


RESEARCH

Open Access



Finite-time H_∞ bounded control of networked control systems with mixed delays and stochastic nonlinearities

Ling Hou^{1*} , Dongyan Chen^{1,2} and Chan He³

*Correspondence:

houling0729@163.com

¹Department of Mathematics, Harbin University of Science and Technology, Harbin, China

Full list of author information is available at the end of the article

Abstract

In this study, we investigate the stochastic finite-time H_∞ bounded control problem for a class of networked control systems (NCSs) subject to mixed delays, stochastic nonlinearities, and randomly missing measurement. The mixed delays consist of discrete time-varying and distributed delays. The stochastic nonlinearities satisfy statistical means. The missing measurement is modeled by a Bernoulli-distributed random variable. By applying stochastic analysis method we present sufficient conditions guaranteeing the stochastic finite-time boundedness of a closed-loop system with desired H_∞ performance level within a finite time interval. Moreover, dynamic output feedback controller can be obtained in terms of a set of matrix inequalities, which can be easily solved by using the cone complementarity linearization method (CCLM). Finally, we provide two numerical examples to illustrate the validity of the proposed design technique.

Keywords: Finite-time H_∞ ; Dynamic output feedback control; Stochastic nonlinearities; Mixed delays; Lyapunov–Krasovskii functional method

1 Introduction

Networked control systems (NCSs), which integrate the development of automatic control technology, network communication technology, and microelectronics technology, have become a hot research topic in the international control field in recent years. Different from the traditional control system, it links sensors, controllers, and actuators distributed in different regions of the control system to form a closed-loop feedback control system through a communication network. The transmission of data and information between each node is realized through the network. For many years now, these kinds of systems can be found in a variety of engineering areas such as automobiles, manufacturing plants, and aircrafts. At present the research on networked control systems is mostly concentrated on continuous systems, but discrete-time networked control systems play important roles in complex control systems and remote control systems, for example, pump-controlled motor speed servo control, pendulum control, remote diagnostic, and troubleshooting control [1–6]. NCSs have many advantages in comparison with traditional control systems, but the introduction of communication networks also bring a series of problems.

© The Author(s) 2020. This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

The communication between sensors, controllers, and actuators is accomplished by sharing network platform after the networks are introduced into the control system. However, due to limited spectrum resources, the problems of network connection interruption, data collision, network congestion, or resource competition are inevitable, which will inevitably lead to network delays, packet dropouts, or data missing phenomena in both sensor-to-controller link and controller-to-actuator link. In addition, for the actual control system, nonlinear factors are inevitable and cannot be ignored. Therefore it is necessary to consider the network delays, missing measurement, and nonlinearity simultaneously in the study of NCSs. In this paper, we focus on the problem of stochastic finite-time H_∞ control for discrete-time NCSs with mixed delays, stochastic nonlinearities, and randomly missing measurement.

Network delays in the NCSs are exposed when the data exchange from the sensor to controller and from the controller to actuator. At present the delays are generally divided into two categories according to the occurrence way, discrete delays and distributed delays. In the recent years, some works have been done on the analysis and synthesis of systems for various types of delays [7–12]. However, most of these studies considered one of the delays mentioned before, whereas few considered two kinds of delays simultaneously, that is, mixed delays (both discrete and distributed). In addition, in the existing literature on mixed delays, continuous systems were considered. Note that infinite distributed delays also exist in discrete case. With the development of applications to digital control systems, it is important to discuss how mixed delays affect the dynamic behavior of discrete NCSs. Some works in this field have been made [13, 14].

On the other hand, the missing measurement is also one of the problems that cause the performance deterioration of NCS indicators. Therefore it is particularly important to consider the impact of missing measurement on the performance of NCSs. The missing measurement is usually described by two forms: the first is to regard the missing measurement as a random variable that satisfies the Bernoulli distribution, and the second is using Markov chains to describe random forms. At present, it is popular describing missing measurement by Bernoulli distribution [15, 16]. Recently, missing measurement has attracted considerable attention, and many results have been obtained; see, for example, [17, 18]. In [17] the quantized recursive filtering problem was further investigated for a class of nonlinear systems. Hu et al. [18] studied event-based filtering problem for a class of nonlinear systems. In addition, it is well known that the nonlinearities, such as delays and missing measurements, also cause instability and poor performance of dynamic systems [19–21]. If the system involves serious nonlinearities, then it is difficult to design a controller with good performance. In recent years, stochastic nonlinearities described by statistical means have become a popular form. Hu et al. [22] investigated the H_∞ filtering problem for a class of stochastic nonlinear systems with time-varying delay and multiple missing measurements. Other related results can be found in [23–26].

It should be pointed out that all the mentioned references on the problems of Lyapunov asymptotical stability for NCSs defined over an infinite time interval, whereas in many practical applications the main concern is not only system dynamic behavior on an infinite time interval but also a bound of system trajectories over a fixed finite-time interval. In this sense, Dorato [27] first proposed the concept of finite-time stability (FTS). Subsequently, by using Lyapunov theory and LMI technique some sufficient conditions have

been established to ensure FTS and finite-time boundedness in various systems [28–30], particularly, in the NCSs [31–36]. Recently, Ma and Song [37, 38] studied a finite-time dissipative problem for a class of singular discrete-time systems and discrete stochastic systems. However, it is worth noting that most of the mentioned references on problems of finite-time control were considered for discrete-time linear NCSs, whereas in the actual systems, most of the controlled objects or processes exhibit nonlinear features. To the best of authors’ knowledge, so far, the problem of stochastic finite-time H_∞ control for discrete-time NCSs with mixed delays, stochastic nonlinearities, and randomly missing measurement has not been studied. This motivates our present research. It should be remarked that it is difficult to analyze this problem owing to two reasons: (1) How to characterize a suitable definition of stochastic finite-time boundedness for discrete-time NCSs with mixed delays, stochastic nonlinearities, and randomly missing measurement? (2) How to deal with mixed delays, stochastic nonlinearities, and randomly missing measurement in the controller design stage.

Our aim is designing a stochastic finite-time output feedback controller by fully taking into account mixed delays, stochastic nonlinearities, and randomly missing measurement. Through such a output feedback controller, both stochastic finite-time boundedness and desired H_∞ performance can be guaranteed. The main contributions of this paper can be itemized as follows: (1) The concept of stochastic finite-time boundedness is extended to more general discrete-time NCSs containing mixed delays, stochastic nonlinearities, and randomly missing measurement. (2) The effects of mixed delays, stochastic nonlinearities, and randomly missing measurement on system performance are considered, which are important in system analysis and synthesis. (3) By constructing a novel Lyapunov–Krasovskii functional and using stochastic analysis method, a more general controller, the output feedback controller, is designed. This controller is more useful than the previous given state feedback controller when the state of the system is not measurable.

Notations \mathbb{R}^n and $\mathbb{R}^{m \times n}$ respectively denote the space of n -dimensional real vectors and $m \times n$ real matrices; I and 0 stand for the identity and zero matrices of compatible dimension, respectively; $\text{Prob}\{\cdot\}$ denotes the occurrence probability of the event; and $\mathbb{E}\{\cdot\}$ is the mathematical expectation operator with respect to the given probability measure. $\text{Diag}\{\cdot\}$ stands for the block diagonal matrix. The notation $X \geq Y$ or $X > Y$ means that $X - Y$ is positive semidefinite or positive definite, where X and Y are both symmetric matrices. λ_{\max} and λ_{\min} are the maximum and minimum eigenvalues of a matrix, respectively. M^T denotes the transpose of matrix M , and $*$ denotes the matrix elements induced by symmetry.

2 Problem formulation

In this paper, we consider the following discrete-time NCS:

$$\begin{cases} x(k+1) = Ax(k) + A_d x(k-d(k)) + A_l \sum_{m=1}^{+\infty} \mu_m x(k-m) \\ \quad + Bu(k) + f(k) + D\omega(k), \\ z(k) = E_1 x(k) + E_2 \omega(k), \\ x(k) = \varphi(k), \quad -\infty < k \leq 0, \end{cases} \tag{1}$$

where $x(k) \in \mathbb{R}^n$ is the state vector, $z(k) \in \mathbb{R}^l$ is the controlled output, $u(k) \in \mathbb{R}^p$ is the control input, $\sum_{m=1}^{+\infty} \mu_m x(k-m)$ is the distributed delay, and $A, A_d, A_l, B, D, E_1, E_2$ are known real constant matrices of appropriate dimensions. For simplicity, we denote $x_d = x(k-d(k))$ and $x_m = x(k-m)$.

Before proceeding the main results, let us introduce the following assumptions.

Assumption 1 $d(k)$ is time-varying communication delay satisfying

$$d_m \leq d(k) \leq d_M, \quad d_m > 0, d_M > 0, \tag{2}$$

where d_m and d_M are given positive numbers.

Assumption 2 $f(k)$ is stochastic nonlinearities cover $x(k), x(k-d(k))$ and $\sum_{m=1}^{+\infty} \mu_m x(k-m)$ in a statistical sense, which is bounded as follows:

$$\begin{aligned} \mathbb{E}[f(k)] &= 0, \\ \mathbb{E}[f(k)f^T(k)] &= \sum_{i=1}^q \rho_i \rho_i^T \left[x^T(k) A_i x(k) + x_d^T A_{di} x_d + \left(\sum_{m=1}^{+\infty} \mu_m x_m \right)^T A_{li} \sum_{m=1}^{+\infty} \mu_m x_m \right], \end{aligned} \tag{3}$$

where A_i, A_{di} , and A_{li} are known positive definite matrices of appropriate dimensions, ρ_i ($i = 1, \dots, q$) are known as column vectors, and $q > 1$ is a known constant. The constants μ_m satisfy the following convergence conditions:

$$\bar{\mu} = \sum_{m=1}^{+\infty} \mu_m \leq \sum_{m=1}^{+\infty} m \mu_m < +\infty. \tag{4}$$

Remark 1 At present, delays have attracted the attention of many scholars. However, most of the existing results are concerned with either discrete delays [32] or distributed delays [8]. Different from [32], on the one hand, we introduce stochastic nonlinearities into the study of finite-time bounded control. It is worth noting that deterministic systems and stochastic systems have different properties. We need to handle this separately. On the other hand, authors in [32] considered only discrete delays, whereas we consider both discrete delays and distributed delays. It is shown that the distributed delays have an important effect on the finite-time boundedness of the system.

Assumption 3 External disturbance input $\omega(k)$ is time varying and satisfies

$$\sum_{k=0}^N \omega^T(k) \omega(k) \leq \bar{\omega}, \quad \bar{\omega} \geq 0, \tag{5}$$

for any given positive number $\bar{\omega}$.

