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Existence and uniqueness results on time scales for fractional nonlocal thermistor problem in the conformable sense

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Abstract
We study a conformable fractional nonlocal thermistor problem on time scales. Under an appropriate nonrestrictive condition on the resistivity function, we establish existence and uniqueness results. The proof is based on the use of Schauder's point fixed theorem.

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1 Introduction

Fractional calculus has aroused keen considerable attention of several researchers with many emerging applications, including memory effects and future dependence, in mathematical physics, biology, dynamical systems, chemistry, population dynamics, and recently in epidemic diseases [2, 3, 6, 9–11, 15, 16, 21, 23, 25, 26].

The theory of time scales is often used for describing models that cannot be considered as exclusively continuous or exclusively discrete processes. It was first introduced by Aulbach and Stefan Hilger [8] with many applications in calculus of variations, economics, quantum calculus [4]. To find out more about the fractional calculus on a time scale, excellent references are situated in the books of Miller and Ross [22] and Podlubny [24]. The monograph of Bohner and Peterson [14] and [4, 5] are considered as good bibliography for calculus on time scales.

In recent years, the notion of conformable fractional derivative, which is a natural extension of the classical derivative, was initiated in [20]. It was after extended to an arbitrary time scales in [13] generalizing the Hilger derivative, preserving the compatibility, and satisfying more standard formulas of the usual derivative. The main advantages and efficiency of the conformable calculus are described in a detailed way in [30]. A nabla conformable fractional calculus was introduced in [12] and more generally in [34] on arbitrary time scales.

A thermistor sensor is a temperature sensing element made of a sintered semiconductor material that is characterized by large changes in resistance proportional to small changes in temperature. Thermistors are made from a combination of metals and metal oxide materials. Once combined, the materials are formed and fired into the desired shape. The thermistors can then be used as disc-shaped thermistors or can be formed and assembled with conductive wires and coatings to form ball-shaped thermistors.

The main qualities of these sensors are: accuracy, nominal value for a given temperature (at 25 Celsius), response time (in seconds), sensitivity or temperature coefficient (variation of resistance according to temperature), extent or measurement range (min. and max. temperature of use), lifespan, stability (variation of the various parameters over time), dimensions, and low cost [17].

Due to its various applications, a great interest is given by researchers to the study of integer order, fractional order, or time scales thermistors models via different approaches [7, 28, 30, 31]. In [28], the question of existence and uniqueness of a positive solution to generalized nonlocal thermistor problems with fractional order derivatives was investigated. Global existence of solutions for a fractional nonlocal thermistor problem in the Caputo sense was addressed in [27]. Existence and uniqueness results for a fractional Riemann–Liouville nonlocal thermistor problem on arbitrary nonempty closed subsets of the real numbers were studied in [29], Dynamics and stability results for Hilfer fractional derivative, which interpolate both the Riemann–Liouville and the Caputo fractional derivative, thermistor problems were investigated in [33]. Existence of a particular tube solution of a nonlocal thermistor problem for fractional derivative in the conformable sense was proved in [32]. In the present work, we are concerned with a conformable fractional thermistor problem and with existence and uniqueness results on time scales.

In order that the paper be complete for beginning researchers wishing to infiltrate the field, we recall in Sect. 2 preliminary basic time scales notation, definitions, and lemmas that are used later. As an illustration, in Sect. 3, existence and uniqueness for a fractional nonlocal thermistor problem on time scales are addressed. The undermentioned model is formulated in the conformable sense. We employ essentially Banach and Schauder's fixed point theorems.

2 Preliminaries

In the beginning we present the following auxiliary theorems that play a crucial role in reaching our existence and uniqueness results.

Theorem 2.1 ([18] Banach fixed point theorem) *Let (X,d) be a complete metric space, and let $F : X \to X$ be a contraction mapping. Then F admits a unique fixed point in X.*

Theorem 2.2 ([18] Schauder fixed point theorem) *Let E be a Banach space. Assume that $F : E \to E$ is a completely continuous operator and the set $V = \{u \in E : u = \mu Fu, 0 \le \mu \le 1\}$ is bounded. Then F has a fixed point in E.*

For convenience, we secondly give some important notations, definitions, and basic results on a fractional calculus on a time scale generalizing the Hilger derivative which can be found, for example, in [14]. A nonempty closed subset $\mathbb{T}$ of $\mathbb{R}$ is called a *time scale*.

