \delta_1^2)^2 \eta^T(t) e_{13} L e_{13}^T \eta(t) - \varepsilon(\delta_2^2 - \delta_1^2)^2 \eta^T(t) e_{13} L e_{13}^T \eta(t)$$

$$- (\delta_2^2 - \delta_1^2)^2 \eta^T(t) e_{12} L e_{12}^T \eta(t) - (1 - \varepsilon)(\delta_2^2 - \delta_1^2)^2 \eta^T(t) e_{12} L e_{12}^T \eta(t), \tag{22}$$

where $\varepsilon = \frac{\delta^2(t) - \delta_1^2}{\delta_2^2 - \delta_1^2}$.

From Lemma 2.3 and condition (7), we obtain

$$-\frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} \dot{w}^T(s) S_1 \dot{w}(s)\, ds\, d\lambda\, d\beta$$

$$\leq \eta^T(t) \begin{bmatrix} e_1 & 2e_{12} + 2e_{13} \end{bmatrix}$$

$$\times \left((\delta_2^2 - \delta_1^2) \bar{Y} + \frac{\delta_2^3 - \delta_1^3}{6} \bar{X} \right) \begin{bmatrix} e_1 & 2e_{12} + 2e_{13} \end{bmatrix}^T \eta(t). \tag{23}$$

From Lemma 2.5, we gain

$$-\frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} w^T(s) S_2 w(s)\, ds\, d\lambda\, d\beta$$

$$\leq -\eta^T(t) e_{18} S_2 e_{18}^T \eta(t) - \alpha \eta^T(t) e_{18} S_2 e_{18}^T \eta(t)$$

$$- \eta^T(t) e_{19} S_2 e_{19}^T \eta(t) - (1 - \alpha) \eta^T(t) e_{19} S_2 e_{19}^T \eta(t), \tag{24}$$

where $\alpha = \frac{\delta^3(t) - \delta_1^3}{\delta_2^3 - \delta_1^3}$.

It follows from Assumptions **(A1)**, **(A2)**, and **(A3)** that

$$2\big(F_p Ww(t) - f\big(Ww(t)\big)\big)^T Z_1\big(f\big(Ww(t)\big) - F_m Ww(t)\big) \ge 0, \tag{25}$$

$$2\big(G_p Ww\big(t - \delta(t)\big) - g\big(Ww\big(t - \delta(t)\big)\big)\big)^T$$
$$\times Z_2\big(g\big(Ww\big(t - \delta(t)\big)\big) - G_m Ww\big(t - \delta(t)\big)\big) \ge 0, \tag{26}$$

$$2\big(H_p Ww(t) - h\big(Ww(t)\big)\big)^T Z_3\big(h\big(Ww(t)\big) - H_m Ww(t)\big) \ge 0. \tag{27}$$

We consider system (1), the following equation is obtained:

$$0 = 2\big[w^T(t)\beta_1 N_1^T + \dot{w}^T(t)\beta_2 N_2^T\big]\bigg[-\dot{w}(t) - Cw(t) + B_0 f\big(Ww(t)\big)$$
$$+ B_1 g\big(Ww\big(t - \delta(t)\big)\big) + B_2 \int_{t-\sigma_2(t)}^{t-\sigma_1(t)} h\big(Ww(s)\big)\,ds + B_3 u(t)\bigg]. \tag{28}$$

By adding the right-hand side of (28) to $\dot{V}(w(t), t)$, we achieve from (9)–(27) that

$$\dot{V}\big(w(t), t\big) \le \bar{\eta}^T(t)\big(\varepsilon \bar{\Xi}^{(1)} + (1-\varepsilon)\bar{\Xi}^{(2)} + \alpha\bar{\Xi}^{(3)} + (1-\alpha)\bar{\Xi}^{(4)}\big)\bar{\eta}(t), \tag{29}$$

where $\bar{\Xi}^{(i)} = \frac{1}{2}\bar{\Xi} + \Xi_i$ $(i = 1, 2)$ and $\bar{\Xi}^{(j)} = \frac{1}{2}\bar{\Xi} + \Xi_j$ $(j = 3, 4)$, $\bar{\Xi} = \Xi + 2e_{26}\beta_1 B_3^T N_1 e_1^T + 2e_{26}\beta_2 B_3^T N_2 e_2^T$, with Ξ and Ξ_i, Ξ_j defined in (4) and (5).

When $u(t) = 0$ (no disturbance), one has from (29) that

$$\dot{V}\big(w(t), t\big) \le \eta^T(t)\big(\varepsilon \Xi^{(1)} + (1-\varepsilon)\Xi^{(2)} + \alpha\Xi^{(3)} + (1-\alpha)\Xi^{(4)}\big)\eta(t),$$

where $\Xi^{(i)} = \frac{1}{2}\Xi + \Xi_i$ $(i = 1, 2)$ and $\Xi^{(j)} = \frac{1}{2}\Xi + \Xi_j$ $(j = 3, 4)$.

The upper bound of $\dot{V}(w(t), t)$ is negative if the condition (6) and the following relations hold simultaneously:

$$\varepsilon \Xi^{(1)} + (1-\varepsilon)\Xi^{(2)} < -c_1 I,$$
$$\alpha\Xi^{(3)} + (1-\alpha)\Xi^{(4)} < c_2 I.$$

The above relations can be rewritten as follows:

$$\varepsilon\big(\Xi^{(1)} + c_1 I\big) + (1-\varepsilon)\big(\Xi^{(2)} + c_1 I\big) < 0, \tag{30}$$

$$\alpha\big(\Xi^{(3)} - c_2 I\big) + (1-\alpha)\big(\Xi^{(4)} - c_2 I\big) < 0. \tag{31}$$

Since $0 \le \varepsilon, \alpha \le 1$, the term $\varepsilon(\Xi^{(1)} + c_1 I) + (1-\varepsilon)(\Xi^{(2)} + c_1 I)$ is a convex combination of $\Xi^{(1)} + c_1 I$ and $\Xi^{(2)} + c_1 I$; and the expression $\alpha(\Xi^{(3)} - c_2 I) + (1-\alpha)(\Xi^{(4)} - c_2 I)$ is a convex combination of $\Xi^{(3)} - c_2 I$ and $\Xi^{(4)} - c_2 I$. These combinations are negative definite only if the vertices become negative; therefore, (30) and (31) are equivalent to (4) and (5), respectively. Then, the system (1) with $u(t) = 0$ is asymptotically stable.   □

Next, we consider the following generalized neural network model as a special case of the system (1):

$$\dot{w}(t) = -Cw(t) + B_0 f\big(Ww(t)\big) + B_1 g\big(Ww(t - \delta(t))\big) + B_3 u(t),$$

$$z(t) = D_1 w(t). \tag{32}$$

Theorem 3.2 *For given scalars δ_1, δ_2, β_1, and β_2, if there exist symmetric positive definite matrices $P, U_1, U_2, T_1, T_2, T_3, L, S_1, S_2 \in \mathbb{R}^{n \times n}$, positive definite matrices $N_1, N_2 \in \mathbb{R}^{n \times n}$, positive diagonal matrices $Z_1, Z_2 \in \mathbb{R}^{n \times n}$, any matrices $X_1, X_2, X_3, X_4, Y_1, Y_2 \in \mathbb{R}^{n \times n}$, and positive scalars b_1, b_2 such that the following LMIs hold:*

$$\Theta + 2\Theta_i + 2b_1 I < 0, \quad i = 1, 2, \tag{33}$$

$$\Theta + 2\Theta_j - 2b_2 I < 0, \quad j = 3, 4, \tag{34}$$

$$b_1 - b_2 > 0, \tag{35}$$

$$\begin{bmatrix} X_1 + X_1^T & X_2 + X_3^T & Y_1 \\ * & X_4 + X_4^T & Y_2 \\ * & * & S_1 \end{bmatrix} \geq 0, \tag{36}$$

where

$$\begin{aligned}
\Theta &= 2e_1 P e_2^T + e_1 U_1 e_1^T - e_3 U_1 e_3^T + e_1 U_2 e_1^T - e_4 U_2 e_4^T + \delta_1^2 e_2 T_1 e_2^T - G_1 T_1 G_1^T \\
&\quad - 3 G_2 T_1 G_2^T - 5 G_3 T_1 G_3^T - 7 G_4 T_1 G_4^T + \delta_2^2 e_2 T_2 e_2^T - G_5 T_2 G_5^T - 3 G_6 T_2 G_6^T \\
&\quad - 5 G_7 T_2 G_7^T - 7 G_8 T_2 G_8^T + (\delta_2 - \delta_1)^2 e_2 T_3 e_2^T - G_9 T_3 G_9^T - 3 G_{10} T_3 G_{10}^T \\
&\quad - 5 G_{11} T_3 G_{11}^T - 7 G_{12} T_3 G_{12}^T - G_{13} T_3 G_{13}^T - 3 G_{14} T_3 G_{14}^T - 5 G_{15} T_3 G_{15}^T \\
&\quad - 7 G_{16} T_3 G_{16}^T + \frac{(\delta_2^2 - \delta_1^2)^2}{4} e_1 L e_1^T - (\delta_2^2 - \delta_1^2)^2 e_{13} L e_{13}^T - (\delta_2^2 - \delta_1^2)^2 e_{12} L e_{12}^T \\
&\quad + \frac{(\delta_2^3 - \delta_1^3)^2}{36} e_2 S_1 e_2^T + G_{17}\left((\delta_2^2 - \delta_1^2) \bar{Y} + \frac{\delta_2^3 - \delta_1^3}{6} \bar{X} \right) G_{17}^T \\
&\quad + \frac{(\delta_2^3 - \delta_1^3)^2}{36} e_1 S_2 e_1^T - e_{18} S_2 e_{18}^T - e_{19} S_2 e_{19}^T + 2 G_{18} Z_1 G_{19}^T \\
&\quad + 2 G_{20} Z_2 G_{21}^T - 2 e_1 \beta_1 N_1^T e_2^T - 2 e_1 \beta_1 N_1^T C e_1^T \\
&\quad + 2 e_1 \beta_1 N_1^T B_0 e_6^T + 2 e_1 \beta_1 N_1^T B_1 e_7^T - 2 e_2 \beta_2 N_2^T e_2^T - 2 e_2 \beta_2 N_2^T C e_1^T \\
&\quad + 2 e_2 \beta_2 N_2^T B_0 e_6^T + 2 e_2 \beta_2 N_2^T B_1 e_7^T,
\end{aligned}$$