In this paper, the randomly missing measurement is described by

$$y(k) = \alpha(k) F_1 x(k) + F_2 \omega(k), \tag{6}$$

where $y(k)$ denotes the measured output vector, F_1, F_2 are parameter matrices of appropriate dimensions, and $\alpha(k)$ is a Bernoulli-distributed stochastic variable, that is,

$$\text{Prob}\{\alpha(k) = 1\} = \bar{\alpha}, \quad \text{Prob}\{\alpha(k) = 0\} = 1 - \bar{\alpha}, \tag{7}$$

where $\bar{\alpha} \in [0, 1]$ is a constant, and the variance $\tilde{\alpha}^2 = \bar{\alpha}(1 - \bar{\alpha})$.

In this paper, we consider the following dynamic output feedback:

$$\begin{cases} \hat{x}(k+1) = A_K \hat{x}(k) + B_K y(k), \\ u(k) = C_K \hat{x}(k), \end{cases} \tag{8}$$

where $\hat{x}(k) \in \mathbb{R}^n$ is the state estimation of system (1), and $A_K, B_K,$ and C_K are the controller gain parameters to be determined.

By the combination of (6) and (8) with (1) the resultant closed-loop system can be written as

$$\begin{cases} \eta(k+1) = \bar{A}\eta(k) + (\alpha(k) - \bar{\alpha})\tilde{A}\eta(k) + \bar{A}_d\eta_d \\ \quad + \bar{A}_l \sum_{m=1}^{+\infty} \mu_m \eta_d + \bar{D}\omega(k) + \bar{G}f(k), \\ z(k) = \bar{E}_1\eta(k) + E_2\omega(k), \end{cases} \tag{9}$$

where

$$\begin{aligned} \eta(k) &= \begin{bmatrix} x(k) \\ \hat{x}(k) \end{bmatrix}, \quad \eta_d = \eta(k-d(k)), \quad \eta_m = \eta(k-m), \quad \bar{A} = \begin{bmatrix} A & DC_K \\ \bar{\alpha}B_KF_1 & A_K \end{bmatrix}, \\ \tilde{A} &= \begin{bmatrix} 0 & 0 \\ B_KF_1 & 0 \end{bmatrix}, \quad \bar{A}_d = \begin{bmatrix} A_d & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_l = \begin{bmatrix} A_l & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{E}_1 = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}^T, \\ \bar{D} &= \begin{bmatrix} D \\ B_KF_2 \end{bmatrix}, \quad \bar{G} = \begin{bmatrix} I \\ 0 \end{bmatrix}. \end{aligned}$$

The objective of this paper is to deal with stochastic finite-time H_∞ control problem for closed-loop dynamic system (9). We introduce the following definitions and lemmas before giving the main results.

Definition 1 For a given symmetric matrix $R > 0$ and positive numbers c_1, c_2 ($c_1 < c_2$), $\bar{\omega}, N$, system (9) is stochastically finite-time bounded (SFTB) with respect to $(c_1, c_2, \bar{\omega}, R, N)$ if

$$\sup_{-\infty < l \leq 0} \mathbb{E}\{x(l)^T R x(l)\} \leq c_1 \quad \Rightarrow \quad \mathbb{E}\{x(k)^T R x(k)\} \leq c_2, \quad k \in \{1, \dots, N\},$$

for all $\omega(k)$ satisfying Assumption 3.

Definition 2 For given symmetric matrix $R > 0$, system (9) is stochastically finite-time H_∞ bounded (SFTH $_\infty$ B) with respect to $(c_1, c_2, \bar{\omega}, \gamma, R, N)$ if the following two conditions hold:

- (1) System (9) is SFTB with respect to $(c_1, c_2, \bar{\omega}, R, N)$.

(2) Under the zero-initial condition, the output $z(k)$ satisfies

$$\mathbb{E} \left\{ \sum_{s=0}^N z^T(s)z(s) \right\} \leq \gamma \sum_{s=0}^N \omega^T(s)\omega(s) \tag{10}$$

for all $\omega(k)$ satisfying Assumption 3.

Lemma 1 (Schur complement [17]) *Let $S_1, S_2,$ and S_3 be constant matrices such that $S_1 = S_1^T$ and $S_2 = S_2^T > 0$. Then $S_1 + S_3^T S_2^{-1} S_3 < 0$ if and only if*

$$\begin{bmatrix} S_1 & S_3^T \\ * & -S_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -S_2 & S_3 \\ * & S_1 \end{bmatrix} < 0.$$

Lemma 2 ([17]) *Let $M \in R^{n \times n}$ be a positive semidefnite matrix, let $x_i \in R^n$, and let α_i ($i = 1, 2, \dots$) be constant series. If the series $\alpha_i \geq 1$ are convergent, then*

$$\left(\sum_{m=1}^{+\infty} \alpha_m x_m \right)^T M \left(\sum_{m=1}^{+\infty} \alpha_m x_m \right) \leq \left(\sum_{m=1}^{+\infty} \alpha_m \right) \sum_{m=1}^{+\infty} \alpha_m x_m^T M x_m. \tag{11}$$

3 Main results

In this section, we first investigate the problem of stochastic finite-time boundedness for dynamic system (9). Furthermore, we design the dynamic gain matrices $A_K, B_K,$ and C_K .

3.1 Stochastic finite-time boundedness analysis

In this subsection, we consider the stochastic finite-time boundedness for discrete-time NCSs with mixed delays, stochastic nonlinearities, and randomly missing measurement.

Theorem 1 *For given scalar $\delta \geq 1$ and symmetric matrix $R > 0$, system (9) is SFTB with respect to $(c_1, c_2, \bar{\omega}, R, N)$ if there exist positive scalars λ_i ($i = 1, 2, 3, 4$) and ξ_i ($i = 1, 2, \dots, q$), and symmetric matrices $P > 0, Q > 0,$ and $Z > 0$ such that the following inequalities hold:*

$$\begin{bmatrix} -\xi_i & \rho_i^T \bar{G}^T \\ * & -P^{-1} \end{bmatrix} < 0 \quad (i = 1, 2, \dots, q), \tag{12}$$

$$\begin{bmatrix} \Phi & 0 & 0 & 0 & \bar{A}^T P & \tilde{\alpha} \tilde{A}^T P \\ * & -\delta^{d_m} Q + \sum_{i=1}^q \xi_i \bar{A}_{di} & 0 & 0 & \bar{A}_d^T P & 0 \\ * & * & -\frac{1}{\mu} Z + \sum_{i=1}^q \xi_i \bar{A}_{li} & 0 & \bar{A}_l^T P & 0 \\ * & * & * & -\frac{\gamma}{\delta^N} I & \bar{D}^T P & 0 \\ * & * & * & * & -P & 0 \\ * & * & * & * & * & -P \end{bmatrix} < 0, \tag{13}$$

$$\lambda_1 R < P < \lambda_2 R, \quad 0 < Q < \lambda_3 R, \quad 0 < Z < \lambda_4 R, \tag{14}$$

$$\begin{bmatrix} \gamma \bar{\omega} - c_2 \delta \lambda_1 & c_1 \delta^{N+1} \lambda_2 & \rho_1 \lambda_3 & \rho_2 \lambda_4 \\ * & -c_1 \delta^{N+1} \lambda_2 & 0 & 0 \\ * & * & -\rho_1 \lambda_3 & 0 \\ * & * & * & -\rho_2 \lambda_4 \end{bmatrix} < 0, \tag{15}$$

where

$$\begin{aligned} \Phi &:= (d_M - d_m + 1)Q + \bar{\mu}Z - \delta P + \sum_{i=1}^q \xi_i \bar{A}_i, \\ \bar{A}_i &= \begin{bmatrix} A_i & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_{di} = \begin{bmatrix} A_{di} & 0 \\ 0 & 0 \end{bmatrix}, \quad \bar{A}_{li} = \begin{bmatrix} A_{li} & 0 \\ 0 & 0 \end{bmatrix}, \\ \rho_1 &:= \delta^{N+d_M-1} \left[d_M \delta + \frac{1}{2}(d_M - d_m)(d_M + d_m - 1) \right], \quad \rho_2 := \delta^{N+1} \sigma, \\ \sigma &= \mu_1 + (1 + \delta)\mu_2 + \dots + (1 + \delta + \dots + \delta^{d_M-1})\mu_{d_M}, \\ \tau_1 &= d_M - d_m + 1, \quad \tau_2 = d_M + d_m - 1. \end{aligned}$$

Proof To show the stochastic finite-time boundedness of system (9), we choose a Lyapunov–Krasovskii functional candidate as follows:

$$V(k) = V_1(k) + V_2(k) + V_3(k), \tag{16}$$

where

$$\begin{aligned} V_1(k) &= \eta^T(k)P\eta(k), \\ V_2(k) &= \sum_{s=k-d(k)}^{k-1} \delta^{k-1-s} \eta^T(s)Q\eta(s) + \sum_{s=-d_M+2}^{-d_m+1} \sum_{t=k+s-1}^{k-1} \delta^{k-1-t} \eta^T(t)Q\eta(t), \\ V_3(k) &= \sum_{m=1}^{+\infty} \mu_m \sum_{l=k-m}^{k-1} \delta^{k-1-l} \eta^T(l)Z\eta(l). \end{aligned}$$

Let us assume that

$$\mathbb{E}\{\eta(k)^T R \eta(k)\} \leq c_1, \quad -\infty < k \leq 0. \tag{17}$$

The goal is to show that $\mathbb{E}\{\eta(k)^T R \eta(k)\} \leq c_2$ for all $k \in \{1, \dots, N\}$ if conditions (12)–(15) hold.