The *forward jump operator* $\sigma : \mathbb{T} \to \mathbb{T}$ is defined by

$$\sigma(t) = \inf\{s \in \mathbb{T} : s > t\} \quad \text{for all } t \in \mathbb{T},$$

while the *backward jump operator* $\rho : \mathbb{T} \to \mathbb{T}$ is defined by

$$\rho(t) = \sup\{s \in \mathbb{T} : s < t\} \quad \text{for all } t \in \mathbb{T},$$

with $\inf \emptyset = \sup \mathbb{T}$ (i.e., $\sigma(M) = M$ if $\mathbb{T}$ has a maximum M) and $\sup \emptyset = \inf \mathbb{T}$ (i.e., $\rho(m) = m$ if $\mathbb{T}$ has a minimum m).

A point $t \in \mathbb{T}$ is called *right-dense*, *right-scattered*, *left-dense*, or *left-scattered* if $\sigma(t) = t$, $\sigma(t) > t$, $\rho(t) = t$, or $\rho(t) < t$, respectively.

The *(forward) graininess function* $\mu : \mathbb{T} \to [0, \infty)$ and the *backward graininess function* $\nu : \mathbb{T} \to [0, \infty)$ are defined by

$$\mu(t) = \sigma(t) - t, \qquad \nu(t) = t - \rho(t) \quad \text{for all } t \in \mathbb{T},$$

respectively.

A closed interval on $\mathbb{T}$ is denoted by $[a, b]_{\mathbb{T}} = \{t \in \mathbb{T}; a \leq t \leq b\}$, where a and b are fixed points of $\mathbb{T}$ with $a < b$. If $\mathbb{T}$ has left-scattered maximum m, then $\mathbb{T}^k = \mathbb{T} - \{m\}$. Otherwise, we set $\mathbb{T}^k = \mathbb{T}$. For the usual definitions of Δ-derivative, Δ-integral of a function, we refer the reader to [14].

Let $h : \mathbb{T} \to \mathbb{R}$ be a real-valued function on a time scale $\mathbb{T}, t \in \mathbb{T}^k$ and $\alpha \in (0, 1]$. In [13], the authors define the conformable fractional(CF) derivative of a function h of order α at t, denoted by $T_\alpha(h)(t)$, the number provided it exists such that, for all $\varepsilon > 0$, there exists a neighborhood $V_t \subseteq \mathbb{T}$ of t such that, for all $s \in V_t$,

$$\left|\left[h\left(\sigma(t)\right) - h(s)\right]t^{\alpha-1} - T_\alpha(h)(t)\left[\sigma(t) - s\right]\right| \leq \varepsilon\left|\sigma(t) - s\right|.$$

We remind the following properties as given in [13].

Lemma 2.3 *Let $\alpha \in (0, 1), t \in \mathbb{T}^k$, and $h : \mathbb{T} \to \mathbb{R}$ be a function. The following properties hold*:

(a) *If h is CF differentiable of order α at $t > 0$, then h is continuous at t.*

(b) *If h is continuous at a right-scattered t, then h is CF differentiable of order α at t with*

$$T_\alpha(h)(t) = \frac{h(\sigma(t)) - h(t)}{\mu(t)} t^{1-\alpha}.$$

(c) *If t is right-dense, then h is CF differentiable of order α at t if and only if $\lim_{t \to s} \frac{h(t)-h(s)}{t-s} t^{1-\alpha}$ exists as a finite number. We then have*

$$T_\alpha(h)(t) = \lim_{t \to s} \frac{h(t) - h(s)}{t - s} t^{1-\alpha}.$$

(d) *If h is CF differentiable of order α at t, then $h(\sigma(t)) = h(t) + \mu(t)t^{1-\alpha}T_\alpha(h)(t)$.*

Let h be a regulated function on $\mathbb{T}$. The indefinite α-CF integral of h is defined by

$$H_\alpha(t) = \int h(t)\Delta^\alpha t = \int h(t)t^{\alpha-1}\Delta t.$$

Finally, we define the Cauchy or definite α-CF integral of h by

$$\int h(t)\Delta^{\alpha} t = H_{\alpha}(b) - H_{\alpha}(a).$$

Other fundamental properties and features of the CF derivative as linearity of conformable fractional derivatives, conformable chain rule, conformable fractional derivative of product, quotient and composite of functions are given in [19].

The following result of the calculus on time scales is also useful [13].