$$\Theta_1 = -(\delta_2^2 - \delta_1^2)^2 e_{13} L e_{13}^T,$$

$$\Theta_2 = -(\delta_2^2 - \delta_1^2)^2 e_{12} L e_{12}^T,$$

$$\Theta_3 = -e_{18} S_2 e_{18}^T,$$

$$\Theta_4 = -e_{19} S_2 e_{19}^T,$$

then, the system (32) *with* $u(t) = 0$ *is asymptotically stable.*

Proof We consider the following Lyapunov–Krasovskii functional candidate for the system (32):

$$V\big(w(t), t\big) = \sum_{i=1}^{9} V_i\big(w(t), t\big), \tag{37}$$

where

$$V_1\big(w(t), t\big) = w^T(t)Pw(t),$$

$$V_2\big(w(t), t\big) = \int_{t-\delta_1}^{t} w^T(s)U_1 w(s)\, ds,$$

$$V_3\big(w(t), t\big) = \int_{t-\delta_2}^{t} w^T(s)U_2 w(s)\, ds,$$

$$V_4\big(w(t), t\big) = \delta_1 \int_{-\delta_1}^{0} \int_{t+s}^{t} \dot{w}^T(\tau)T_1 \dot{w}(\tau)\, d\tau\, ds,$$

$$V_5\big(w(t), t\big) = \delta_2 \int_{-\delta_2}^{0} \int_{t+s}^{t} \dot{w}^T(\tau)T_2 \dot{w}(\tau)\, d\tau\, ds,$$

$$V_6\big(w(t), t\big) = (\delta_2 - \delta_1) \int_{-\delta_2}^{-\delta_1} \int_{t+s}^{t} \dot{w}^T(\tau)T_3 \dot{w}(\tau)\, d\tau\, ds,$$

$$V_7\big(w(t), t\big) = \frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} w^T(s)Lw(s)\, ds\, d\lambda\, d\beta,$$

$$V_8\big(w(t), t\big) = \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{\lambda}^{0} \int_{t+\varphi}^{t} \dot{w}^T(s)S_1 \dot{w}(s)\, ds\, d\varphi\, d\lambda\, d\beta,$$

$$V_9\big(w(t), t\big) = \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{\lambda}^{0} \int_{t+\varphi}^{t} w^T(s)S_2 w(s)\, ds\, d\varphi\, d\lambda\, d\beta.$$

Time derivatives of $V_i(w(t), t)$, $i = 1, 2, \ldots, 9$, along the trajectories of (32) are as follows:

$$\dot{V}_1\big(w(t), t\big) = w^T(t)P\dot{w}(t) + \dot{w}^T(t)Pw(t), \tag{38}$$

$$\dot{V}_2\big(w(t), t\big) = w^T(t)U_1 w(t) - w^T(t - \delta_1)U_1 w(t - \delta_1), \tag{39}$$

$$\dot{V}_3\big(w(t), t\big) = w^T(t)U_2 w(t) - w^T(t - \delta_2)U_2 w(t - \delta_2), \tag{40}$$

$$\dot{V}_4\big(w(t), t\big) = \delta_1 \int_{-\delta_1}^{0} \big[\dot{w}^T(t)T_1 \dot{w}(t) - \dot{w}^T(t + s)T_1 \dot{w}(t + s) \big]\, ds$$

$$= \delta_1^2 \dot{w}^T(t)T_1 \dot{w}(t) - \delta_1 \int_{t-\delta_1}^{t} \dot{w}^T(\alpha)T_1 \dot{w}(\alpha)\, d\alpha, \tag{41}$$

$$\dot{V}_5\big(w(t), t\big) = \delta_2 \int_{-\delta_2}^{0} \big[\dot{w}^T(t)T_2 \dot{w}(t) - \dot{w}^T(t + s)T_2 \dot{w}(t + s) \big]\, ds$$

$$= \delta_2^2 \dot{w}^T(t)T_2 \dot{w}(t) - \delta_2 \int_{t-\delta_2}^{t} \dot{w}^T(\alpha)T_2 \dot{w}(\alpha)\, d\alpha, \tag{42}$$

$$\dot{V}_6\big(w(t), t\big) = (\delta_2 - \delta_1) \int_{-\delta_2}^{-\delta_1} \big[\dot{w}^T(t)T_3 \dot{w}(t) - \dot{w}^T(t + s)T_3 \dot{w}(t + s) \big]\, ds$$

$$= (\delta_2 - \delta_1)^2 \dot{w}^T(t) T_3 \dot{w}(t)$$

$$- (\delta_2 - \delta_1) \int_{t-\delta_2}^{t-\delta_1} \dot{w}^T(\alpha) T_3 \dot{w}(\alpha) \, d\alpha, \tag{43}$$

$$\dot{V}_7\big(w(t), t\big) = \frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \big[w^T(t) L w(t) - w^T(t+\lambda) L w(t+\lambda) \big] \, d\lambda \, d\beta$$

$$= \frac{(\delta_2^2 - \delta_1^2)^2}{4} w^T(t) L w(t)$$

$$- \frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{t+\beta}^{t} w^T(s) L w(s) \, ds \, d\beta, \tag{44}$$

$$\dot{V}_8\big(w(t), t\big) = \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{\lambda}^{0} \big[\dot{w}^T(t) S_1 \dot{w}(t)$$

$$- \dot{w}^T(t+\varphi) S_1 \dot{w}(t+\varphi) \big] \, d\varphi \, d\lambda \, d\beta$$

$$= \frac{(\delta_2^3 - \delta_1^3)^2}{36} \dot{w}^T(t) S_1 \dot{w}(t)$$

$$- \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} \dot{w}^T(s) S_1 \dot{w}(s) \, ds \, d\lambda \, d\beta, \tag{45}$$

$$\dot{V}_9\big(w(t), t\big) = \frac{(\delta_2^3 - \delta_1^3)^2}{36} w^T(t) S_2 w(t)$$

$$- \frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} w^T(s) S_2 w(s) \, ds \, d\lambda \, d\beta. \tag{46}$$

Utilizing Lemma 2.4, the following relations are easily obtained:

$$-\delta_1 \int_{t-\delta_1}^{t} \dot{w}^T(\alpha) T_1 \dot{w}(\alpha) \, d\alpha$$

$$\leq -\varsigma^T(t)(e_1 - e_3) T_1 (e_1 - e_3)^T \varsigma(t)$$

$$- 3\varsigma^T(t)(e_1 + e_3 - 2e_8) T_1 (e_1 + e_3 - 2e_8)^T \varsigma(t)$$

$$- 5\varsigma^T(t)(e_1 - e_3 + 6e_8 - 12e_{14}) T_1 (e_1 - e_3 + 6e_8 - 12e_{14})^T \varsigma(t)$$

$$- 7\varsigma^T(t)(e_1 + e_3 - 12e_8 + 60e_{14} - 120e_{20})$$

$$\times T_1 (e_1 + e_3 - 12e_8 + 60e_{14} - 120e_{20})^T \varsigma(t), \tag{47}$$

$$-\delta_2 \int_{t-\delta_2}^{t} \dot{w}^T(\alpha) T_2 \dot{w}(\alpha) \, d\alpha$$

$$\leq -\varsigma^T(t)(e_1 - e_4) T_2 (e_1 - e_4)^T \varsigma(t)$$

$$- 3\varsigma^T(t)(e_1 + e_4 - 2e_9) T_2 (e_1 + e_4 - 2e_9)^T \varsigma(t)$$

$$- 5\varsigma^T(t)(e_1 - e_4 + 6e_9 - 12e_{15}) T_2 (e_1 - e_4 + 6e_9 - 12e_{15})^T \varsigma(t)$$

$$- 7\varsigma^T(t)(e_1 + e_4 - 12e_9 + 60e_{15} - 120e_{21})$$

$$\times T_2 (e_1 + e_4 - 12e_9 + 60e_{15} - 120e_{21})^T \varsigma(t), \tag{48}$$

$$-(\delta_2 - \delta_1) \int_{t-\delta_2}^{t-\delta_1} \dot{w}^T(\alpha) T_3 \dot{w}(\alpha) \, d\alpha$$

$$= -(\delta_2 - \delta_1) \int_{t-\delta_2}^{t-\delta(t)} \dot{w}^T(\alpha) T_3 \dot{w}(\alpha) \, d\alpha - (\delta_2 - \delta_1) \int_{t-\delta(t)}^{t-\delta_1} \dot{w}^T(\alpha) T_3 \dot{w}(\alpha) \, d\alpha$$

$$\leq -\varsigma^T(t)(e_5 - e_4)T_3(e_5 - e_4)^T\varsigma(t)$$

$$- 3\varsigma^T(t)(e_5 + e_4 - 2e_{11})T_3(e_5 + e_4 - 2e_{11})^T\varsigma(t)$$

$$- 5\varsigma^T(t)(e_5 - e_4 + 6e_{11} - 12e_{16})T_3(e_5 - e_4 + 6e_{11} - 12e_{16})^T\varsigma(t)$$

$$- 7\varsigma^T(t)(e_5 + e_4 - 12e_{11} + 60e_{16} - 120e_{22})$$

$$\times T_3(e_5 + e_4 - 12e_{11} + 60e_{16} - 120e_{22})^T\varsigma(t),$$

$$- \varsigma^T(t)(e_3 - e_5)T_3(e_3 - e_5)^T\varsigma(t)$$

$$- 3\varsigma^T(t)(e_3 + e_5 - 2e_{10})T_3(e_3 + e_5 - 2e_{10})^T\varsigma(t)$$

$$- 5\varsigma^T(t)(e_3 - e_5 + 6e_{10} - 12e_{17})T_3(e_3 - e_5 + 6e_{10} - 12e_{17})^T\varsigma(t)$$

$$- 7\varsigma^T(t)(e_3 + e_5 - 12e_{10} + 60e_{17} - 120e_{23})$$

$$\times T_3(e_3 + e_5 - 12e_{10} + 60e_{17} - 120e_{23})^T\varsigma(t). \tag{49}$$