Firstly, we calculate $\mathbb{E}\{V_i(k + 1) - \delta V_i(k)\}$ ($i = 1, 2, 3$) along the solution of (9):

$$\begin{aligned} &\mathbb{E}\{V_1(k + 1) - \delta V_1(k)\} \\ &= \mathbb{E}\{\eta^T(k + 1)P\eta(k + 1) - \delta \eta^T(k)P\eta(k)\} \\ &= \mathbb{E}\left\{ \eta^T(k)\bar{A}^T P \bar{A} \eta(k) + 2\eta^T(k)\bar{A}^T P \bar{A}_d \eta_d + 2\eta^T(k)\bar{A}^T P \bar{A}_l \sum_{m=1}^{+\infty} \mu_m \eta_m \right. \\ &\quad + 2\eta^T(k)\bar{A}^T P \bar{G} f(k) + 2\eta^T(k)\bar{A}^T P \bar{D} \omega(k) + 2(\alpha(k) - \bar{\alpha})\eta^T(k)\bar{A}^T P \bar{A}_l \sum_{m=1}^{+\infty} \mu_m \eta_m \\ &\quad + 2(\alpha(k) - \bar{\alpha})\eta^T(k)\bar{A}^T P \bar{A}_d \eta_d + (\alpha(k) - \bar{\alpha})^2 \eta^T(k)\bar{A}^T P \bar{A} \eta(k) \\ &\quad \left. + 2(\alpha(k) - \bar{\alpha})\eta^T(k)\bar{A}^T P \bar{G} f(k) + 2(\alpha(k) - \bar{\alpha})\eta^T(k)\bar{A}^T P \bar{D} \omega(k) \right\} \end{aligned}$$

$$\begin{aligned}
 &+ \eta_d^T \bar{A}_d^T P \bar{A}_d \eta_d + 2\eta_d^T \bar{A}_d^T P \bar{A}_l \sum_{m=1}^{+\infty} \mu_m \eta_m + 2\eta_d^T \bar{A}_d^T P \bar{G} f(k) + 2\eta_d^T \bar{A}_d^T P \bar{D} \omega(k) \\
 &+ \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \bar{A}_l^T P \bar{A}_l \sum_{m=1}^{+\infty} \mu_m \eta_m + 2 \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \bar{A}_l^T P \bar{G} f(k) \\
 &+ 2 \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \bar{A}_l^T P \bar{D} \omega(k) + f^T(k) \bar{G}^T P \bar{G} f(k) + 2f^T(k) \bar{G}^T P \bar{D} \omega(k) \\
 &+ \omega^T(k) \bar{D}^T P \bar{D} \omega(k) - \delta \eta^T(k) P \eta(k) \Big\}.
 \end{aligned}$$

By Assumption 2 and inequality (12) we have

$$\begin{aligned}
 &\mathbb{E}\{f^T(k) \bar{G}^T P \bar{G} f(k)\} \\
 &= \mathbb{E}\{\text{tr}[\bar{G}^T P \bar{G} f(k) f^T(k)]\} \\
 &= \sum_{i=1}^q \text{tr}(\bar{G}^T P \bar{G} \rho_i \rho_i^T) \left[\eta^T(k) \bar{A}_i \eta(k) + \eta_d^T \bar{A}_{di} \eta_d + \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \bar{A}_{li} \sum_{m=1}^{+\infty} \mu_m \eta_m \right] \\
 &< \sum_{i=1}^q \xi_i \left[\eta^T(k) \bar{A}_i \eta(k) + \eta_d^T \bar{A}_{di} \eta_d + \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \bar{A}_{li} \sum_{m=1}^{+\infty} \mu_m \eta_m \right]. \tag{18}
 \end{aligned}$$

Thus

$$\begin{aligned}
 &\mathbb{E}\{V_1(k+1) - \delta V_1(k)\} \\
 &< \eta^T(k) \left(\bar{A}^T P \bar{A} \tilde{\alpha}^2 \bar{A}^T P \bar{A} + \sum_{i=1}^q \xi_i \bar{A}_i - \delta P \right) \eta(k) + 2\eta^T(k) \bar{A}^T P \bar{A}_d \eta_d \\
 &+ 2\eta^T(k) \bar{A}^T P \bar{A}_l \sum_{m=1}^{+\infty} \mu_m \eta_m + 2\eta^T(k) \bar{A}^T P \bar{D} \omega(k) \\
 &+ \eta_d^T \left(\bar{A}_d^T P \bar{A}_d + \sum_{i=1}^q \xi_i \bar{A}_{di} \right) \eta_d \\
 &+ 2\eta_d^T \bar{A}_d^T P \bar{A}_l \sum_{m=1}^{+\infty} \mu_m \eta_m + 2\eta_d^T \bar{A}_d^T P \bar{D} \omega(k) \\
 &+ \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \left(\bar{A}_l^T P \bar{A}_l + \sum_{i=1}^q \xi_i \bar{A}_i \right) \sum_{m=1}^{+\infty} \mu_m \eta_m \\
 &+ 2 \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \bar{A}_l^T P \bar{D} \omega(k) + \omega^T(k) \bar{D}^T P \bar{D} \omega(k), \tag{19}
 \end{aligned}$$

$$\begin{aligned}
 &\mathbb{E}\{V_2(k+1) - \delta V_2(k)\} \\
 &= \sum_{s=k+1-d(k+1)}^k \delta^{k-s} \eta^T(s) Q \eta(s) - \sum_{s=k-d(k)}^{k-1} \delta^{k-s} \eta^T(s) Q \eta(s)
 \end{aligned}$$

$$\begin{aligned}
 & + \sum_{s=-d_M+2}^{-d_m+1} \sum_{t=k+s}^k \delta^{k-t} \eta^T(t) Q \eta(t) - \sum_{s=-d_M+2}^{-d_m+1} \sum_{t=k+s-1}^{k-1} \delta^{k-t} \eta^T(t) Q \eta(t) \\
 & \leq (d_M - d_m + 1) \eta^T(k) Q \eta(k) - \delta^{d_m} \eta_d^T Q \eta_d, \tag{20}
 \end{aligned}$$

$$\begin{aligned}
 & \mathbb{E}\{V_3(k+1) - \delta V_3(k)\} \\
 & = \sum_{m=1}^{+\infty} \mu_m \left[\sum_{l=k+1-m}^k \delta^{k-l} \eta^T(l) Z \eta(l) - \sum_{l=k-m}^{k-1} \delta^{k-l} \eta^T(l) Z \eta(l) \right] \\
 & = \sum_{m=1}^{+\infty} \mu_m [\eta^T(k) Z \eta(k) - \delta^m \eta_m^T Z \eta_m]. \tag{21}
 \end{aligned}$$

Noting that $\bar{\mu} = \sum_{m=1}^{+\infty} \mu_m$ and $\delta \geq 1$, we get

$$\mathbb{E}\{V_3(k+1) - \delta V_3(k)\} \leq \bar{\mu} \eta^T(k) Z \eta(k) - \sum_{m=1}^{+\infty} \mu_m \eta_m^T Z \eta_m. \tag{22}$$

Furthermore, from Lemma 2 we obtain

$$- \sum_{m=1}^{+\infty} \mu_m \eta_m^T Z \eta_m \leq -\frac{1}{\bar{\mu}} \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T Z \sum_{m=1}^{+\infty} \mu_m \eta_m. \tag{23}$$

Then

$$\mathbb{E}\{V_3(k+1) - \delta V_3(k)\} \leq \bar{\mu} \eta^T(k) Z \eta(k) - \frac{1}{\bar{\mu}} \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T Z \sum_{m=1}^{+\infty} \mu_m \eta_m. \tag{24}$$

Thus

$$\mathbb{E}\{V(k+1) - \delta V(k)\} < \xi^T(k) \Pi \xi(k),$$

where

$$\begin{aligned}
 \xi^T(k) & = \left[\eta^T(k) \quad \eta_d^T \quad \left(\sum_{m=1}^{+\infty} \mu_m \eta_m \right)^T \quad \omega^T(k) \right], \\
 \Pi & = \begin{bmatrix} \Pi_{11} & \bar{A}^T P \bar{A}_d & \bar{A}^T P \bar{A}_l & \bar{A}^T P \bar{D} \\ * & \Pi_{22} & \bar{A}_d^T P \bar{A}_l & \bar{A}_d^T P \bar{D} \\ * & * & \Pi_{33} & \bar{A}_l^T P \bar{D} \\ * & * & * & \bar{D}^T P \bar{D} \end{bmatrix}, \\
 \Pi_{11} & = \bar{A}^T P \bar{A} + \tilde{\alpha}^2 \bar{A}^T P \bar{A} + (d_M - d_m + 1) Q + \bar{\mu} Z + \sum_{i=1}^q \xi_i \bar{A}_i - \delta P, \\
 \Pi_{22} & = \bar{A}_d^T P \bar{A}_d + \sum_{i=1}^q \xi_i \bar{A}_{di} - \delta^{d_m} Q, \\
 \Pi_{33} & = \bar{A}_l^T P \bar{A}_l + \sum_{i=1}^q \xi_i \bar{A}_{li} - \frac{1}{\bar{\mu}} Z.
 \end{aligned}$$

Hence

$$\mathbb{E}\{V(k+1) - \delta V(k)\} \leq \xi^T(k) \bar{\Pi} \xi(k) + \frac{\gamma}{\delta^N} \omega^T(k) \omega(k), \tag{25}$$

where

$$\bar{\Pi} = \begin{bmatrix} \Pi_{11} & \bar{A}^T P \bar{A}_d & \bar{A}^T P \bar{A}_l & \bar{A}^T P \bar{D} \\ * & \Pi_{22} & \bar{A}_d^T P \bar{A}_l & \bar{A}_d^T P \bar{D} \\ * & * & \Pi_{33} & \bar{A}_l^T P \bar{D} \\ * & * & * & \bar{D}^T P \bar{D} - \frac{\gamma}{\delta^N} I \end{bmatrix}.$$

Then from (13) it follows that $\bar{\Pi} < 0$, and subsequently

$$\mathbb{E}\{V(k+1) - \delta V(k)\} < \frac{\gamma}{\delta^N} \omega^T(k) \omega(k), \quad \forall k \in \mathbb{Z}_+, \tag{26}$$

that is,

$$\mathbb{E}\{V(k+1)\} < \delta \mathbb{E}\{V(k)\} + \frac{\gamma}{\delta^N} \omega^T(k) \omega(k), \quad \forall k \in \mathbb{Z}_+. \tag{27}$$

We can obtain the following inequality from Assumption 3:

$$\begin{aligned} \mathbb{E}\{V(k)\} &< \delta^k \mathbb{E}\{V(0)\} + \frac{\gamma}{\delta^N} \sum_{s=0}^{k-1} \delta^{k-1-s} \{\omega^T(s) \omega(s)\} \\ &< \delta^N \mathbb{E}\{V(0)\} + \frac{\gamma}{\delta^N} \delta^{N-1} \sum_{s=0}^{N-1} \{\omega^T(s) \omega(s)\} \\ &< \delta^N \mathbb{E}\{V(0)\} + \frac{\gamma}{\delta} \bar{\omega}. \end{aligned} \tag{28}$$