Proposition 2.4 *Let $[a,b] \subseteq \mathbb{T}$ and h be an increasing continuous function on the time-scale interval $[a,b]$. If H is the extension of h to the real interval $[a,b]$ defined by*

$$H(s) := \begin{cases} h(s) & \text{if } s \in \mathbb{T}, \\ h(t) & \text{if } s \in (t, \sigma(t)) \notin \mathbb{T}, \end{cases}$$

then

$$\int_a^b h(t)\triangle t \leq \int_a^b H(t)\,dt,$$

The above inequality can be extended to the case of double integral as follows:

$$\int_a^b \int_c^d h(t)\triangle t \Delta s \leq \int_a^b \int_c^d H(t)\,dt\,ds.$$

3 Main results

In this section, we are concerned with the conformable fractional nonlocal thermistor problem governed by

$$-T_{\alpha}\bigl(T_{\alpha}\bigl(u(t)\bigr)\bigr) = \frac{\lambda f(t,u(t))}{(\int_{\rho(0)}^{T} f(x,u(x))\triangle x^{\alpha})^2} = g\bigl(t,u(t)\bigr), \quad t \in [0,T], \tag{1}$$

with the boundary value conditions

$$\begin{aligned} &T_{\alpha}\bigl(u\bigl(\rho(0)\bigr)\bigr) - \beta T_{\alpha}\bigl(u(\eta)\bigr) = 0, \\ &u(T) - \beta u(\eta) = 0, \end{aligned} \tag{2}$$

where $0 < \alpha < 1$, T_{α} is the CF derivative. λ is a dimensionless real parameter. In case $\alpha = 1$, problem (1) describes the diffusion of heat through a thermistor. $f(u)$ is the temperature-dependent resistivity of the conductor; and β is a heat transfer coefficient verifying $0 < \beta < 1$. $\eta \in (0,T)_{\mathbb{T}}$, T is real fixed. Many existence results for different forms of thermistor problem were investigated using various fixed point theorems [28, 30]. Different to our earlier contributions, we consider the nonlocal thermistor problem in the conformable sense on a time scale. Further, we establish the main results under the following nonrestrictive hypothesis:

(H_1) $f : \mathbb{T} \times \mathbb{R} \to \mathbb{R}^{+*}$ is a continuous function with respect to the first variable and a locally Lipschitzian function with respect to the second variable.

$\mathbb{C}(\mathbb{T},\mathbb{R})$ will denote the Banach space of all continuous real-valued functions defined on the time scale $\mathbb{T}$ with the supremum norm.

Lemma 3.1 *Assume that hypothesis* $(H1)$ *is verified. Then* u *is a solution to* (1)–(2) *if and only if* u *is a solution of the representation integral equation*

$$u(t) = u\big(\rho(0)\big) - \frac{\beta}{1-\beta}\int_{\rho(0)}^{t}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s - \int_{\rho(0)}^{t}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s,$$

where

$$\begin{aligned} &u\big(\rho(0)\big) \\ &\quad = -\frac{\beta^2}{(1-\beta)^2}\int_{\rho(0)}^{\eta}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s - \frac{\beta}{1-\beta}\int_{\rho(0)}^{\eta}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s \\ &\qquad + \frac{\beta}{(1-\beta)^2}\int_{\rho(0)}^{T}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s + \frac{1}{1-\beta}\int_{\rho(0)}^{T}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s. \end{aligned}$$

Proof Suppose that $u(t)$ verifies (1), then after taking the α-CF integral of this equation from $\rho(0)$ to s, we get

$$\int_{\rho(0)}^{s} T_{\alpha}\big(T_{\alpha}\big(u(\tau)\big)\big)\triangle\tau^{\alpha} = -\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle\tau^{\alpha}.$$

Using the property of inverse integral, we have

$$T_{\alpha}\big(u(s)\big) = T_{\alpha}\big(u\big(\rho(0)\big)\big) - \int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau. \tag{3}$$

The boundary condition implies

$$\begin{aligned} T_{\alpha}\big(u\big(\rho(0)\big) &= \beta T_{\alpha}\big(u(\eta)\big) \\ &= \beta T_{\alpha}\big(u\big(\rho(0)\big)\big) - \int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau, \end{aligned}$$

we then get

$$T_{\alpha}\big(u\big(\rho(0)\big) = \frac{-\beta}{1-\beta}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau.$$

We can then substitute in (3) and have

$$T_{\alpha}\big(u(s)\big) = \frac{-\beta}{1-\beta}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau - \int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau.$$