On the other hand, we have the following inequality from Lemma 2.2:

$$-\frac{(\delta_2^2 - \delta_1^2)}{2} \int_{-\delta_2}^{-\delta_1} \int_{t+\beta}^{t} w^T(s)Lw(s)\,ds\,d\beta$$

$$\leq -(\delta_2^2 - \delta_1^2)^2 \varsigma^T(t)e_{13}Le_{13}^T\varsigma(t) - \varepsilon(\delta_2^2 - \delta_1^2)^2 \varsigma^T(t)e_{13}Le_{13}^T\varsigma(t)$$

$$- (\delta_2^2 - \delta_1^2)^2 \varsigma^T(t)e_{12}Le_{12}^T\varsigma(t) - (1 - \varepsilon)(\delta_2^2 - \delta_1^2)^2 \varsigma^T(t)e_{12}Le_{12}^T\varsigma(t), \tag{50}$$

where $\varepsilon = \frac{\delta^2(t) - \delta_1^2}{\delta_2^2 - \delta_1^2}$.
From Lemma 2.3 and condition (36), we obtain

$$-\frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} \dot{w}^T(s)S_1\dot{w}(s)\,ds\,d\lambda\,d\beta$$

$$\leq \varsigma^T(t)\begin{bmatrix} e_1 & 2e_{12} + 2e_{13} \end{bmatrix}$$

$$\times \left((\delta_2^2 - \delta_1^2)\bar{Y} + \frac{\delta_2^3 - \delta_1^3}{6}\bar{X} \right)\begin{bmatrix} e_1 & 2e_{12} + 2e_{13} \end{bmatrix}^T\varsigma(t). \tag{51}$$

From Lemma 2.5, we get

$$-\frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} w^T(s)S_2w(s)\,ds\,d\lambda\,d\beta$$

$$\leq -\varsigma^T(t)e_{18}S_2e_{18}^T\varsigma(t) - \alpha\varsigma^T(t)e_{18}S_2e_{18}^T\varsigma(t)$$

$$- \varsigma^T(t)e_{19}S_2e_{19}^T\varsigma(t) - (1 - \alpha)\varsigma^T(t)e_{19}S_2e_{19}^T\varsigma(t), \tag{52}$$

where $\alpha = \frac{\delta^3(t) - \delta_1^3}{\delta_2^3 - \delta_1^3}$.
It follows from Assumptions **(A1)** and **(A2)** that

$$2(F_pWw(t) - f(Ww(t)))^T Z_1(f(Ww(t)) - F_mWw(t)) \geq 0, \tag{53}$$

$$2(G_pWw(t - \delta(t)) - g(Ww(t - \delta(t))))^T$$

$$\times Z_2(g(Ww(t - \delta(t))) - G_mWw(t - \delta(t))) \geq 0. \tag{54}$$

We consider system (32), and the following equation is obtained:

$$0 = 2\left[w^T(t)\beta_1 N_1^T + \dot{w}^T(t)\beta_2 N_2^T\right]\left[-\dot{w}(t) - Cw(t) + B_0 f\left(Ww(t)\right)\right.$$
$$\left. + B_1 g\left(Ww(t - \delta(t))\right) + B_3 u(t)\right]. \tag{55}$$

By adding the right-hand side of (55) to $\dot{V}(t)$, we achieve from (38)–(54) that

$$\dot{V}\left(w(t), t\right) \le \bar{\xi}^T(t)\left(\varepsilon\bar{\Theta}^{(1)} + (1 - \varepsilon)\bar{\Theta}^{(2)} + \alpha\bar{\Theta}^{(3)} + (1 - \alpha)\bar{\Theta}^{(4)}\right)\bar{\xi}(t), \tag{56}$$

where $\bar{\Theta}^{(i)} = \frac{1}{2}\bar{\Theta} + \Theta_i$ $(i = 1, 2)$ and $\bar{\Theta}^{(j)} = \frac{1}{2}\bar{\Theta} + \Theta_j$ $(j = 3, 4)$, $\bar{\Theta} = \Theta + 2e_{24}\beta_1 B_3^T N_1 e_1^T + 2e_{24}\beta_2 B_3^T N_2 e_2^T$, with Θ and Θ_i, Θ_j defined in (33) and (34).

When $u(t) = 0$ (no disturbance), one has from (56) that

$$\dot{V}\left(w(t), t\right) \le \varsigma^T(t)\left(\varepsilon\Theta^{(1)} + (1 - \varepsilon)\Theta^{(2)} + \alpha\Theta^{(3)} + (1 - \alpha)\Theta^{(4)}\right)\varsigma(t),$$

where $\Theta^{(i)} = \frac{1}{2}\Theta + \Theta_i$ $(i = 1, 2)$ and $\Theta^{(j)} = \frac{1}{2}\Theta + \Theta_j$ $(j = 3, 4)$.

The upper bound of $\dot{V}(w(t), t)$ is negative if the condition (35) and the following relations hold simultaneously:

$$\varepsilon\Theta^{(1)} + (1 - \varepsilon)\Theta^{(2)} < -b_1 I,$$
$$\alpha\Theta^{(3)} + (1 - \alpha)\Theta^{(4)} < b_2 I.$$

The above relations can be rewritten as follows:

$$\varepsilon\left(\Theta^{(1)} + b_1 I\right) + (1 - \varepsilon)\left(\Theta^{(2)} + b_1 I\right) < 0, \tag{57}$$
$$\alpha\left(\Theta^{(3)} - b_2 I\right) + (1 - \alpha)\left(\Theta^{(4)} - b_2 I\right) < 0. \tag{58}$$

Since $0 \le \varepsilon, \alpha \le 1$, the term $\varepsilon(\Theta^{(1)} + b_1 I) + (1 - \varepsilon)(\Theta^{(2)} + b_1 I)$ is a convex combination of $\Theta^{(1)} + b_1 I$ and $\Theta^{(2)} + b_1 I$; and the expression $\alpha(\Theta^{(3)} - b_2 I) + (1 - \alpha)(\Theta^{(4)} - b_2 I)$ is a convex combination of $\Theta^{(3)} - b_2 I$ and $\Theta^{(4)} - b_2 I$. These combinations are negative definite only if the vertices become negative; therefore, (57) and (58) are equivalent to (33) and (34), respectively. Then, the system (32) with $u(t) = 0$ is asymptotically stable. □

3.2 Extended dissipativity analysis for generalized neural networks

In this section, new extended dissipativity criteria for the generalized neural networks (1), and their special case, are obtained based on the stability conditions that were developed in Theorems 3.1 and 3.2.

Theorem 3.3 *For given scalars δ_1, δ_2, σ_1, σ_2, β_1, β_2, and a positive scalar $d < 1$, if there exist symmetric positive definite matrices $P, U_1, U_2, T_1, T_2, T_3, L, S_1, S_2, Q \in \mathbb{R}^{n \times n}$, positive definite matrices $N_1, N_2 \in \mathbb{R}^{n \times n}$, positive diagonal matrices $Z_1, Z_2, Z_3 \in \mathbb{R}^{n \times n}$, any matrices $X_1, X_2, X_3, X_4, Y_1, Y_2 \in \mathbb{R}^{n \times n}$, and positive scalars c_1, c_2 such that the following LMIs hold*:

$$\tilde{\Xi} + 2\Xi_i + 2c_1 I < 0, \quad i = 1, 2, \tag{59}$$
$$\tilde{\Xi} + 2\Xi_j - 2c_2 I < 0, \quad j = 3, 4, \tag{60}$$

$$c_1 - c_2 > 0, \tag{61}$$

$$\begin{bmatrix} X_1 + X_1^T & X_2 + X_3^T & Y_1 \\ * & X_4 + X_4^T & Y_2 \\ * & * & S_1 \end{bmatrix} \geq 0, \tag{62}$$

$$\begin{bmatrix} dP - D_1^T \Gamma_4 D_1 & -D_1^T \Gamma_4 D_2 \\ * & (1-d)P - D_2^T \Gamma_4 D_2 \end{bmatrix} \geq 0, \tag{63}$$

where

$$
\begin{aligned}
\tilde{\Xi} = {} & \bar{\Xi} - e_1 D_1^T \Gamma_1 D_1 e_1^T - e_1 D_1^T \Gamma_1 D_2 e_5^T - e_1 D_1^T \Gamma_1 D_3 e_{26}^T - e_5 D_2^T \Gamma_1 D_1 e_1^T \\
& - e_5 D_2^T \Gamma_1 D_2 e_5^T - e_5 D_2^T \Gamma_1 D_3 e_{26}^T - e_{26} D_3^T \Gamma_1 D_1 e_1^T - e_{26} D_3^T \Gamma_1 D_2 e_5^T \\
& - 2 e_1 D_1^T \Gamma_2 e_{26}^T - 2 e_5 D_2^T \Gamma_2 e_{26}^T - e_{26} \big(D_3^T \Gamma_1 D_3 + 2 D_3^T \Gamma_2 + \Gamma_3 \big) e_{26}^T, \\
\bar{\Xi} = {} & \Xi + 2 e_{26} \beta_1 B_3^T N_1 e_1^T + 2 e_{26} \beta_2 B_3^T N_2 e_2^T,
\end{aligned}
$$

then, the system (1) *is asymptotically stable and extended dissipative.*

Proof To show that the GNNs system (1) is extended dissipative, first, we use the LKFs candidate (8) and the following performance index for the GNNs (1).

Using inequality (29) in Theorem 3.1, equation (3), and LMIs (59)–(63), we obtain

$$
\begin{aligned}
\dot{V}\big(w(t), t\big) - J(t) &\leq \bar{\eta}^T(t) \big(\varepsilon \tilde{\Xi}^{(1)} + (1-\varepsilon) \tilde{\Xi}^{(2)} + \alpha \tilde{\Xi}^{(3)} + (1-\alpha) \tilde{\Xi}^{(4)} \big) \bar{\eta}(t) \leq 0, \\
\dot{V}\big(w(t), t\big) &\leq \bar{\eta}^T(t) \big(\varepsilon \tilde{\Xi}^{(1)} + (1-\varepsilon) \tilde{\Xi}^{(2)} + \alpha \tilde{\Xi}^{(3)} + (1-\alpha) \tilde{\Xi}^{(4)} \big) \bar{\eta}(t) + J(t) \\
&\leq J(t). \tag{64}
\end{aligned}
$$

Then we integrate both sides of the inequality (64) from 0 to $t \geq 0$ and, letting $\lambda \leq -V(w(0), 0)$, get

$$\int_0^t J(s)\, ds \geq V\big(w(t), t\big) - V\big(w(0), 0\big) \geq w^T(t) P w(t) + \lambda. \tag{65}$$

Next, we consider two cases:

Case I. $\Gamma_4 = 0$. For this case, from inequality (65) we obtain

$$\int_0^{t_f} J(s)\, ds \geq \lambda. \tag{66}$$

This matches Definition 2.1 with $\Gamma_4 = 0$.