By condition (14) we have

$$\begin{aligned} \mathbb{E}\{V(0)\} &= \eta^T(0) P \eta(0) + \sum_{s=-d(0)}^{-1} \delta^{-1-s} \eta^T(s) Q \eta(s) \\ &\quad + \sum_{s=-d_M+2}^{-d_m+1} \sum_{t=s-1}^{-1} \delta^{-1-t} \eta^T(t) Q \eta(t) + \sum_{m=1}^{+\infty} \mu_m \sum_{l=-m}^{-1} \delta^{-1-l} \eta^T(l) Z \eta(l) \\ &< \lambda_2 \eta^T(0) R \eta(0) + \lambda_3 \delta^{d_M-1} \sum_{s=-d_M}^{-1} \eta^T(s) R \eta(s) \\ &\quad + \lambda_3 \delta^{d_M+2} \sum_{s=-d_M+2}^{-d_m+1} \sum_{t=-1+s}^{-1} \eta^T(t) R \eta(t) \\ &\quad + \lambda_4 \sum_{m=1}^{+\infty} \mu_m \sum_{l=-m}^{-1} \delta^{-1-l} \eta^T(l) R \eta(l). \end{aligned}$$

Since

$$\begin{aligned}
 & \sum_{m=1}^{+\infty} \mu_m \sum_{l=-m}^{-1} \delta^{-1-l} \eta^T(l) R \eta(l) \\
 &= \mu_1 \eta^T(-1) R \eta(-1) + \mu_2 [\delta \eta^T(-2) R \eta(-2) + \eta^T(-1) R \eta(-1)] \\
 & \quad + \mu_3 [\delta^2 \eta^T(-3) R \eta(-3) + \delta \eta^T(-2) R \eta(-2) + \eta^T(-1) R \eta(-1)] \\
 & \quad + \dots + \mu_m [\delta^{m-1} \eta^T(-m) R \eta(-m) + \delta^{m-2} \eta^T(-m+1) R \eta(-m+1) \\
 & \quad + \dots + \eta^T(-1) R \eta(-1)] + \dots \\
 &= \mu_1 \eta^T(-1) R \eta(-1) + \mu_2 [\delta \eta^T(-2) R \eta(-2) + \eta^T(-1) R \eta(-1)] \\
 & \quad + \mu_3 [\delta^2 \eta^T(-3) R \eta(-3) + \delta \eta^T(-2) R \eta(-2) + \eta^T(-1) R \eta(-1)] \\
 & \quad + \dots + \mu_{d_M} [\delta^{d_M-1} \eta^T(-d_M) R \eta(-d_M) + \delta^{d_M-2} \eta^T(-d_M+1) R \eta(-d_M+1) \\
 & \quad + \dots + \eta^T(-1) R \eta(-1)] \\
 & \leq c_1 [\mu_1 + (1 + \delta) \mu_2 + \dots + (1 + \delta + \dots + \delta^{d_M-1}) \mu_{d_M}] \\
 & \triangleq c_1 \sigma,
 \end{aligned}$$

we get

$$\begin{aligned}
 \mathbb{E}\{V(0)\} &< \left[\lambda_2 + \lambda_3 d_M \delta^{d_M-1} + \lambda_3 \frac{(d_M - d_m)(d_M + d_m - 1)}{2} \delta^{d_M-2} + \lambda_4 \sigma \right] c_1 \\
 &\triangleq \varsigma_1.
 \end{aligned} \tag{29}$$

Taking (28) and (29) into account, we have

$$\mathbb{E}\{V(k)\} < \delta^N \varsigma_1 + \frac{\gamma}{\delta} \bar{\omega}, \quad \forall k \in \mathbb{Z}_+. \tag{30}$$

On the other hand, we can deduce that

$$\mathbb{E}\{V(k)\} \geq \eta^T(k) P \eta(k) > \lambda_1 \eta^T(k) R \eta(k), \quad \forall k \in \mathbb{Z}_+. \tag{31}$$

Consequently, from (30) and (31) it is easy to get that

$$\eta^T(k) R \eta(k) < \frac{1}{\lambda_1 \delta} (\delta^{N+1} \varsigma_1 + \gamma \bar{\omega}). \tag{32}$$

Note that

$$\begin{aligned}
 & \delta^{N+1} \varsigma_1 + \gamma \bar{\omega} - c_2 \delta \lambda_1 < 0 \\
 & \Leftrightarrow \gamma \bar{\omega} - c_2 \delta \lambda_1 + c_1 \delta^{N+1} \lambda_2 + c_1 \rho_1 \lambda_3 + c_1 \rho_2 \lambda_4 < 0 \\
 & \Leftrightarrow \gamma \bar{\omega} - c_2 \delta \lambda_1 + \begin{bmatrix} c_1 \delta^{N+1} \lambda_2 & \rho_1 \lambda_3 & \rho_2 \lambda_4 \end{bmatrix} \Gamma \begin{bmatrix} c_1 \delta^{N+1} \lambda_2 & \rho_1 \lambda_3 & \rho_2 \lambda_4 \end{bmatrix}^T < 0,
 \end{aligned} \tag{33}$$

where

$$\Gamma = \begin{bmatrix} c_1\delta^{N+1}\lambda_2 & 0 & 0 \\ 0 & \rho_1\lambda_3 & 0 \\ 0 & 0 & \rho_2\lambda_4 \end{bmatrix}^{-1}.$$

By Lemma 1 inequality (33) is equivalent to LMI (15), so we have

$$\eta^T(k)R\eta(k) < \frac{1}{\lambda_1\delta}(\delta^{N+1}\varsigma_1 + \gamma\bar{\omega}) < c_2.$$

By Definition 1 system (9) is SFTB with respect to $(c_1, c_2, \bar{\omega}, R, N)$, which completes the proof of Theorem 1. \square

3.2 Stochastic finite-time H_∞ boundedness analysis

In this subsection, we provide sufficient conditions guaranteeing the stochastic finite-time H_∞ boundedness of system (9).

Theorem 2 For given scalar $\delta \geq 1$ and symmetric matrix $R > 0$, system (9) is SFTH $_{\infty}B$ with respect to $(c_1, c_2, \bar{\omega}, \gamma, R, N)$ if there exist positive scalars λ_i ($i = 1, 2, 3, 4$) and ξ_i ($i = 1, 2, \dots, q$) and symmetric matrices $P > 0$, $Q > 0$, and $Z > 0$ such that (16), (17), and the following inequalities hold:

$$\begin{bmatrix} -\xi_i & \rho_i^T \bar{G}^T \\ * & -P^{-1} \end{bmatrix} < 0, \quad i = 1, \dots, q, \tag{34}$$

$$\begin{bmatrix} \Phi & 0 & 0 & 0 & \bar{A}^T P & \tilde{\alpha} \tilde{A}^T P & \bar{E}_1^T \\ * & -\delta^{dm} Q + \sum_{i=1}^q \xi_i \bar{A}_{di} & 0 & 0 & \bar{A}_d^T P & 0 & 0 \\ * & * & -\frac{1}{\mu} Z + \sum_{i=1}^q \xi_i \bar{A}_{li} & 0 & \bar{A}_l^T P & 0 & 0 \\ * & * & * & -\frac{\gamma}{\delta^N} I & \bar{D}^T P & 0 & E_2^T \\ * & * & * & * & -P & 0 & 0 \\ * & * & * & * & * & -P & 0 \\ * & * & * & * & * & * & -I \end{bmatrix} < 0, \tag{35}$$

with the remaining parameters as in Theorem 1.

Proof By Lemma 1 and Theorem 1 conditions (34) and (35) can ensure that system (9) is SFTB with respect to $(c_1, c_2, \bar{\omega}, R, N)$. In the following, our objective is to prove inequality (10). By using the same Lyapunov–Krasovskii functional as in Theorem 1 from (25) it follows that

$$\begin{aligned} & \mathbb{E}\{V(k+1) - \delta V(k)\} \\ & < \xi^T(k) \tilde{\Pi} \xi(k) + \frac{\gamma}{\delta^N} \omega^T(k) \omega(k) + \mathbb{E}\{z^T(k) z(k)\} - \mathbb{E}\{z^T(k) z(k)\}, \quad \forall k \in \mathbb{Z}_+. \end{aligned}$$

Then

$$\mathbb{E}\{V(k+1) - \delta V(k)\} < \xi^T(k) \tilde{\Pi} \xi(k) + \frac{\gamma}{\delta^N} \omega^T(k) \omega(k) - \mathbb{E}\{z^T(k) z(k)\}, \quad \forall k \in \mathbb{Z}_+,$$

where

$$\tilde{\Pi} = \begin{bmatrix} \Pi_{11} + \bar{E}_1^T \bar{E}_1 & \bar{A}^T P \bar{A}_d & \bar{A}^T P \bar{A}_l & \bar{A}^T P \bar{D} + \bar{E}_1^T E_2 \\ * & \Pi_{22} & \bar{A}_d^T P \bar{A}_l & \bar{A}_d^T P \bar{D} \\ * & * & \Pi_{33} & \bar{A}_l^T P \bar{D} \\ * & * & * & \bar{D}^T P \bar{D} + E_2^T E_2 - \frac{\gamma}{\delta^N} I \end{bmatrix}.$$

From condition (35) we have

$$\mathbb{E}\{V(k+1)\} < \delta \mathbb{E}\{V(k)\} + \frac{\gamma}{\delta^N} \omega^T(k)\omega(k) - \mathbb{E}\{z^T(k)z(k)\}, \quad \forall k \in \mathbb{Z}_+.$$

Thus

$$0 \leq \mathbb{E}\{V(k)\} < \delta^k \mathbb{E}\{V(0)\} + \sum_{s=0}^{k-1} \delta^{k-1-s} \left[\frac{\gamma}{\delta^N} \omega^T(s)\omega(s) - \mathbb{E}\{z^T(s)z(s)\} \right]. \tag{36}$$

Therefore, under the zero initial condition, (36) implies

$$\sum_{s=0}^{k-1} \delta^{k-1-s} \mathbb{E}\{z^T(s)z(s)\} < \sum_{s=0}^{k-1} \left[\delta^{k-1-s} \frac{\gamma}{\delta^N} \omega^T(s)\omega(s) \right].$$

Letting $k = N + 1$, we have

$$\sum_{s=0}^N \delta^{N-s} \mathbb{E}\{z^T(s)z(s)\} < \sum_{s=0}^N \delta^{N-s} \left[\frac{\gamma}{\delta^N} \omega^T(s)\omega(s) \right]. \tag{37}$$

Note that $1 \leq \delta^{N-s} \leq \delta^N$ for $s \in \{0, 1, \dots, N\}$, which, together with (37), yields

$$\sum_{s=0}^N \mathbb{E}\{z^T(s)z(s)\} < \gamma \sum_{s=0}^N \omega^T(s)\omega(s).$$

This completes the proof. □

3.3 Stochastic finite-time output feedback control

In this subsection, we give the design procedure of the controller gains.