By taking the α-CF integral

$$\int_{\rho(0)}^{t} T_{\alpha}\big(u(s)\big)\triangle^{\alpha}s = \frac{-\beta}{1-\beta}\int_{\rho(0)}^{t}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s - \int_{\rho(0)}^{t}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s.$$

Thus

$$u(t) = u\big(\rho(0)\big) - \frac{\beta}{1-\beta}\int_{\rho(0)}^{t}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s - \int_{\rho(0)}^{t}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s, \quad (4)$$

which together with the second boundary value condition and (4) yields

$$\begin{aligned} u\big(\rho(0)\big) &= u(T) + \frac{\beta}{1-\beta}\int_{\rho(0)}^{T}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s + \int_{\rho(0)}^{T}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s \\ &= \beta u(\eta) + \frac{\beta}{1-\beta}\int_{\rho(0)}^{T}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s + \int_{\rho(0)}^{T}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s \\ &= \beta u\big(\rho(0)\big) - \frac{\beta^2}{1-\beta}\int_{\rho(0)}^{\eta}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s \\ &\quad - \beta\int_{\rho(0)}^{\eta}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s \\ &\quad + \frac{\beta}{1-\beta}\int_{\rho(0)}^{T}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s + \int_{\rho(0)}^{T}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s. \end{aligned}$$

It follows that

$$\begin{aligned} &u\big(\rho(0)\big) \\ &\quad = -\frac{\beta^2}{(1-\beta)^2}\int_{\rho(0)}^{\eta}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s - \frac{\beta}{1-\beta}\int_{\rho(0)}^{\eta}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s \\ &\qquad + \frac{\beta}{(1-\beta)^2}\int_{\rho(0)}^{T}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s + \frac{1}{1-\beta}\int_{\rho(0)}^{T}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s. \end{aligned}$$

Therefore, we have the intended result. □

Theorem 3.2 *There exists a unique solution for a well-chosen parameter* λ.

Proof Let $S \subseteq \mathbb{C}(\mathbb{T},\mathbb{R})$. Define the subset S(r) as

$$S(r) = \big\{v \in S, \|v\| \le r\big\},$$

where r is described below. First, it is clear that the set $S(r)$ is a nonempty, closed, convex, and bounded set. Define the operator F by

$$\begin{aligned} &F\big(u(t)\big) \qquad (5) \\ &\quad = u\big(\rho(0)\big) - \frac{\beta}{1-\beta}\int_{\rho(0)}^{t}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s - \int_{\rho(0)}^{t}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^{\alpha}\tau\triangle^{\alpha}s. \end{aligned}$$

It is clear that $u(t)$ is a solution of (1)–(2) if it is a fixed point of the operator Fu. Since f is a continuous function, there exist two positive constants such that $M_1 \le f(t,u(t)) \le M_2$. Then

$$\frac{M_1}{\alpha}\big(T^{\alpha} - \rho^{\alpha}(0)\big) \le \int_{\rho(0)}^{T} f\big(x,u(x)\big)\triangle^{\alpha}x = \int_{\rho(0)}^{T} f\big(x,u(x)\big)x^{\alpha-1}\triangle x$$

$$\begin{aligned}
&\le M_2 \int_{\rho(0)}^{T} x^{\alpha-1} \triangle x \\
&\le M_2 \int_{\rho(0)}^{T} x^{\alpha-1}\, dx \\
&\le \frac{M_2}{\alpha} T^{\alpha} = M_3.
\end{aligned}$$

Setting $u(\rho(0)) = \mu$. Then, by using Proposition 2.4, we get

$$\begin{aligned}
\bigl|F\bigl(u(t)\bigr)\bigr| &\le \mu + \frac{\beta}{1-\beta} M_3 \int_{\rho(0)}^{T}\int_{\rho(0)}^{T} \tau^{\alpha-1}s^{\alpha-1}\, d\tau\, ds + M_3 \int_{\rho(0)}^{T}\int_{\rho(0)}^{T} \tau^{\alpha-1}s^{\alpha-1}\, d\tau\, ds \\
&\le \mu + \frac{M_3}{1-\beta} \int_{\rho(0)}^{T}\int_{\rho(0)}^{T} \tau^{\alpha-1}s^{\alpha-1}\, d\tau\, ds \\
&\le \mu + \frac{M_3}{1-\beta}\frac{T^{2\alpha}}{\alpha^2}.
\end{aligned}$$