Case II. $\Gamma_4 \neq 0$. From Assumption (H1), it is clear that $\Gamma_1 = 0$, $\Gamma_2 = 0$, $\Gamma_3 > 0$, and $D_3 = 0$. Then, for any $0 \leq t \leq t_f$ and $0 \leq t - \delta(t) \leq t_f$, (65) leads to

$$\int_0^{t_f} J(s)\, ds \geq \int_0^t J(s)\, ds \geq w^T(t) P w(t) + \lambda$$

and

$$\int_0^{t_f} J(s)\, ds \geq \int_0^{t-\delta(t)} J(s)\, ds \geq w^T\big(t - \delta(t)\big) P w\big(t - \delta(t)\big) + \lambda,$$

respectively. On the other hand, for $t - \delta(t) \leq 0$, it can be shown that

$$
\begin{aligned}
w^T\big(t - \delta(t)\big)Pw\big(t - \delta(t)\big) + \lambda &\leq \|P\|\big|w\big(t - \delta(t)\big)\big|^2 + \lambda \\
&\leq \|P\| \sup_{-\delta_2 \leq \theta \leq 0} \big|\phi(\theta)\big|^2 + \lambda \\
&\leq -V\big(w(0), 0\big) \\
&\leq \int_0^{t_f} J(s)\, ds.
\end{aligned}
$$

Thus, there exists a positive scalar $d < 1$ such that

$$
\int_0^{t_f} J(s)\, ds \geq \lambda + dw^T(t)Pw(t) + (1 - d)w^T\big(t - \delta(t)\big)Pw\big(t - \delta(t)\big).
$$

By the relationship of output $z(t)$ with (63),

$$
\begin{aligned}
z(t)^T \Gamma_4 z(t) = &-\begin{bmatrix} w(t) \\ w(t - \delta(t)) \end{bmatrix}^T \begin{bmatrix} dP - D_1^T\Gamma_4 D_1 & -D_1^T\Gamma_4 D_2 \\ * & (1 - d)P - D_2^T\Gamma_4 D_2 \end{bmatrix} \\
&\times \begin{bmatrix} w(t) \\ w(t - \delta(t)) \end{bmatrix} + dw^T(t)Pw(t) + (1 - d)w^T\big(t - \delta(t)\big)Pw\big(t - \delta(t)\big).
\end{aligned}
$$

So, it is clear that for any t satisfying $0 \leq t \leq t_f$,

$$
\int_0^{t_f} J(s)\, ds \geq z(t)^T \Gamma_4 z(t) + \lambda. \tag{67}
$$

Taking the supremum over t in inequalities (66) and (67), system (1) is extended dissipative. This completes the proof. □

Theorem 3.4 *For given scalars δ_1, δ_2, β_1, and β_2, if there exist symmetric positive definite matrices $P, U_1, U_2, T_1, T_2, T_3, L, S_1, S_2 \in \mathbb{R}^{n \times n}$, positive definite matrices $N_1, N_2 \in \mathbb{R}^{n \times n}$, positive diagonal matrices $Z_1, Z_2 \in \mathbb{R}^{n \times n}$, any matrices $X_1, X_2, X_3, X_4, Y_1, Y_2 \in \mathbb{R}^{n \times n}$, and positive scalars b_1, b_2 such that the following LMIs hold:*

$$
\tilde{\Theta} + 2\Theta_i + 2b_1 I < 0, \quad i = 1, 2, \tag{68}
$$

$$
\tilde{\Theta} + 2\Theta_j - 2b_2 I < 0, \quad j = 3, 4, \tag{69}
$$

$$
b_1 - b_2 > 0, \tag{70}
$$

$$
\begin{bmatrix} X_1 + X_1^T & X_2 + X_3^T & Y_1 \\ * & X_4 + X_4^T & Y_2 \\ * & * & S_1 \end{bmatrix} \geq 0, \tag{71}
$$

$$
P - D_1^T \Gamma_4 D_1 \geq 0, \tag{72}
$$

where

$$
\tilde{\Theta} = \bar{\Theta} - e_1 D_1^T \Gamma_1 D_1 e_1^T - 2e_{24}\Gamma_2^T D_1 e_1^T - e_{24}\Gamma_3 e_{24}^T,
$$

$$\bar{\Theta} = \Theta + 2e_{24}\beta_1 B_3^T N_1 e_1^T + 2e_{24}\beta_2 B_3^T N_2 e_2^T,$$

then, the system (32) is asymptotically stable and extended dissipative.

Proof To show that the GNNs system (32) is extended dissipative, first, we use the LKFs candidate (37) and the following performance index for the GNNs (32).

Using inequality (56) in Theorem 3.2, equation (3), and LMIs (68)–(72), we get

$$\dot{V}(w(t), t) - J(t) \le \bar{\xi}^T(t)\big(\varepsilon\tilde{\Theta}^{(1)} + (1 - \varepsilon)\tilde{\Theta}^{(2)} + \alpha\tilde{\Theta}^{(3)} + (1 - \alpha)\tilde{\Theta}^{(4)}\big)\xi(t) \le 0,$$

$$\dot{V}(w(t), t) \le \bar{\xi}^T(t)\big(\varepsilon\tilde{\Theta}^{(1)} + (1 - \varepsilon)\tilde{\Theta}^{(2)} + \alpha\tilde{\Theta}^{(3)} + (1 - \alpha)\tilde{\Theta}^{(4)}\big)\xi(t) + J(t)$$

$$\le J(t). \tag{73}$$

Then we integrate both sides of the inequality (73) from 0 to $t \ge 0$ and, letting $0 = \lambda \le -V(w(0), 0)$, get

$$\int_0^t J(s)\,ds \ge V(w(t), t) - V(w(0), 0) \ge w^T(t)Pw(t). \tag{74}$$

Next, we consider two cases:

Case I. $\Gamma_4 = 0$. For this case, from inequality (74) we get

$$\int_0^{t_f} J(s)\,ds \ge 0. \tag{75}$$

This matches Definition 2.1 with $\Gamma_4 = 0$.

Case II. $\Gamma_4 \ne 0$. From Assumption (H1), it is clear that $\Gamma_1 = 0$, $\Gamma_2 = 0$, and $\Gamma_3 > 0$. Then, for any $0 \le t \le t_f$, (74) leads to

$$\int_0^{t_f} J(s)\,ds \ge \int_0^t J(s)\,ds \ge w^T(t)Pw(t).$$

By the relationship of output with (72),

$$z(t)^T \Gamma_4 z(t) = -w^T(t)\big(P - D_1^T \Gamma_4 D_1\big)w(t) + w^T(t)Pw(t) \le w^T(t)Pw(t).$$

So, it is clear that for any t satisfying $0 \le t \le t_f$,

$$\int_0^{t_f} J(s)\,ds \ge z(t)^T \Gamma_4 z(t). \tag{76}$$

Taking the supremum over t in inequalities (75) and (76), system (32) is extended dissipative. This completes the proof. □

Remark 3 Recently, many problems have been investigated to analyze extended dissipativity for neural networks with time delays, and the results have been reported in [11, 34, 35]. Moreover, the extended dissipative analysis was studied for the GNNs with interval time-varying delay signals [37]. Unfortunately, most of research does not include the distributed time-varying delay in the GNNs systems. We know that the distributed time-varying delay

is a delay resulting from the transmission of distributed nerve impulses in a diversity of axon sizes and lengths, which is considered difficult to avoid. Therefore, the extended dissipativity analysis for the GNNs with interval discrete and distributed time-varying delays was addressed in our research.

Remark 4 In recent years, the WSII was developed in [40] to estimate the derivatives of LKFs with a single integral term. Several studies have used the improved WSII to estimate the derivative of LKFs, for example, in [42], the authors have obtained criteria of finite-time passivity for neutral-type neural networks with time-varying delays by using the improved WSII with Jensen's inequality. In 2018, a novel triple integral inequality was constructed in [39] to estimate the derivative of LKFs with the triple integral term, moreover, the authors achieved an improved delay-dependent exponential stability criterion by using a novel triple integral inequality with the extended reciprocally convex technique. On the other hand, in this work, we use the improved WSII to estimate four terms and the triple integral inequality to estimate $-\frac{(\delta_2^3 - \delta_1^3)}{6} \int_{-\delta_2}^{-\delta_1} \int_{\beta}^{0} \int_{t+\lambda}^{t} \dot{w}^T(s) S_1 \dot{w}(s) \, ds \, d\lambda \, d\beta$. By applying the improved WSII, a novel triple integral inequality, and convex combination technique, we gain less conservative results when compared with the other works [6, 7, 43–45].

Remark 5 In this work, the Lyapunov–Krasovskii functional contains single, double, triple, and quadruple integral terms, in which full information on the delays δ_1, δ_2, σ_1, σ_2, and a state variable is used. Moreover, more information on activation functions has been taken into the stability and performance analysis, that is, $F_i^- \leq \frac{f_i(W_i w(t))}{W_i w(t)} \leq F_i^+$, $G_i^- \leq \frac{g_i(W_i w(t-\delta(t)))}{W_i w(t-\delta(t))} \leq G_i^+$, and $H_i^- \leq \frac{h_i(W_i w(t))}{W_i w(t)} \leq H_i^+$ are addressed in the proof. Therefore, the construction and the technique for computation of the Lyapunov–Krasovskii functional are the main keys to improve results of this work. In the proof of Theorems 3.1–3.4, improved Wirtinger's single integral inequality [40], a novel triple integral inequality [39], and convex combination technique are used to bound the derivative of Lyapunov–Krasovskii functional, which provide tighter bound than the inequalities in [6, 7, 43–45]. All of these lead to a reduction of the conservatism of our results compared to those in some exiting works and, in particular, numerical examples. However, the complex computation of the Lyapunov–Krasovskii functional leads to the LMI derived in this work which contains much information about the system. Hence, for further work, it is interesting for researchers to improve the technique for a simple Lyapunov–Krasovskii functional and also achieve better results.