Theorem 3 *For given scalar $\delta \geq 1$ and symmetric matrix $R > 0$, system (1) is SFTH_∞B with respect to $(c_1, c_2, \bar{\omega}, \gamma, R, N)$ if there exist positive scalars λ'_i ($i = 1, 2, 3, 4$) and ξ_i ($i = 1, 2, \dots, q$), symmetric matrices $S > 0, T > 0, \hat{Q}_1 > 0, \hat{Q}_3 > 0, \hat{Z}_1 > 0$, and $\hat{Z}_3 > 0$, nonsingular matrices Y_{12} and Y_{22} , and real matrices $\hat{A}, \hat{B}, \hat{C}, \hat{Q}_2$, and \hat{Z}_2 such that the following inequalities hold:*

$$\begin{bmatrix} -\xi_i & \rho_i^T & 0 \\ * & -S & -Y_{12} \\ * & * & -Y_{22} \end{bmatrix} < 0, \quad i = 1, \dots, q, \tag{38}$$

$$\begin{bmatrix} \Omega_{11} & \Omega_{12} \\ * & \Omega_{22} \end{bmatrix} < 0, \tag{39}$$

$$\delta^{N+1}S_2 + \gamma\bar{\omega} - c_2\delta\lambda'_1 < 0, \tag{40}$$

$$\lambda'_1R_1 \leq S \leq \lambda'_2R_1, \quad \hat{Q}_3 \leq \lambda'_3R_1, \quad \hat{Z}_3 \leq \lambda'_4R_1. \tag{41}$$

Furthermore, the corresponding gain matrices of the controller can be given by

$$A_K = X_{12}^{-1}T\hat{A}S(X_{12}Y_{12}^T)^{-1}X_{12}, \quad B_K = X_{12}^{-1}T\hat{B}, \quad C_K = \hat{C}S(X_{12}Y_{12}^T)^{-1}X_{12},$$

where

$$S_2 = \left[\lambda'_2 + \lambda'_3d_M\mu^{d_M-1} + \frac{1}{2}\lambda'_3(\tau_1 - 1)\tau_2\mu^{d_M-2} + \lambda'_4\sigma \right] c_1,$$

$$\Omega_{11} = \text{diag}\{\Omega'_{11}, \Omega'_{22}, \Omega'_{33}, \Omega'_{44}\},$$

$$\Omega_{12} = \begin{bmatrix} A^T + \hat{C}^TD^T & A^T + \hat{C}^TD^T + \bar{\alpha}F_1^T\hat{B}^T + \hat{A} & 0 & \bar{\alpha}F_1^T\hat{B}^T & E_1^T \\ A^T & A^T + \bar{\alpha}F_1^T\hat{B}^T & 0 & \bar{\alpha}F_1^T\hat{B}^T & E_1^T \\ A_d^T & A_d^T & 0 & 0 & 0 \\ A_d^T & A_d^T & 0 & 0 & 0 \\ A_l^T & A_l^T & 0 & 0 & 0 \\ A_l^T & A_l^T & 0 & 0 & 0 \\ D^T & D^T + F_2^T\hat{B}^T & 0 & 0 & E_2^T \end{bmatrix},$$

$$\Omega_{22} = \begin{bmatrix} -S & -T^{-1} & 0 & 0 & 0 \\ * & -T^{-1} & 0 & 0 & 0 \\ * & * & -S & -T^{-1} & 0 \\ * & * & * & -T^{-1} & 0 \\ * & * & * & * & -I \end{bmatrix},$$

$$\Omega'_{11} = \begin{bmatrix} -\delta S^{-1} + \tau\hat{Q}_1 + \bar{\mu}\hat{Z}_1 + \sum_{i=1}^q \xi_i\bar{A}_i & -\delta S^{-1} + \tau\hat{Q}_2^T + \bar{\mu}\hat{Z}_2^T + \sum_{i=1}^q \xi_i\bar{A}_i \\ * & -\delta T + \tau\hat{Q}_3 + \bar{\mu}\hat{Z}_3 + \sum_{i=1}^q \xi_i\bar{A}_i \end{bmatrix},$$

$$\Omega'_{22} = \begin{bmatrix} -\delta^{d_m}\hat{Q}_1 + \sum_{i=1}^q \xi_i\bar{A}_{di} & -\delta^{d_m}\hat{Q}_2^T + \sum_{i=1}^q \xi_i\bar{A}_{di} \\ * & -\delta^{d_m}\hat{Q}_3^T + \sum_{i=1}^q \xi_i\bar{A}_{di} \end{bmatrix},$$

$$\Omega'_{33} = \begin{bmatrix} -\frac{1}{\bar{\mu}}\hat{Z}_1 + \sum_{i=1}^q \xi_i\bar{A}_{li} & -\frac{1}{\bar{\mu}}\hat{Z}_2^T + \sum_{i=1}^q \xi_i\bar{A}_{li} \\ * & -\frac{1}{\bar{\mu}}\hat{Z}_3 + \sum_{i=1}^q \xi_i\bar{A}_{li} \end{bmatrix}, \quad \Omega'_{44} = -\frac{\gamma}{\delta^N}I,$$

X_{12} is a nonsingular matrix satisfying $X_{12}Y_{12}^T = I - TS$, and the remaining parameters are as in Theorem 1.

Proof Firstly, we define P, Q, Z , and R in Theorem 2 as follows:

$$P = \begin{bmatrix} T & X_{12} \\ X_{12}^T & X_{22} \end{bmatrix}, \quad P^{-1} = \begin{bmatrix} S & Y_{12} \\ Y_{12}^T & Y_{22} \end{bmatrix}, \quad Q = \begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix},$$

$$Z = \begin{bmatrix} Z_1 & Z_2 \\ Z_2^T & Z_3 \end{bmatrix}, \quad R = \begin{bmatrix} R_1 & 0 \\ 0 & R_2 \end{bmatrix}, \quad T_1 = \begin{bmatrix} S & I \\ Y_{12}^T & 0 \end{bmatrix}, \quad T_2 = \begin{bmatrix} I & T \\ 0 & X_{12}^T \end{bmatrix},$$

where the partitioning of P , P^{-1} , Q , and Z is compatible with that of \bar{A} , \tilde{A} , \bar{A}_d , and \bar{A}_l , which implies that

$$PT_1 = T_2, \tag{42}$$

$$\begin{aligned} T_1^T Q T_1 &= \begin{bmatrix} SQ_1 S + SQ_2 Y_{12}^T + Y_{12} Q_2^T S + Y_{12} Q_3 Y_{12}^T & SQ_1 + Y_{12} Q_2^T \\ * & Q_1 \end{bmatrix} \\ &= \begin{bmatrix} S\hat{Q}_1 S & S\hat{Q}_2^T \\ * & \hat{Q}_3 \end{bmatrix}, \end{aligned} \tag{43}$$

$$T_1^T Z T_1 = \begin{bmatrix} S\hat{Z}_1 S & S\hat{Z}_2^T \\ * & \hat{Z}_3 \end{bmatrix}, \tag{44}$$

where

$$\begin{aligned} \hat{Q}_1 &= Q_1 + Q_2 Y_{12}^T S^{-1} + S^{-1} Y_{12} Q_2^T + S^{-1} Y_{12} Q_3 Y_{12}^T S^{-1}, \\ \hat{Q}_2 &= Q_1 + Q_2 Y_{12}^T S^{-1}, \quad \hat{Q}_3 = Q_1, \\ \hat{Z}_1 &= Z_1 + Z_2 Y_{12}^T S^{-1} + S^{-1} Y_{12} Z_2^T + S^{-1} Y_{12} Z_3 Y_{12}^T S^{-1}, \\ \hat{Z}_2 &= Z_1 + Z_2 Y_{12}^T S^{-1}, \quad \hat{Z}_3 = Z_1. \end{aligned}$$

It is easy to see that (38) and (34) are equivalent. To obtain (39), pre- and postmultiplying (35) by ψ^T and ψ , we obtain

$$\begin{bmatrix} \hat{\Pi}_{11} & \hat{\Pi}_{12} \\ * & \hat{\Pi}_{22} \end{bmatrix} < 0, \tag{45}$$

where

$$\begin{aligned} \psi &= \text{diag}\{T_1, T_1, T_1, I, T_1, T_1, I\}, \quad \hat{\Pi}_{11} = \text{diag}\{\hat{\Pi}'_{11}, \hat{\Pi}'_{22}, \hat{\Pi}'_{33}, \hat{\Pi}'_{44}\}, \\ \hat{\Pi}_{12} &= \begin{bmatrix} SA^T + Y_{12} C_K^T D^T & \Delta_1 & 0 & \tilde{\alpha} S F_1^T B_K^T X_{12}^T & S E_1^T \\ A^T & A^T T + \tilde{\alpha} F_1^T B_K^T X_{12}^T & 0 & \tilde{\alpha} F_1^T B_K^T X_{12}^T & E_1^T \\ SA_d^T & SA_d^T T & 0 & 0 & 0 \\ A_d^T & A_d^T T & 0 & 0 & 0 \\ SA_l^T & SA_l^T T & 0 & 0 & 0 \\ A_l^T & A_l^T T & 0 & 0 & 0 \\ D^T & D^T T + F_2^T B_K^T X_{12}^T & 0 & 0 & E_2^T \end{bmatrix}, \\ \hat{\Pi}_{22} &= \begin{bmatrix} -S & -I & 0 & 0 & 0 \\ * & -T & 0 & 0 & 0 \\ * & * & -S & -I & 0 \\ * & * & * & -T & 0 \\ * & * & * & * & -I \end{bmatrix}, \\ \hat{\Pi}'_{11} &= \begin{bmatrix} -\delta S + \tau S \hat{Q}_1 S + \bar{\mu} S \hat{Z}_1 S + \sum_{i=1}^q \xi_i S A_i S & -\delta I + \tau S \hat{Q}_2^T + \bar{\mu} S \hat{Z}_2^T + \sum_{i=1}^q \xi_i S A_i \\ * & -\delta T + \tau \hat{Q}_3 + \bar{\mu} \hat{Z}_3 + \sum_{i=1}^q \xi_i A_i \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} \hat{\Pi}'_{22} &= \begin{bmatrix} -\delta^{d_m} S \hat{Q}_1 S + \sum_{i=1}^q \xi_i S A_{di} S & -\delta^{d_m} S \hat{Q}_2^T + \sum_{i=1}^q \xi_i S A_{di} \\ * & -\delta^{d_m} \hat{Q}_3^T + \sum_{i=1}^q \xi_i A_{di} \end{bmatrix}, \\ \hat{\Pi}'_{33} &= \begin{bmatrix} -\frac{1}{\mu} S \hat{Z}_1 S + \sum_{i=1}^q \xi_i S A_{li} S & -\frac{1}{\mu} S \hat{Z}_2^T + \sum_{i=1}^q \xi_i S A_{li} \\ * & -\frac{1}{\mu} \hat{Z}_3 + \sum_{i=1}^q \xi_i A_{li} \end{bmatrix}, \\ \hat{\Pi}'_{44} &= \Omega'_{44}, \quad \Delta_1 = SA^T T + Y_{12} C_K^T D^T T + \bar{\alpha} S F_1^T B_K^T X_{12}^T + Y_{12} A_K^T X_{12}^T. \end{aligned}$$