With the equality $\mu + \frac{M_3}{1-\beta}\frac{T^{2\alpha}}{\alpha^2} = r$, we conclude that F is an operator from $S(r)$ to $S(r)$. We prove that F is contractive, we can easily see that

$$\begin{aligned}
\bigl|F\bigl(u(t)\bigr) - F\bigl(v(t)\bigr)\bigr| \le{}& \frac{\beta}{1-\beta} \int_{\rho(0)}^{t}\int_{\rho(0)}^{\eta} \bigl|g\bigl(\tau, u(\tau)\bigr) - g\bigl(\tau, v(\tau)\bigr)\bigr| \triangle^{\alpha}\tau \triangle^{\alpha} s \\
&+ \int_{\rho(0)}^{t}\int_{\rho(0)}^{s} \bigl|g\bigl(\tau, u(\tau)\bigr) - g\bigl(\tau, v(\tau)\bigr)\bigr| \triangle^{\alpha}\tau \triangle^{\alpha} s.
\end{aligned}$$

On the other hand, we have

$$\begin{aligned}
&g\bigl(s, u(s)\bigr) - g\bigl(s, v(s)\bigr) \\
&\quad= \frac{\lambda f(s,u(s))}{(\int_{\rho(0)}^{T} f(x,u(x))\triangle^{\alpha}x)^2} - \frac{\lambda f(s,v(s))}{(\int_{\rho(0)}^{T} f(x,v(x))\triangle^{\alpha}x)^2} \\
&\quad= \frac{\lambda}{(\int_{\rho(0)}^{T} f(x,u(x))\triangle^{\alpha}x)^2}\bigl(f\bigl(s,u(s)\bigr) - f\bigl(s,v(s)\bigr)\bigr) \\
&\qquad+ \lambda f\bigl(s,v(s)\bigr)\biggl(\frac{1}{(\int_{\rho(0)}^{T} f(x,u(x))\triangle^{\alpha}x)^2} - \frac{1}{(\int_{a}^{T} f(x,v(x))\triangle^{\alpha}x)^2}\biggr) \\
&\quad= I_1 + I_2.
\end{aligned}$$

Using that f is locally Lipschitzian, we have

$$\begin{aligned}
|I_1| &\le \frac{\lambda}{(\frac{M_1}{\alpha}(T^{\alpha} - \rho^{\alpha}(0)))^2}\bigl|f(s,u(s) - f\bigl(s,v(s)\bigr)\bigr| \\
&\le \frac{\lambda L_f}{(\frac{M_1}{\alpha}(T^{\alpha} - \rho^{\alpha}(0)))^2}\bigl|u(s) - v(s)\bigr| \\
&\le \frac{\lambda L_f}{(\frac{M_1}{\alpha}(T^{\alpha} - \rho^{\alpha}(0)))^2}\|u - v\| = \lambda M_4\|u - v\|.
\end{aligned}$$

Furthermore, we have

$$
\begin{aligned}
|I_2| &\le \frac{\lambda M_2 |(\int_{\rho(0)}^T f(x,u(x))\triangle^\alpha x)^2 - (\int_{\rho(0)}^T f(x,v(x))\triangle^\alpha x)^2|}{(\int_{\rho(0)}^T f(x,u(x))\triangle^\alpha x)^2 (\int_{\rho(0)}^T f(x,v(x))\triangle^\alpha x)^2} \\
&\le \frac{\lambda M_2}{(\frac{M_1}{\alpha}(T^\alpha - \rho^\alpha(0)))^4} \left| \left(\int_{\rho(0)}^T \big(f\big(x,u(x)\big) - f\big(x,v(x)\big)\big)\triangle^\alpha \tau \right) \right. \\
&\quad \times \left. \left(\int_{\rho(0)}^T \big(f\big(x,u(x)\big) + f\big(x,v(x)\big)\big)\triangle^\alpha x \right) \right| \\
&\le \frac{2\lambda M_2^2}{(\frac{M_1}{\alpha}(T^\alpha - \rho^\alpha(0)))^4} \frac{T^\alpha}{\alpha} \left(\int_{\rho(0)}^T \big| f\big(x,u(x)\big) - f\big(x,v(x)\big)\big| \triangle^\alpha x \right) \\
&\le \frac{2\lambda M_2^2}{(\frac{M_1}{\alpha}(T^\alpha - \rho^\alpha(0)))^4} \frac{T^\alpha}{\alpha} \left(\int_{\rho(0)}^T \big| f\big(x,u(x)\big) - f\big(x,v(x)\big)\big| x^{\alpha-1}\,dx \right) \\
&\le L_f \frac{2\lambda M_2^2}{(\frac{M_1}{\alpha}(T^\alpha - \rho^\alpha(0)))^4} \left(\frac{T^\alpha}{\alpha} \right)^2 \|u - v\| \\
&\le \lambda M_5 \|u - v\|.
\end{aligned}
$$