4 Numerical examples

In this section, five numerical examples are presented to illustrate the effectiveness of our results.

Example 4.1 Consider the generalized neural network (32) with the following matrices:

$$C = \text{diag}\{7.3458, 6.9987, 5.5949\}, \qquad W = \begin{bmatrix} 13.6014 & -2.9616 & -0.6936 \\ 7.4736 & 21.6810 & 3.2100 \\ 0.7290 & -2.6334 & -20.1300 \end{bmatrix}$$

$$B_1 = I, \qquad B_0 = F_m = G_m = 0, \quad \text{and} \quad F_p = G_p = \text{diag}\{0.3680, 0.1795, 0.2876\}.$$

Table 1 The maximum allowable values of δ_2 for $\delta_1 = 0.5$ and different values of μ in Example 4.1

Methods	$\mu = 0.9$	Unknown μ
Yang et al. (2014) [43]	0.6053	0.6053
Lee et al. (2015) [44]	–	0.6169
Saravanakumar et al. (2017) [45]	0.6547	0.6547
Theorem 3.2	–	0.7442

Table 2 The maximum allowable values of δ_2 for $\delta_1 = 1$ and different values μ in Example 4.2

Methods	$\mu = 0.8$	$\mu = 0.9$	Unknown μ
Chen et al. (2010) [6]	1.4692	1.2948	1.1774
Wang et al. (2015) [7] ($N = 2$)	1.6201	1.3955	1.2363
Wang et al. (2015) [7] ($N = 3$)	1.6895	1.4385	1.2617
Saravanakumar et al. (2017) [45]	1.7732	1.6457	1.4765
Theorem 3.2	–	–	1.9423

By taking parameters $\beta_1 = \beta_2 = 1$ and solving Example 4.1 using LMIs in Theorem 3.2, we obtain maximum allowable values of δ_2 for $\delta_1 = 0.5$ without the upper bound of differentiable delay (μ) as shown in Table 1. This table shows that the results derived in this research are less conservative than those in [43–45].

Example 4.2 Consider the generalized neural network (32) with the following matrices:

$$C = W = I, \qquad B_0 = \begin{bmatrix} -1 & 0.5 \\ 0.5 & -1.5 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -2 & 0.5 \\ 0.5 & -2 \end{bmatrix},$$

$$F_m = G_m = 0, \quad \text{and} \quad F_p = G_p = \mathrm{diag}\{0.4, 0.8\}.$$

By taking parameters $\beta_1 = \beta_2 = 1$ and solving Example 4.2 using LMIs in Theorem 3.2, we obtain maximum allowable values of δ_2 for $\delta_1 = 1$ without the upper bound of differentiable delay (μ) as shown in Table 2. This table shows that the results derived in this research are less conservative than those in [6, 7, 45].

Example 4.3 Consider the generalized neural network (1) with $\delta_1 = 0.3$, $\delta_2 = 1$, $\sigma_1 = 0.01$, $\sigma_2 = 0.4$, $\beta_1 = 0.9$, $\beta_2 = 1$,

$$C = I, \qquad B_0 = \begin{bmatrix} 0.2 & -0.1 \\ -0.5 & 0.1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.5 & 0 \\ -0.3 & -0.2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.15 & 0.1 \\ 0 & -0.3 \end{bmatrix}, \qquad W = 0.5I, \qquad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$D_1 = D_2 = D_3 = 0, \qquad F_p = G_p = H_p = 0.5I, \quad \text{and} \quad F_m = G_m = H_m = 0.$$

When the LMIs of (4) and (5) in Theorem 3.1 are solved, we obtain

$$P = 10^{-3} \times \begin{bmatrix} 0.0814 & -0.0082 \\ -0.0082 & 0.4716 \end{bmatrix}, \qquad U_1 = 10^{-4} \times \begin{bmatrix} 0.1293 & -0.0131 \\ -0.0131 & 0.8506 \end{bmatrix},$$

$$U_2 = 10^{-4} \times \begin{bmatrix} 0.0851 & -0.0155 \\ -0.0155 & 0.8611 \end{bmatrix}, \qquad T_1 = 10^{-6} \times \begin{bmatrix} 0.2360 & 0.0002 \\ 0.0002 & 0.2268 \end{bmatrix},$$

$$T_2 = 10^{-6} \times \begin{bmatrix} 0.2350 & 0.0002 \\ 0.0002 & 0.2268 \end{bmatrix}, \qquad T_3 = 10^{-6} \times \begin{bmatrix} 0.7593 & 0.0118 \\ 0.0118 & 0.6331 \end{bmatrix},$$

$$L = 10^{-3} \times \begin{bmatrix} 0.0882 & -0.0089 \\ -0.0089 & 0.9342 \end{bmatrix}, \qquad S_1 = \begin{bmatrix} 0.0010 & -0.0001 \\ -0.0001 & 0.0041 \end{bmatrix},$$

$$S_2 = \begin{bmatrix} 0.0004 & 0 \\ 0 & 0.0030 \end{bmatrix}, \qquad Q = \begin{bmatrix} 0.0014 & 0 \\ 0 & 0.0032 \end{bmatrix},$$

$$Z_1 = 10^{-3} \times \begin{bmatrix} 0.4624 & 0 \\ 0 & 0.4624 \end{bmatrix}, \qquad Z_2 = 10^{-4} \times \begin{bmatrix} 0.2162 & 0 \\ 0 & 0.2162 \end{bmatrix},$$

$$Z_3 = 10^{-3} \times \begin{bmatrix} 0.5730 & 0 \\ 0 & 0.5730 \end{bmatrix}, \qquad X_1 = 10^7 \times \begin{bmatrix} 0 & 1.7712 \\ -1.7712 & 0 \end{bmatrix},$$

$$X_2 = 10^8 \times \begin{bmatrix} -0.6297 & -0.4719 \\ 0.8100 & -5.6813 \end{bmatrix}, \qquad X_3 = 10^8 \times \begin{bmatrix} 0.6297 & -0.8100 \\ 0.4719 & 5.6813 \end{bmatrix},$$

$$X_4 = 10^5 \times \begin{bmatrix} 0 & 1.1347 \\ -1.1347 & 0 \end{bmatrix}, \qquad Y_1 = 10^{-3} \times \begin{bmatrix} -0.9288 & 0.0009 \\ 0.0002 & -0.9876 \end{bmatrix},$$

$$Y_2 = 10^{-3} \times \begin{bmatrix} 0.9194 & -0.0003 \\ 0.0001 & 0.8781 \end{bmatrix}, \qquad c_1 = 2.0007 \times 10^{-8}, \quad \text{and}$$

$$c_2 = 1.1845 \times 10^{-8}.$$

The maximum allowable values of δ_2 for different values of δ_1 are shown in Table 3. Figure 1 shows the response solution $w(t)$ in Example 4.3 where $u(t) = 0$ and the initial condition

Table 3 The maximum allowable values of δ_2 for different values of δ_1 in Example 4.3

Method	$\delta_1 = 0$	$\delta_1 = 0.3$	$\delta_1 = 1.0$	$\delta_1 = 1.5$	$\delta_1 = 2.0$
Theorem 3.1	1.2085	1.2086	1.3483	1.9039	2.0993

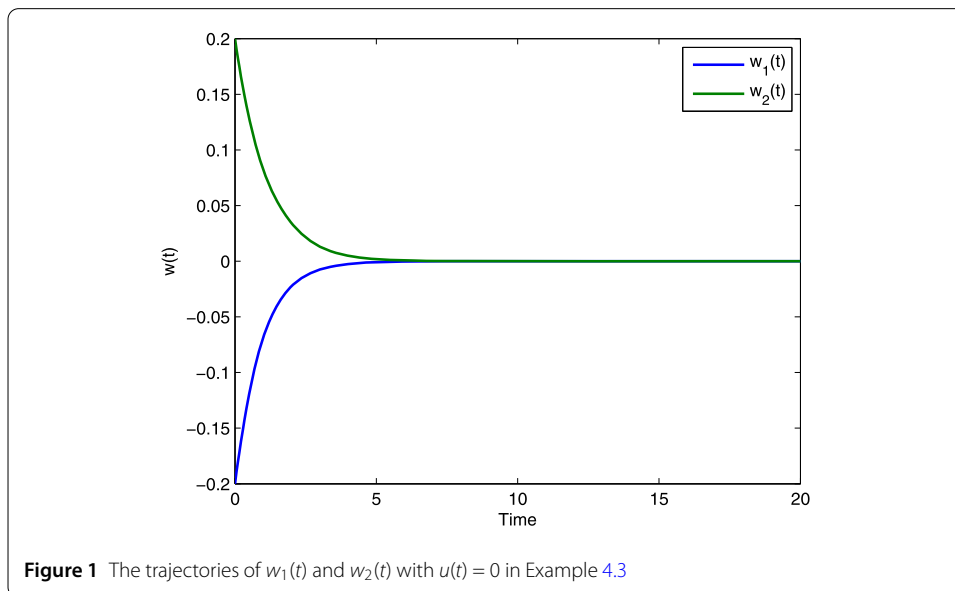**Figure 1** The trajectories of $w_1(t)$ and $w_2(t)$ with $u(t) = 0$ in Example 4.3