Pre- and postmultiplying (45) by $\text{diag}\{S^{-1}, I, S^{-1}, I, S^{-1}, I, I, I, T^{-1}, I, T^{-1}, I\}$, we can conclude that

$$\begin{bmatrix} \check{\Pi}_{11} & \check{\Pi}_{12} \\ * & \check{\Pi}_{22} \end{bmatrix} < 0, \tag{46}$$

where

$$\begin{aligned} \check{\Pi}_{11} &= \Omega_{11}, \\ \check{\Pi}_{12} &= \begin{bmatrix} A^T + S^{-1} Y_{12} C_K^T D^T & \Delta_2 & 0 & \bar{\alpha} F_1^T B_K^T X_{12}^T T^{-1} & E_1^T \\ A^T & A^T + \bar{\alpha} F_1^T B_K^T X_{12}^T T^{-1} & 0 & \bar{\alpha} F_1^T B_K^T X_{12}^T T^{-1} & E_1^T \\ A_d^T & A_d^T & 0 & 0 & 0 \\ A_d^T & A_d^T & 0 & 0 & 0 \\ A_l^T & A_l^T & 0 & 0 & 0 \\ A_l^T & A_l^T & 0 & 0 & 0 \\ D^T & D^T + F_2^T B_K^T X_{12}^T T^{-1} & 0 & 0 & E_2^T \end{bmatrix}, \\ \check{\Pi}_{22} &= \Omega_{22}, \\ \Delta_2 &= A^T + S^{-1} Y_{12} C_K^T D^T + \bar{\alpha} F_1^T B_K^T X_{12}^T T^{-1} + S^{-1} Y_{12} A_K^T X_{12}^T T^{-1}. \end{aligned}$$

Letting $\hat{A} = T^{-1} X_{12} A_K Y_{12}^T S^{-1}$, $\hat{B} = T^{-1} X_{12} B_K$, and $\hat{C} = C_K Y_{12}^T S^{-1}$, we get inequality (39). Since P is a positive definite matrix, we have

$$T_1^T P T_1 = \begin{bmatrix} S & I \\ I & T \end{bmatrix} > 0 \quad \text{or} \quad TS - I > 0. \tag{47}$$

From (47) it follows that $I - TS = X_{12} Y_{12}^T < 0$. So there exist two nonsingular matrices X_{12} and Y_{12} such that $X_{12} Y_{12}^T < 0$. This completes the proof. \square

Remark 2 So far, we have investigated the problem of the stochastic finite-time H_∞ bounded control for a class of NCSs with mixed delays, stochastic nonlinearities, and randomly missing measurement. Also, some sufficient conditions reflecting the impacts from the involved phenomena are given. Some criterion of the error analysis of the proposed theoretical result will be obtained in the near future if the error analysis becomes necessary.

Remark 3 In Theorem 3, sufficient conditions guaranteeing the stochastic finite-time boundedness of the system are obtained. It is easy to see that the obtained results of this

paper are influenced by all system parameters, such as the bounds on the delay. Thus the results of this paper are more general and less conservative than the existing results. In addition, we also give sufficient conditions for the existence of the output feedback controller in Theorem 3. However, this condition is not expressed in the form of LMI, and we cannot directly use the standard LMI toolbox to obtain it. Based on the literature [36], we present the following controller design method.

Let us denote $U = S^{-1}$ and $V = T^{-1}$. Then (41) can be rewritten

$$\begin{bmatrix} \bar{\Omega}_{11} & \bar{\Omega}_{12} \\ * & \bar{\Omega}_{22} \end{bmatrix} < 0, \tag{48}$$

where

$$\begin{aligned} \bar{\Omega}_{11} &= \text{diag}\{\bar{\Omega}'_{11}, \bar{\Omega}'_{22}, \bar{\Omega}'_{33}, \bar{\Omega}'_{44}\}, & \bar{\Omega}_{12} &= \Omega_{12}, \\ \bar{\Omega}_{22} &= \begin{bmatrix} -S & -V & 0 & 0 & 0 \\ * & -V & 0 & 0 & 0 \\ * & * & -S & -V & 0 \\ * & * & * & -V & 0 \\ * & * & * & * & -I \end{bmatrix}, \\ \bar{\Omega}'_{11} &= \begin{bmatrix} -\delta U + \tau \hat{Q}_1 + \bar{\mu} \hat{Z}_1 + \sum_{i=1}^q \xi_i A_i & & & & \\ & * & & & \\ & & -\delta U + \tau \hat{Q}_2 + \bar{\mu} \hat{Z}_2 + \sum_{i=1}^q \xi_i A_i & & \\ & & & * & \\ & & & & -\delta T + \tau \hat{Q}_3 + \bar{\mu} \hat{Z}_3 + \sum_{i=1}^q \xi_i A_i \end{bmatrix}, \\ \bar{\Omega}'_{22} &= \Omega'_{22}, & \bar{\Omega}'_{33} &= \Omega'_{33}, & \bar{\Omega}'_{44} &= \Omega'_{44}. \end{aligned}$$

Consequently, we can transform the original nonconvex problem into the following minimization problem involving LMI conditions, which can be solved by CCLM:

$$\begin{aligned} \min_{V>0, S>0, T>0} \text{tr}\{US + VT\} \\ \text{s.t. (48), } \begin{bmatrix} U & I \\ I & S \end{bmatrix} > 0, \begin{bmatrix} V & I \\ I & T \end{bmatrix} > 0. \end{aligned}$$

Remark 4 For given scalars $\delta \geq 1, \gamma, c_1, c_2, \bar{\omega}, N$ and symmetric positive definite matrix R , we can get the following algorithm for calculating the output feedback controller by CCLM [36].

- Step 1. Given the maximum iteration \bar{N} and fixed scalars $\delta \geq 1, \gamma, c_1, \bar{\omega}, N$.
- Step 2. Determine an initial value of c_2 .
- Step 3. Find a feasible solution S_0, U_0, T_0, V_0 satisfying (49) and let $S_k = S_0, U_k = U_0, T_k = T_0, V_k = V_0$. If there are none, then exit. Set $k = 0$.
- Step 4. If $|\text{tr}(U_{k+1}S_k + S_{k+1}U_k + \phi_{k+1}R_k + \phi_k R_{k+1}) - 4n| < c_2$, then exit. Otherwise, set $k = k + 1$ and go to Step 2.
- Step 5. If $k > \bar{N}$, then stop.

Step 6. If problem (49) is unfeasible, then the initial value for c_2 need to be increased. Otherwise, we decrease c_2 until we get its minimum value.

$$\begin{aligned} & \min_{\substack{V>0,S>0,T>0,Y_{12}>0, \\ Y_{22}>0,\tilde{Z}_1,\tilde{Z}_2,\tilde{Z}_3,\tilde{Q}_1,\tilde{Q}_2,\tilde{Q}_3}} \text{tr}(US_k + ST_k + VT_k + TV_k) + \gamma \\ & \text{s.t. (38), (40), (41), and (48).} \end{aligned} \tag{49}$$

where γ is the disturbance attenuation, and c_2 is the prescribed bound of $\mathbb{E}\{x^T R_1 x\}$.

4 Numerical examples

In this section, we give two numerical examples to show the validity of the proposed theoretical results for stochastic nonlinear NCSs (1) with mixed delays and randomly missing measurement.

Example 1 To demonstrate that our methods are better than some existing results, we consider the same example as in [29] with the following system parameters:

$$\begin{aligned} A &= \begin{bmatrix} 0.4 & 0.1 \\ 0.3 & 0.5 \end{bmatrix}, & A_d &= \begin{bmatrix} 0.2 & -0.15 \\ 0.15 & 0.1 \end{bmatrix}, & B &= \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, & D &= \begin{bmatrix} 0.25 \\ 0.3 \end{bmatrix}, \\ R_1 &= \begin{bmatrix} 1.2 & 0 \\ 0 & 1.3 \end{bmatrix}, & E_1 &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, & E_2 &= 0.1, & F_1 &= \begin{bmatrix} 2.738 \\ 2.287 \end{bmatrix}, & F_2 &= 0.2. \end{aligned}$$

Without loss of generality, we assume that the noise $\omega(k)$ in system (1) is selected as $\omega(k) = \frac{1}{0.1+k^2}$, the time-varying communication delay satisfies $1 \leq d(k) \leq 12$, the initial states of NCSs and the estimators are $x(0) = [0.5 \ -0.2]^T$ and $\hat{x}(0) = [0 \ 0]^T$, respectively, $N = 40$, $\bar{\omega} = 2$, $c_1 = 1$, $\delta = 1.00027$, and $f(k) = 0$. We easily check that the initial condition $\mathbb{E}\{x^T(0)R_1x(0)\} \leq c_1$ is satisfied.

Firstly, we let $A_l = 0$. By applying the method in this paper and the method in [29], we can obtain the minimum values of γ and c_2 for different values of d_M in Table 1. From Table 1 we can obviously observe that the obtained minimum values of this paper are smaller than that in [29]. Since c_2 is the prescribed bound of $\mathbb{E}\{x^T R_1 x\}$, the smaller c_2 is, the better the state trajectory of the system converges. Since γ is the level of disturbance attenuation, the smaller the γ , the better the system performance. This shows that the obtained results of this paper are better than that in [29].