Figure 2 The trajectories of $w_1(t)$ and $w_2(t)$ in Example 4.3
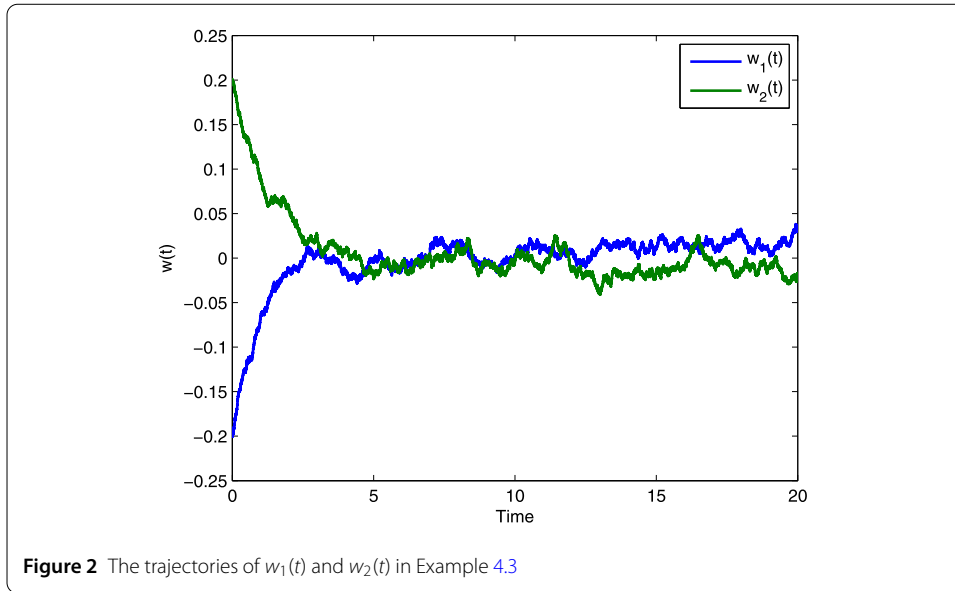
Table 4 Minimum γ for Case I and Case II in Example 4.4 with $\delta_1 = 0.5$, and various δ_2

Methods	$\delta_2 = 0.6$	$\delta_2 = 0.7$	$\delta_2 = 0.8$	$\delta_2 = 0.9$
Case I. L_2–L_∞	2.0751	2.1393	2.2210	2.3226
Case II. Passivity	4.3061	4.5764	4.9328	5.3942

$\phi(t) = [-0.2\ 0.2]^T$. Figure 2 shows the response solution $w(t)$ in Example 4.3 where $u(t)$ is Gaussian noise with mean 0 and variance 1, and the initial condition is $\phi(t) = [-0.2\ 0.2]^T$.

Example 4.4 In this example, the extended dissipativity performance of the generalized neural networks (32) is considered, which links all of the famous and important performance notions such as the L_2–L_∞, H_∞, passivity, and dissipativity performances. We consider the GNNs (32) with the following parameters:

$$C = 5I, \qquad B_0 = \begin{bmatrix} 2 & -0.1 \\ -5 & 2 \end{bmatrix}, \qquad W = 0.3I, \qquad \beta_1 = \beta_2 = 1,$$

$$B_1 = \begin{bmatrix} -1.5 & -0.1 \\ -0.2 & -1.5 \end{bmatrix}, \qquad B_3 = D_1 = I, \qquad F_m = G_m = 0, \quad \text{and} \quad F_p = G_p = I.$$

When we solve Example 4.4 by using LMIs of (68), (69) in Theorem 3.4, we obtain four cases:

Case I. L_2–L_∞ performance. By using the LMIs in Theorem 3.4 and letting $\Gamma_1 = 0$, $\Gamma_2 = 0$, $\Gamma_3 = \gamma^2 I$, and $\Gamma_4 = I$, the extended dissipativity performance is converted into the L_2–L_∞ performance. The L_2–L_∞ performance index γ can be achieved for $\delta_1 = 0.5$, and different δ_2, which are shown in Table 4. Figure 3 shows the plot of $L(t) = \sqrt{z^T(t)z(t)/\int_0^t u^T(s)u(s)\,ds}$ versus time with the initial condition $\phi(t) = [-0.1\ 0.1]^T$. Clearly, $\sup_t L(t) = 0.0265$, which is less than the prescribed L_2–L_∞ performance index 2.0751 in Table 4.

Case II. Passivity performance. By applying the LMIs in Theorem 3.4 and taking $\Gamma_1 = 0$, $\Gamma_2 = I$, $\Gamma_3 = \gamma I$, and $\Gamma_4 = 0$, the extended dissipativity performance degenerates to the passivity performance. The passivity performance index γ can be gained for $\delta_1 = $

Figure 3 The trajectory of $L(t)$ in Example 4.4

Figure 4 The trajectory of $P(t)$ in Example 4.4
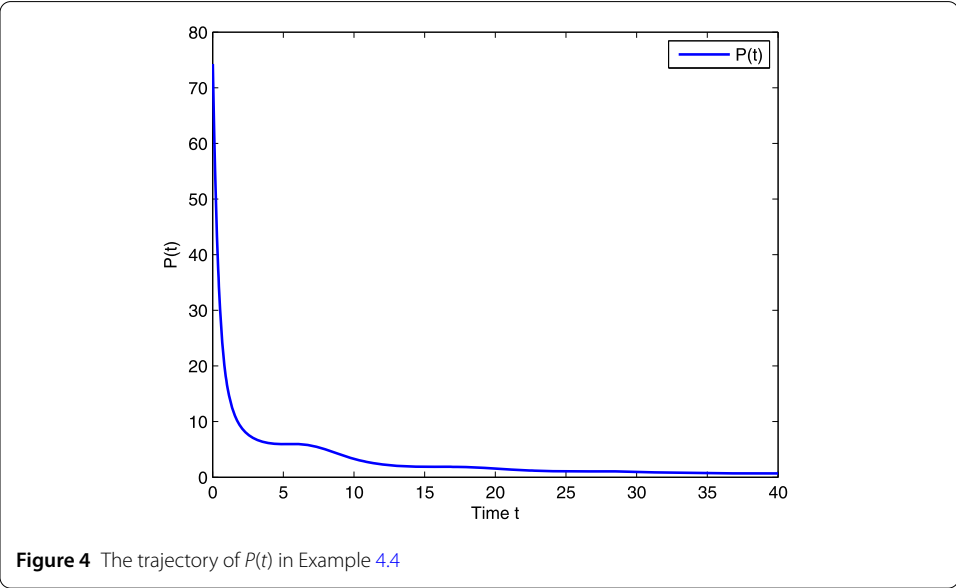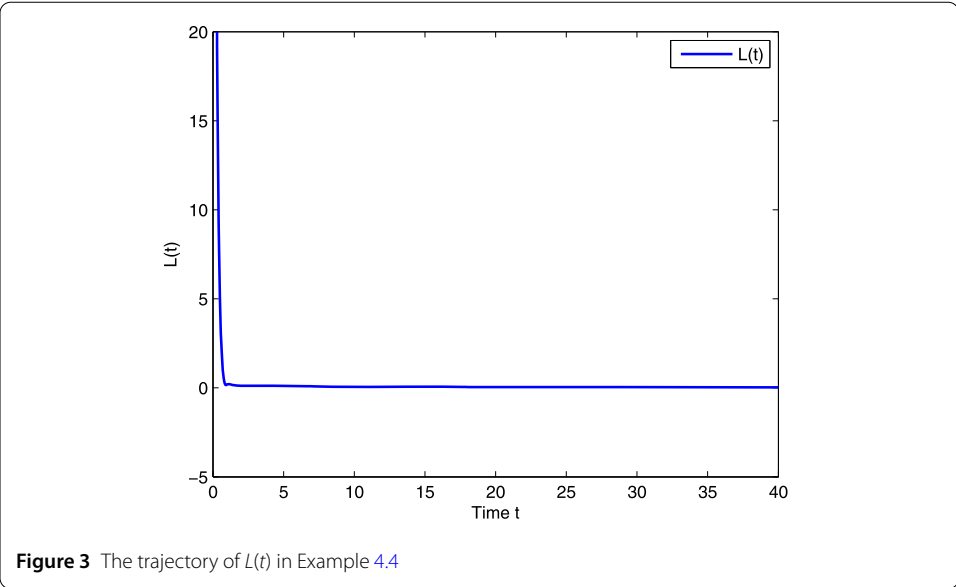
0.5, and various δ_2, which are presented in Table 4. Figure 4 shows the plot of $P(t) = -2\int_0^t z^T(s)u(s)\,ds/\int_0^t u^T(s)u(s)\,ds$ versus time with the initial condition $\phi(t) = [-0.1\ 0.1]^T$. Clearly, $P(t)$ converges to 0.6817, which is less than the prescribed passivity performance index 4.3061 in Table 4.

Case III. H_∞ performance. By using the LMIs in Theorem 3.4 and letting $\Gamma_1 = -I$, $\Gamma_2 = 0$, $\Gamma_3 = \gamma^2 I$, and $\Gamma_4 = 0$, the extended dissipativity performance becomes the H_∞ performance. The maximum allowable values of δ_2 with various γ can be obtained for $\delta_1 = 0.5$, which are depicted in Table 5. Figure 5 shows the plot of $H(t) = \sqrt{\int_0^t z^T(s)z(s)\,ds/\int_0^t u^T(s)u(s)\,ds}$ versus time with the initial condition $\phi(t) = [-0.1\ 0.1]^T$. Clearly, $H(t)$ converges to 3.0415.

Case IV. Dissipativity performance. By applying the LMIs in Theorem 3.4 and taking $\Gamma_1 = -I$, $\Gamma_2 = I$, $\Gamma_3 = \mathcal{R} - \gamma I$, $\mathcal{R} = 8I$, and $\Gamma_4 = 0$, the extended dissipativity performance

Table 5 The maximum allowable values of δ_2 for Case III and Case IV in Example 4.4 with $\delta_1 = 0.5$, and various γ
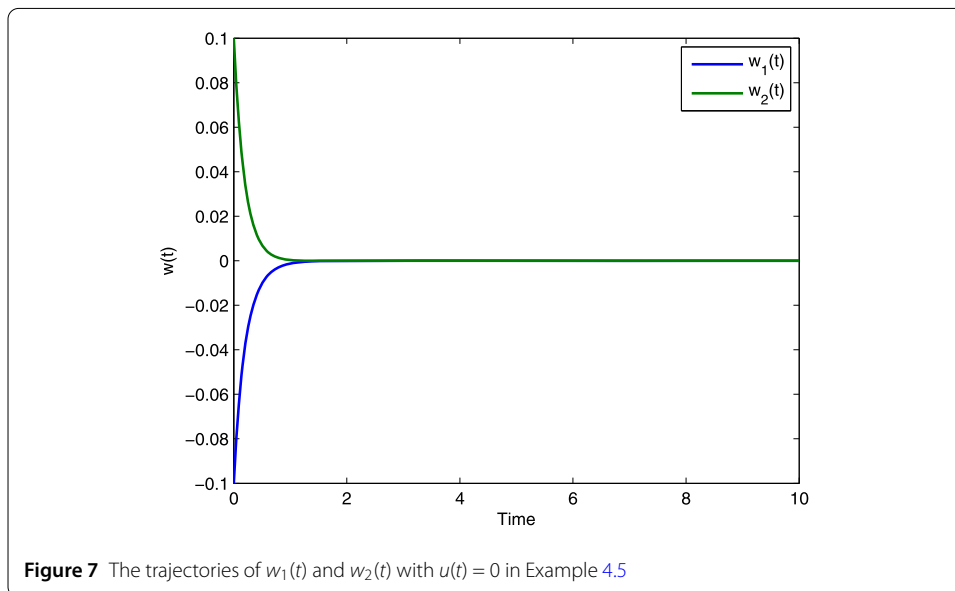
Methods	$\gamma = 2.1$	$\gamma = 2.3$	$\gamma = 2.5$	$\gamma = 2.7$
Case III. H_∞	0.6419	0.8795	1.0383	1.1621
Case IV. Dissipativity	0.9871	0.9548	0.9198	0.8815

Figure 5 The trajectory of $H(t)$ in Example 4.4

Figure 6 The trajectory of $D(t)$ in Example 4.4
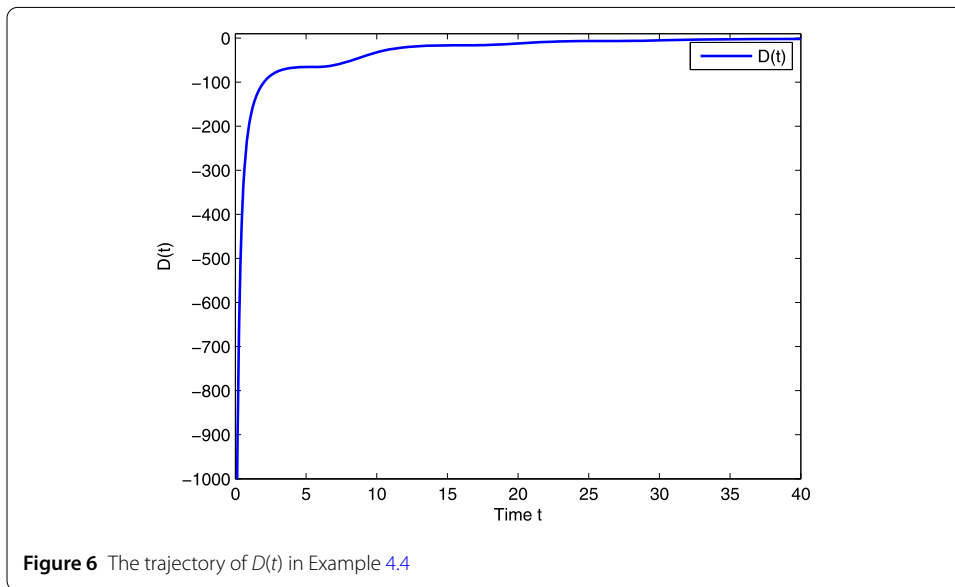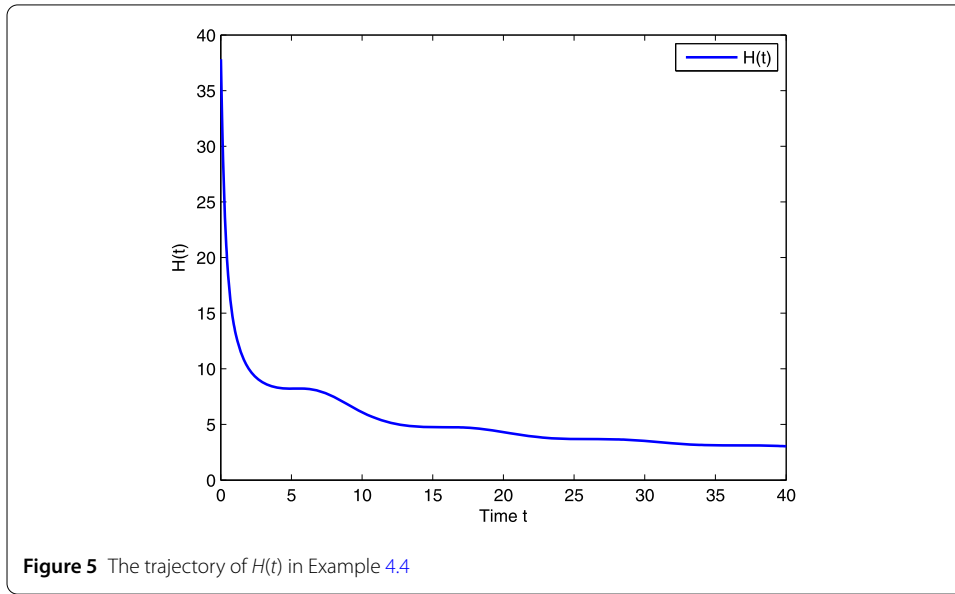
determines the dissipativity performance. The maximum allowable values of δ_2 with various γ can be achieved for $\delta_1 = 0.5$, which are shown in Table 5. Figure 6 shows the plot of $D(t) = (\int_0^t -z^T(s)z(s) + 2z^T(s)u(s) + 8u^T(s)u(s)\,ds)/(\int_0^t u^T(s)u(s)\,ds)$ versus time with the initial condition $\phi(t) = [-0.1\ 0.1]^T$. Clearly, $D(t)$ converges to -1.9323.

Table 6 The maximum allowable values of δ_2 for different values of δ_1 in Example 4.5

Method	$\delta_1 = 0.1$	$\delta_1 = 0.3$	$\delta_1 = 0.5$	$\delta_1 = 0.7$
Theorem 3.3	0.5031	0.5121	0.6134	0.8246

Figure 7 The trajectories of $w_1(t)$ and $w_2(t)$ with $u(t) = 0$ in Example 4.5

Example 4.5 Consider the neural network (1) with $\sigma_1 = 0.1$, $\sigma_2 = 0.5$, $\beta_1 = 2$, $\beta_2 = 3$,

$$C = 5I, \qquad B_0 = \begin{bmatrix} 0.2 & -0.1 \\ -0.5 & 0.1 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} -0.5 & 0 \\ -0.3 & -0.2 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.15 & 0.1 \\ 0 & -0.3 \end{bmatrix}, \qquad B_3 = I, \qquad W = -0.4I,$$

$$D_1 = D_2 = D_3 = 0.1I, \qquad F_p = G_p = H_p = I, \quad \text{and} \quad F_m = G_m = H_m = 0.$$

Then, the extended dissipativity performance of system (1) is determined by choosing $\Gamma_1 = -I$, $\Gamma_2 = I$, $\Gamma_3 = (8 - \gamma)I$, and $\Gamma_4 = I$. Here solving LMIs of (59) and (60) in Theorem 3.3, we obtain the maximum allowable values of δ_2 for different values of δ_1 which are shown in Table 6. Figure 7 shows the response solution $w(t)$ in Example 4.5 where $u(t) = 0$ and the initial condition $\phi(t) = [-0.1 \ 0.1]^T$. Figure 8 shows the response solution $w(t)$ in Example 4.5 where $u(t)$ is Gaussian noise with mean 0 and variance 1, and the initial condition $\phi(t) = [-0.1 \ 0.1]^T$.

5 Conclusions

In this paper, we focus on the problem of asymptotic stability and extended dissipativity analysis for the generalized neural networks with interval discrete and distributed time-varying delays. Firstly, we obtain new asymptotic stability criteria for the generalized neural networks and also achieve an improved asymptotic stability criterion for a special case of the generalized neural networks by using a suitable Lyapunov–Krasovskii functional (LKF), an improved Wirtinger single integral inequality, a novel triple integral inequality, and convex combination technique. Then, the asymptotic stability results are applied

Figure 8 The trajectories of $w_1(t)$ and $w_2(t)$ in Example 4.5
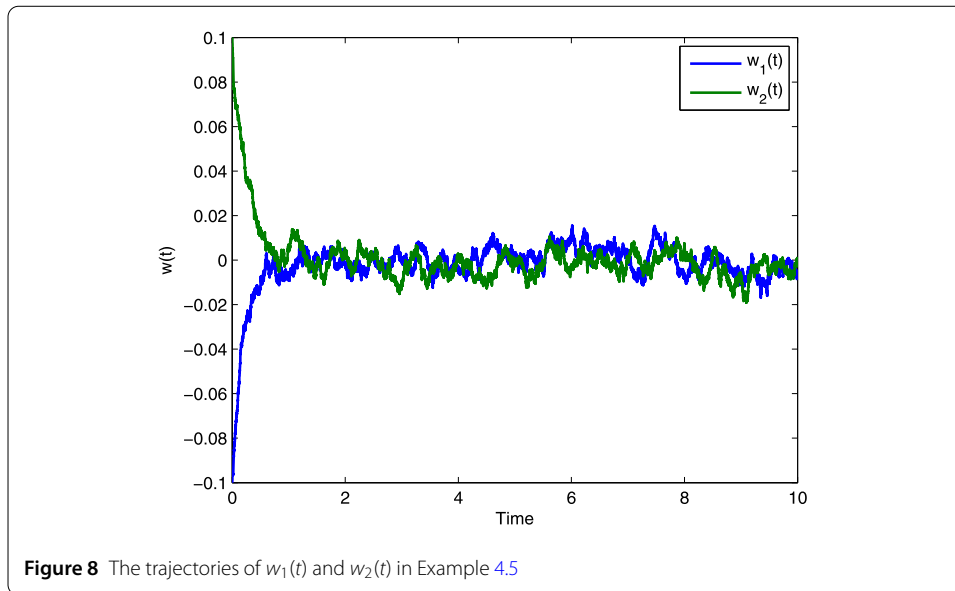
to extended dissipativity analysis that covers H_∞, L_2-L_∞, passivity, and dissipativity performance by setting parameters in the general performance index. Finally, we demonstrate numerical examples that are less conservative than in other references. Moreover, we present numerical examples for asymptotic stability and extended dissipativity performance of the generalized neural networks, including a special case of the generalized neural networks. In the future work, the derived results and methods in this paper are expected to be applied to other systems such as fuzzy generalized neural networks, generalized neural networks with Markovian switching, complex dynamical networks, and so on [10, 32, 46].

Availability of data and materials
Not applicable.

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
The authors claim to have contributed significantly and equally to this work. All authors read and approved the final manuscript.