Table 1 Optimal values for different time-delay upper bound d_M when $\bar{\mu} = 0$

d_M	4	6	8	12
Theorem 3 in [29]	$\gamma_{\min} = 0.4243$ $c_{2 \min} = 2.3383$	$\gamma_{\min} = 0.5901$ $c_{2 \min} = 3.2000$	$\gamma_{\min} = 0.7751$ $c_{2 \min} = 4.2679$	$\gamma_{\min} = 1.2997$ $c_{2 \min} = 6.8833$
Theorem 3	$\gamma_{\min} = 0.2208$ $c_{2 \min} = 1.5068$	$\gamma_{\min} = 0.2636$ $c_{2 \min} = 1.8734$	$\gamma_{\min} = 0.3072$ $c_{2 \min} = 2.2580$	$\gamma_{\min} = 0.3131$ $c_{2 \min} = 2.7284$

Table 2 Optimal values for different upper bound d_M and $\bar{\mu}$

	$d_M = 4, \bar{\mu} = \frac{1}{4}$	$d_M = 6, \bar{\mu} = \frac{1}{6}$	$d_M = 8, \bar{\mu} = \frac{1}{8}$	$d_M = 12, \bar{\mu} = \frac{1}{12}$
γ_{\min}	0.2356	0.2625	0.2995	0.3138
$c_{2 \min}$	1.4281	1.7283	2.0821	2.5549

In the following, we choose $A_l = \begin{bmatrix} 0.2 & 0 \\ 0.3 & 0.4 \end{bmatrix}$. The constant sequence $\{\mu_m\} \in [0, +\infty)$ is chosen as $\mu_m = 3^{-(3+m)}$, and the stochastic nonlinear function is selected as follows:

$$f(k) = \begin{bmatrix} 0.3 \\ 0.3 \end{bmatrix} \left[0.2x_1(k)v_1(k) + 0.2x_2(k)v_2(k) + 0.1x_1(k-d(k))\xi_1(k) + 0.2x_2(k-d(k))\xi_2(k) \right],$$

where $x_i(k)$ ($i = 1, 2$) stand for the i th element of the system state, $v_i(k)$ and $\xi_i(k)$ ($i = 1, 2$) represent the mutually uncorrelated Gaussian white noise sequences with unity variances:

$$\mathbb{E}\{v_i(k)\} = 0, \quad \mathbb{E}\{v_i(k)^2\} = 1, \quad \mathbb{E}\{\xi_i(k)\} = 0, \quad \mathbb{E}\{\xi_i(k)^2\} = 1.$$

For different upper bounds of discrete delay and distributed delays, the optimal values of γ and c_2 are listed in Table 2. From Table 2 we can easily find that as upper bounds of discrete delay and distributed delays increase, the optimal values of γ and c_2 get bigger. Thus, when upper bounds of discrete delay and distributed delays increase, the system performance becomes worse, which means that the mixed delays have important influence on the system performance. Thus the results proposed in this paper are less conservative than those obtained in [29].

Example 2 In the following, we consider system (1) with following system parameters:

$$A = \begin{bmatrix} -0.9226 & -0.4330 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \quad A_d = \begin{bmatrix} 0.2 & 0 & 0 \\ 0 & -0.3 & 0.4 \\ 0 & 0 & 0.4 \end{bmatrix},$$

$$A_l = \begin{bmatrix} -0.1 & 0.2 & 0 \\ 0 & 0.3 & 0 \\ 0.2 & 0.1 & 0.4 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 0.5 \\ 0 \\ -1.01 \end{bmatrix},$$

$$R_1 = \begin{bmatrix} 1.2 & 0 & 0 \\ 0 & 1.3 & 0 \\ 0 & 0 & 1.4 \end{bmatrix}, \quad E_1 = \begin{bmatrix} 0 \\ 0.2 \\ 0.1 \end{bmatrix}^T, \quad E_2 = \begin{bmatrix} 2.738 \\ 2.287 \\ 0 \end{bmatrix}^T, \quad F_2 = 0.1,$$

$$F_2 = 0.2, \quad N = 60, \quad d(k) = 1 + \sin(0.5k\pi)^2, \quad c_1 = 1, \quad \delta = 1.03.$$

We consider the stochastic nonlinear function

$$f(k) = \begin{bmatrix} 0.3 \\ 0.3 \\ 0.3 \end{bmatrix} \left[0.2x_1(k)v_1(k) + 0.2x_2(k)v_2(k) + 0.2x_3(k)v_3(k) + 0.1x_1(k-d(k))\xi_1(k) + 0.2x_2(k-d(k))\xi_2(k) + 0.1x_3(k-d(k))\xi_3(k) \right].$$

We easily obtain that

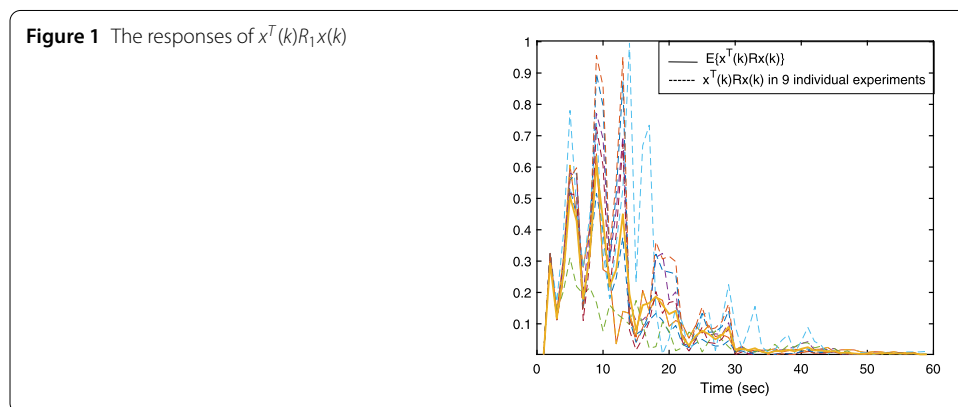
$$\begin{aligned} \mathbb{E}[f(k)f^T(k)] &= \begin{bmatrix} 0.3 \\ 0.3 \\ 0.3 \end{bmatrix} \begin{bmatrix} 0.3 \\ 0.3 \\ 0.3 \end{bmatrix}^T \begin{bmatrix} x^T(k) \begin{bmatrix} 0.04 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.04 \end{bmatrix} x(k) \\ + x^T(k-d(k)) \begin{bmatrix} 0.01 & 0 & 0 \\ 0 & 0.04 & 0 \\ 0 & 0 & 0.01 \end{bmatrix} x(k-d(k)) \end{bmatrix}. \end{aligned}$$

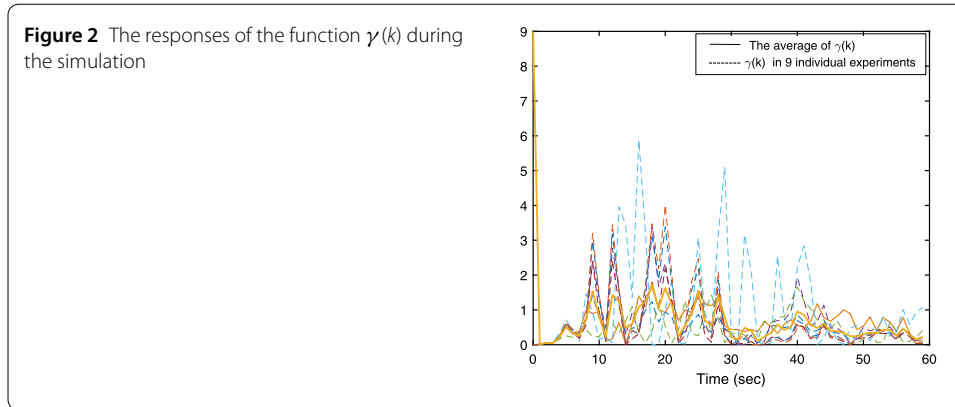
The initial conditions are supposed to be $x(0) = [-0.2 \ 0.5 \ 0.2]^T$, $\hat{x}(0) = [0 \ 0 \ 0]^T$. The external disturbance is chosen as $\omega(k) = \frac{1}{k^2}$. We easily check that the initial condition $\mathbb{E}\{x^T(0)R_1x(0)\} \leq c_1$ is satisfied. The corresponding lower and upper bounds of discrete delay are $d_m = 1$ and $d_M = 2$, respectively. Other parameters are chosen as those in Example 1.

Solving the minimization problem given in Remark 4 by using Matlab LMI toolbox, it follows that the optimal values of $\gamma_{\min} = 3.0314$ and $c_{2\min} = 1.5660$ can be obtained with the desired controller as

$$\begin{aligned} A_K &= \begin{bmatrix} -0.6051 & 0.0572 & 0.0045 \\ 2.1737 & -0.0815 & -0.0052 \\ -0.4089 & -0.1916 & -0.0020 \end{bmatrix}, & B_K &= \begin{bmatrix} 0.0947 \\ -0.1708 \\ -1.0978 \end{bmatrix}, \\ C_K &= \begin{bmatrix} -0.4020 \\ -0.2370 \\ -0.0035 \end{bmatrix}^T. \end{aligned}$$

In addition, the simulation results of $x^T(k)R_1x(k)$ are shown in Fig. 1. Figure 1 depicts the trajectories of $x^T(k)R_1x(k)$ along nine individual experiments. From Fig. 1 we can conclude that the state trajectories of $\mathbb{E}\{x^T(k)R_1x(k)\}$ remain within the obtained optimal value $c_{2\min} = 1.5660$ over the fixed interval despite facing mixed delays, stochastic nonlinearities, and randomly missing measurement, which means that the systems are both SFTB under nine individual experiments. Furthermore, by applying the method in [36] we define the following function $\gamma(k)$ to show the effect on the output energy $z(k)$ from the





disturbance input energy $\omega(k)$:

$$\gamma(k) = \frac{\sum_{s=0}^k \mathbb{E}\{z^T(s)z(s)\}}{\sum_{s=0}^k \omega^T(s)\omega(s)}, \quad k = 1, \dots, N.$$

The corresponding curves of $\gamma(k)$ are plotted in Fig. 2. We can obtain that $\mathbb{E}\{\gamma(60)\} \approx \frac{1}{9} \sum_{i=1}^9 [\gamma(60)] = 0.4693 < \gamma_{\min} = 3.0314$. Thus the developed output feedback controller is a finite-time H_∞ controller for the discrete-time NCSs with mixed delays, stochastic nonlinearities, and randomly missing measurement (1) according to Definition 2.

Remark 5 In [29, 32, 34] the problem of stochastic finite-time H_∞ control for discrete time NCSs has been investigated. However, they did not consider mixed delays, stochastic nonlinearity, and randomly missing measurement. Thus our proposed results are more general than those given in [29, 32, 34].