Author details
[1] Department of Mathematics, Khon Kaen University, 40002 Khon Kaen, Thailand. [2] Department of Mathematics, University of Pha Yao, 56000 Pha Yao, Thailand.

Publisher's Note
Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

References

1. Cichoki, A., Unbehauen, R.: Neural Networks for Optimization and Signal Processing. Wiley, Chichester (1993)
2. Watta, P.B., Wang, K., Hassoun, M.H.: Recurrent neural nets as dynamical Boolean systems with applications to associative memory. IEEE Trans. Neural Netw. **8**, 1268–1280 (1997)
3. Cohen, M.A., Grossberg, S.: Absolute stability of global pattern formation and parallel memory storage by competitive neural networks. IEEE Trans. Syst. Man Cybern. **13**, 815–826 (1983)
4. Meng, X.Z., Zhao, S.N., Zhang, W.Y.: Adaptive dynamics analysis of a predator–prey model with selective disturbance. Appl. Math. Comput. **266**, 946–958 (2015)
5. Chen, Y., Zheng, W.X.: Stochastic state estimation for neural networks with distributed delays and Markovian jump. Neural Netw. **25**, 14–20 (2012)
6. Chen, J., Sun, J., Liu, G.P., Rees, D.: New delay-dependent stability criteria for neural networks with time-varying interval delay. Phys. Lett. A **374**, 4397–4405 (2010)
7. Wang, J.A., Ma, X.H., Wen, X.Y.: Less conservative stability criteria for neural networks with interval time-varying delay based on delay-partitioning approach. Neurocomputing **155**, 146–152 (2015)
8. Lv, X., Li, X.: Delay-dependent dissipativity of neural networks with mixed non-differentiable interval delays. Neurocomputing **267**, 85–94 (2017)
9. Yu, H.J., He, Y., Wu, M.: Delay-dependent state estimation for neural networks with time-varying delay. Neurocomputing **275**, 881–887 (2018)
10. Li, Q., Liang, J.: Improved stabilization results for Markovian switching CVNNs with partly unknown transition rates. Neural Process. Lett. **52**(2), 1189–1205 (2020)
11. Zeng, H.B., Park, J.H., Zhang, C.F., Wang, W.: Stability and dissipativity analysis of static neural networks with interval time-varying delay. J. Franklin Inst. **352**, 1284–1295 (2015)
12. Zhang, X.M., Han, Q.L.: Global asymptotic stability for a class of generalized neural networks with interval time-varying delays. IEEE Trans. Neural Netw. **22**, 1180–1192 (2011)
13. Lin, W.J., He, Y., Zhang, C.K., Long, F., Wu, M.: Dissipativity analysis for neural networks with two-delay components using an extended reciprocally convex matrix inequality. Inf. Sci. **450**, 169–181 (2018)
14. Manivannan, R., Samidurai, R., Cao, J., Alsaedi, A., Alsaadi, F.E.: Global exponential stability and dissipativity of generalized neural networks with time-varying delay signals. Neural Netw. **87**, 149–159 (2017)
15. Liu, Y., Lee, S.M., Kwon, O.M., Park, J.H.: New approach to stability criteria for generalized neural networks with interval time-varying delays. Neurocomputing **149**, 1544–1551 (2015)
16. Rakkiyappan, R., Sivasamy, R., Park, J.H., Lee, T.H.: An improved stability criterion for generalized neural networks with additive time-varying delays. Neurocomputing **171**, 615–624 (2016)
17. Shu, Y., Liu, X.G., Qiu, S., Wang, F.: Dissipativity analysis for generalized neural networks with Markovian jump parameters and time-varying delay. Nonlinear Dyn. **89**, 2125–2140 (2017)
18. Li, Z., Bai, Y., Huang, C., Yan, H., Mu, S.: Improved stability analysis for delayed neural networks. IEEE Trans. Neural Netw. Learn. Syst. **29**, 4535–4541 (2018)
19. Du, Y., Liu, X., Zhong, S.: Robust reliable H_∞ control for neural networks with mixed time delays. Chaos Solitons Fractals **91**, 1–8 (2016)
20. Rajavel, S., Samidurai, R., Cao, J., Alsaedi, A., Ahmad, B.: Finite-time non-fragile passivity control for neural networks with time-varying delay. Appl. Math. Comput. **297**, 145–158 (2017)
21. Thuan, M.V., Trinh, H., Hien, L.V.: New inequality-based approach to passivity analysis of neural networks with interval time-varying delay. Neurocomputing **194**, 301–307 (2016)
22. Maharajan, C., Raja, R., Cao, J., Rajchakit, G., Alsaedi, A.: Novel results on passivity and exponential passivity for multiple discrete delayed neutral-type neural networks with leakage and distributed time-delays. Chaos Solitons Fractals **115**, 268–282 (2018)
23. Senthilraj, S., Raja, R., Zhu, Q., Samidurai, R., Yao, Z.: Exponential passivity analysis of stochastic neural networks with leakage, distributed delays and Markovian jumping parameters. Neurocomputing **175**, 401–410 (2015)
24. Yotha, N., Botmart, T., Mukdasai, K., Weera, W.: Improved delay-dependent approach to passivity analysis for uncertain neural networks with discrete interval and distributed time-varying delays. Vietnam J. Math. **45**(4), 721–736 (2017)
25. Cao, J., Manivannan, R., Chong, K.T., Lv, X.: Enhanced L_2–L_∞ state estimation design for delayed neural networks including leakage term via quadratic-type generalized free-matrix-based integral inequality. J. Franklin Inst. **356**, 7371–7392 (2019)
26. Kumar, S.V., Anthoni, S.M., Raja, R.: Dissipative analysis for aircraft flight control systems with randomly occurring uncertainties via non-fragile sampled-data control. Math. Comput. Simul. **155**, 217–226 (2019)
27. Raja, R., Karthik Raja, U., Samidurai, R., Leelamani, A.: Improved stochastic dissipativity of uncertain discrete-time neural networks with multiple delays and impulses. Int. J. Mach. Learn. Cybern. **6**(2), 289–305 (2015)
28. Willems, J.C.: Dissipative dynamical systems part I: general theory. Arch. Ration. Mech. Anal. **45**(5), 321–351 (1972)
29. Niu, Y., Wang, X., Lu, J.: Dissipative-based adaptive neural control for nonlinear systems. J. Control Theory Appl. **2**, 126–130 (2004)
30. Jeltsema, D., Scherpen, J.M.A.: Tuning of passivity-preserving controllers for switched-mode power converters. IEEE Trans. Autom. Control **49**, 1333–1344 (2004)
31. Senthilraj, S., Raja, R., Cao, J., Fardoun, H.M.: Dissipativity analysis of stochastic fuzzy neural networks with randomly occurring uncertainties using delay dividing approach. Nonlinear Anal., Model. Control **24**(4), 561–581 (2019)
32. Li, Q., Liang, J.: Dissipativity of the stochastic Markovian switching CVNNs with randomly occurring uncertainties and general uncertain transition rates. Int. J. Syst. Sci. **51**(6), 1102–1118 (2020)
33. Zhang, B., Zheng, W.X., Xu, S.: Filtering of Markovian jump delay systems based on a new performance index. IEEE Trans. Circuits Syst. I, Regul. Pap. **60**, 1250–1263 (2013)
34. Lee, T.H., Park, M.J., Park, J.H., Kwon, O.M., Lee, S.M.: Extended dissipative analysis for neural networks with time-varying delays. IEEE Trans. Neural Netw. Learn. Syst. **25**, 1936–1941 (2014)
35. Wang, X., She, K., Zhong, S., Cheng, J.: On extended dissipativity analysis for neural networks with time-varying delay and general activation functions. Adv. Differ. Equ. **2016**, 1 (2016)
36. Lin, W.J., He, Y., Zhang, C.K., Wu, M., Shen, J.: Extended dissipativity analysis for Markovian jump neural networks with time-varying delay via delay-product-type functionals. IEEE Trans. Neural Netw. Learn. Syst. **30**(8), 2527–2537 (2019)

37. Manivannan, R., Mahendrakumar, G., Samidurai, R., Cao, J., Alsaedi, A.: Exponential stability and extended dissipativity criteria for generalized neural networks with interval time-varying delay signals. J. Franklin Inst. **354**, 4353–4376 (2017)

38. Sun, J., Liu, G.P., Chen, J.: Delay-dependent stability and stabilization of neutral time-delay systems. Int. J. Robust Nonlinear Control **19**, 1364–1375 (2009)

39. Xie, W., Zhu, H., Zhong, S., Zhang, D., Shi, K., Cheng, J.: Extended dissipative estimator design for uncertain switched delayed neural networks via a novel triple integral inequality. Appl. Math. Comput. **335**, 82–102 (2018)

40. Park, P.G., Lee, W.I., Lee, S.Y.: Auxiliary function-based integral/summation inequalities: application to continuous/discrete time-delay systems. Int. J. Control. Autom. Syst. **14**, 3–11 (2016)

41. Farnam, A., Esfanjani, R.M.: Improved linear matrix inequality approach to stability analysis of linear systems with interval time-varying delays. J. Comput. Appl. Math. **294**, 49–56 (2016)

42. Saravanan, S., Ali, M.S., Alsaedi, A., Ahmad, B.: Finite-time passivity for neutral-type neural networks with time-varying delays—via auxiliary function-based integral inequalities. Nonlinear Anal., Model. Control **25**(2), 206–224 (2020)

43. Yang, Q., Ren, Q., Xie, X.: New delay dependent stability criteria for recurrent neural networks with interval time-varying delay. ISA Trans. **53**, 994–999 (2014)

44. Lee, W.I., Lee, S.Y., Park, P.: Improved stability criteria for recurrent neural networks with interval time-varying delays via new Lyapunov functionals. Neurocomputing **155**, 128–134 (2015)

45. Saravanakumar, R., Ali, M.S., Ahn, C.K., Karimi, H.R., Shi, P.: Stability of Markovian jump generalized neural networks with interval time-varying delays. IEEE Trans. Neural Netw. Learn. Syst. **28**, 1840–1850 (2017)

46. Niamsup, P., Botmart, T., Weera, W.: Modified function projective synchronization of complex dynamical networks with mixed time-varying and asymmetric coupling delays via new hybrid pinning adaptive control. Adv. Differ. Equ. **2017**(1), 1 (2017)