5 Conclusions

In this paper, we investigated the problem of stochastic finite-time H_∞ control for stochastic nonlinear NCSs with mixed delays and randomly missing measurement. Based on a novel Lyapunov–Krasovskii functional and stochastic analysis method, we provided the sufficient conditions for stochastic finite-time boundedness of discrete-time NCSs. Then we designed an H_∞ output feedback controller ensuring that discrete-time NCS (1) is SFTB. Although the derived results are for a nonconvex feasibility problem, we can turn it into LMI feasibility problem by CCLM. In addition, we analyzed effects of mixed delays on dynamic performance of NCSs and the effective of the developed controller by two numerical examples. Note that the main results proposed in this paper can be extended to the stochastic finite-time filtering design problem. In addition, other factors can be considered in the finite-time H_∞ control issues for discrete-time NCSs with mixed delays in the future. For example, we can consider actuator saturation [37], stochastic fading measurement [38], and event-triggered scheme [39].

Acknowledgements

The authors greatly appreciate the reviewers' suggestions and the editors' encouragement. Thank you very much for reviewing the paper.

Funding

This work was supported in part by the National Natural Science Foundation of China (NSFC) under Grant Nos. 11271103 and 11371114 and Natural Science Foundation of Heilongjiang Province under Grant no. A2018007.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

LH contributed significantly to analyze and prepare the manuscript. She performed the data analysis and wrote the manuscript. DC helped to perform the analysis with constructive discussions. CH helped to complete the simulation of the examples. All authors read and approved the final manuscript.

Author details

¹Department of Mathematics, Harbin University of Science and Technology, Harbin, China. ²Heilongjiang Provincial Key Laboratory of Optimization Control and Intelligent Analysis for Complex Systems, Harbin University of Science and Technology, Harbin, China. ³Department of Mathematics, North University of China, Taiyuan, China.

Publisher's Note

Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

Received: 1 July 2019 Accepted: 8 January 2020 Published online: 28 February 2020

References

1. Angel, C., Unnati, O., Lian, S.J., Chow, M.Y.: A non-uniform multi-rate control strategy for a Markov chain-driven networked control system. *Inf. Sci.* **321**, 31–47 (2015)
2. Ge, X., Han, Q.L.: Distributed event-triggered H_∞ filtering over sensor networks with communication delays. *Inf. Sci.* **291**, 128–142 (2015)
3. Ge, X., Han, Q.L.: Distributed H_∞ filtering over sensor networks with heterogeneous Markovian coupling inter communication delays. *IET Control Theory Appl.* **9**(1), 82–90 (2015)
4. Yan, H., Yan, S., Zhang, H., Shi, H.: L_2 control design of event-triggered networked control systems with quantization. *J. Franklin Inst.* **2352**, 332–345 (2015)
5. Wan, X.B., Wang, Z.D., Wu, M., Liu, X.H.: H_∞ state estimation for discrete-time nonlinear singularly perturbed complex networks under the round-robin protocol. *IEEE Trans. Neural Netw. Learn. Syst.* **30**(2), 415–426 (2019)
6. Wu, Y., Meng, X., Xie, L., Lu, R., Su, H.: An input-based triggering approach to leader-following problems. *Automatica* **75**, 221–228 (2017)
7. Hu, J., Wang, Z.D., Gao, H.J.: Joint state and fault estimation for uncertain time-varying nonlinear systems with randomly occurring faults and sensor saturations. *Automatica* **97**, 150–160 (2018)
8. Wang, Z.D., Liu, Y., Wei, G., Liu, X.: A note on control of a class of discrete-time stochastic systems with distributed delays and nonlinear disturbances. *Automatica* **46**, 543–548 (2014)
9. Wan, X.B., Wang, Z.D., Wu, M., Liu, X.H.: State estimation for discrete time-delayed genetic regulatory networks with stochastic noises under the round-robin protocols. *IEEE Trans. Nanobiosci.* **17**(2), 145–154 (2018)
10. Tian, E., Yue, D., Peng, C.: Reliable control for networked control systems with probabilistic actuator fault and random delays. *J. Franklin Inst.* **347**, 1907–1926 (2010)
11. Zhang, H.X., Hu, J., Liu, H.J., Yu, X.Y., Liu, F.Q.: Recursive state estimation for time-varying complex networks subject to missing measurements and stochastic inner coupling under random access protocol. *Neurocomputing* **346**(21), 48–57 (2019)
12. Wan, X.B., Fang, H., Fu, S.: Observer-based fault detection for networked discrete time infinite-distributed delay systems with packet dropouts. *Appl. Math. Model.* **36**, 270–278 (2012)
13. Wang, Z.D., Wang, Y., Liu, Y.R.: Global synchronization for discrete-time stochastic complex networks with randomly occurred nonlinearities and mixed time delays. *IEEE Trans. Neural Netw.* **21**(1), 11–25 (2010)
14. Hu, J., Wang, Z.D., Gao, H.J.: Robust sliding mode control for discrete stochastic systems with mixed time delays, randomly occurring uncertainties, and randomly occurring nonlinearities. *IEEE Trans. Ind. Electron.* **59**(7), 3008–3015 (2012)
15. Wang, Z.D., Ho, D., Liu, Y., Liu, X.: Robust H_∞ control for a class of nonlinear discrete time-delay stochastic systems with missing measurements. *Automatica* **45**(3), 684–691 (2012)
16. Hu, J., Wang, Z.D., Liu, S., Gao, H.J.: A variance-constrained approach to recursive state estimation for time-varying complex networks with missing measurements. *Automatica* **64**, 155–162 (2016)
17. Hu, J., Wang, Z.D., Shen, B., Gao, H.J.: Quantised recursive filtering for a class of nonlinear systems with multiplicative noises and missing measurements. *Int. J. Control* **86**(4), 650–663 (2013)
18. Hu, J., Wang, Z.D., Alsaadi, F.E., Hayat, T.: Event-based filtering for time-varying nonlinear systems subject to multiple missing measurements with uncertain missing probabilities. *Inf. Fusion* **38**, 74–83 (2017)
19. Wang, Z.D., Shen, B., Liu, X.: H_∞ filtering with randomly occurring sensor saturations and missing measurements. *Automatica* **48**(3), 556–562 (2012)
20. Hou, L., Cheng, J., Wang, H.: Finite-time stochastic boundedness of discrete-time Markovian jump neural networks with boundary transition probabilities and randomly varying nonlinearities. *Neurocomputing* **174**(22), 773–779 (2016)
21. Liu, Y., Wang, Z.D., Liu, X.: Robust H_∞ control for a class of nonlinear stochastic systems with mixed time-delay. *Int. J. Robust Nonlinear Control* **17**(16), 1525–1551 (2007)
22. Hu, J., Wang, Z.D., Gao, H.J., Stergioulas, L.K.: Extended Kalman filtering with stochastic nonlinearities and multiple missing measurements. *Automatica* **48**(9), 2007–2015 (2012)
23. Alavi, S., Saif, M.: Fast state estimation subject to random data loss in discrete-time nonlinear stochastic systems. *Int. J. Control* **86**(12), 302–2314 (2013)
24. Wei, G., Wang, Z.D., Shu, H.: Robust filtering with stochastic nonlinearities and multiple missing measurements. *Automatica* **45**, 836–841 (2009)

25. Wang, Z.D., Liu, Y., Liu, X.: H_∞ filtering for uncertain stochastic time-delay systems with sector-bounded nonlinearities. *Automatica* **44**(5), 1268–1277 (2008)
26. Liu, Y., Wang, Z.D., Wang, W.: Robust reliable control for discrete-time-delay systems with stochastic nonlinearities and multiplicative noises. *Optim. Control Appl. Methods* **32**, 285–297 (2011)
27. Dorato, P.: Short-time stability in linear time-varying systems. *IRE Int. Conv. Rec.* **5**, 83–87 (1961)
28. Wan, X.B., Wang, Z.D., Han, Q.L., Wu, M.: Finite-time H_∞ state estimation for discrete time-delayed genetic regulatory networks under stochastic communication protocols. *IEEE Trans. Circuits Syst. I, Regul. Pap.* **65**(10), 3481–3491 (2018)
29. Tuan, L.A., Phat, V.N.: Finite-time stability and H_∞ control of linear discrete-time delay systems with norm-bounded disturbances. *Acta Math. Vietnam.* **41**, 481–493 (2016)
30. Zhao, J., Shen, H., Li, B., Wang, J.: Finite-time H_∞ control for a class of Markovian jump delayed systems with input saturation. *Nonlinear Dyn.* **73**(1), 1099–1110 (2013)
31. Liu, Y., Yu, X., Zhou, X.: Finite-time H_∞ control for linear systems with semi-Markovian switching. *Nonlinear Dyn.* **85**(4), 2297–2308 (2016)
32. Song, J., Niu, Y.G., Zou, Y.Y.: Robust finite-time bounded control for discrete-time stochastic systems with communication constraint. *IET Control Theory Appl.* **9**(13), 2015–2021 (2015)
33. Hua, C., Yu, S., Guan, X.: Finite-time control for a class of networked control systems with short time-varying delays and sampling jitter. *Int. J. Autom. Comput.* **12**(4), 448–454 (2015)
34. Elahi, A., Alfi, A.: Finite-time H_∞ control of uncertain networked control systems with randomly varying communication delays. *ISA Trans.* **69**, 65–88 (2017)
35. Mathyalagan, K., Park, J., Sakthivel, R.: Finite-time boundedness and dissipativity analysis of networked cascade control systems. *Nonlinear Dyn.* **84**(4), 2149–2160 (2016). Arezou E., Alireza A.: Finite-time H_∞ control of uncertain networked control systems with randomly varying communication delays. *ISA Trans.* **69**, 65–88 (2017)
36. Song, J., Niu, Y.G.: Resilient finite-time stabilization of fuzzy stochastic systems with randomly occurring uncertainties and randomly occurring gain fluctuations. *Neurocomputing* **171**, 444–451 (2016)
37. Ma, Y.C., Jia, X.R., Liu, D.Y.: Finite-time dissipative control for singular discrete-time Markovian jump systems with actuator saturation and partly unknown transition rates. *Appl. Math. Model.* **53**, 49–70 (2018)
38. Song, J., Niu, Y.G., Wang, S.B.: Robust finite-time dissipative control subject to randomly occurring uncertainties and stochastic fading measurements. *J. Franklin Inst.* **354**(10), 3706–3723 (2017)
39. Shen, H., Li, F., Yan, H., Karimi, H.R., Lam, H.K.: Finite-time event-triggered H_∞ control for T-S fuzzy Markov jump systems. *IEEE Trans. Fuzzy Syst.* **26**(5), 3122–3135 (2018)

Submit your manuscript to a SpringerOpen[®] journal and benefit from:

- Convenient online submission
- Rigorous peer review
- Open access: articles freely available online
- High visibility within the field
- Retaining the copyright to your article

Submit your next manuscript at ► [springeropen.com](https://www.springeropen.com)