Then

$$
\big| g\big(s,u(s)\big) - g\big(s,v(s)\big)\big| \le \lambda(M_4 + M_5)\|u - v\|.
$$

Hence

$$
\begin{aligned}
\big| F\big(u(t)\big) - F\big(v(t)\big)\big| &\le \lambda(M_4 + M_5)\left(\frac{\beta}{1-\beta} + 1 \right)\|u - v\| \int_{\rho(0)}^T \int_{\rho(0)}^T \tau^{\alpha-1}s^{\alpha-1}\,d\tau\,ds \\
&\le \lambda(M_4 + M_5)\frac{1}{1-\beta}\left(\frac{T}{\alpha} \right)^2 \|u - v\|.
\end{aligned}
$$

By the assumption that $\frac{\lambda}{1-\beta}(M_4 + M_5)(\frac{T}{\alpha})^2 < 1$, we conclude that F is contractive. It implies that (1)–(2) has a unique solution by the Banach fixed point theorem. □

Theorem 3.3 *Under hypothesis* $(H1)$, (1)–(2) *has a solution on* $\mathbb{T}$.

Step 1 We prove that F is continuous. Let u_n be a sequence which converges to u. In a similar way as above, one can have, for all $t \in \mathbb{T}$,

$$
\big| F\big(u_n(t)\big) - F\big(u(t)\big)\big| \le \lambda(M_4 + M_5)\frac{1}{1-\beta}\left(\frac{T}{\alpha} \right)^2 \|u_n - u\|.
$$

Then

$$
\big\| F(u_n) - F(u)\big\| \le \lambda(M_4 + M_5)\frac{1}{1-\beta}\left(\frac{T}{\alpha} \right)^2 \|u_n - u\|.
$$

The above inequality permits to conclude that whenever $u_n \to u$, then $Fu_n \to Fu$. This proves the continuity of F.

Step 2 The map F sends bounded sets into bounded sets in $\mathbb{C}(\mathbb{T},\mathbb{R})$. Furthermore, we obtain for all $u \in S(r)$ any ball centered in the origin

$$\left|F\big(u(t)\big)\right| \le \mu + \frac{M_3}{1-\beta}\frac{T^{2\alpha}}{\alpha^2}.$$

Step 3 To prove that $F(S(r))$ resides in a relatively compact subset of $\mathbb{C}(\mathbb{T},\mathbb{R})$. Let $t_1 < t_2$, we have

$$\begin{aligned}
&\left|Fu(t_2) - Fu(t_1)\right| \\
&\quad = \left|\frac{\beta}{1-\beta}\left(\int_{\rho(0)}^{t_2}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s - \int_{\rho(0)}^{t_1}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s\right)\right. \\
&\qquad \left. + \left(\int_{\rho(0)}^{t_2}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s - \int_{\rho(0)}^{t_1}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s\right)\right| \\
&\quad \le \left|\frac{\beta}{1-\beta}\int_{t_1}^{t_2}\int_{\rho(0)}^{\eta} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s + \int_{t_1}^{t_2}\int_{\rho(0)}^{s} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s\right| \\
&\quad \le \left|\frac{\beta}{1-\beta}\int_{t_1}^{t_2}\int_{\rho(0)}^{T} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s + \int_{t_1}^{t_2}\int_{\rho(0)}^{T} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s\right| \\
&\quad \le \left|\frac{1}{1-\beta}\int_{t_1}^{t_2}\int_{\rho(0)}^{T} g\big(\tau,u(\tau)\big)\triangle^\alpha\tau\triangle^\alpha s\right| \\
&\quad \le \frac{M_2}{1-\beta}\int_{t_1}^{t_2}\int_{\rho(0)}^{T} \tau^{\alpha-1}s^{\alpha-1}\,d\tau\,ds \\
&\quad \le \frac{M_2}{1-\beta}\frac{T^\alpha - \rho^\alpha(0)}{\alpha^2}\big(t_2^\alpha - t_1^\alpha\big).
\end{aligned}$$

This implies that F is a bounded and uniformly Cauchy subset of $\mathbb{C}(\mathbb{T},\mathbb{R})$. Thus, by virtue of the Arzela–Ascoli's theorem, we conclude that F is relatively compact.

Step 4 Now, it remains to show that the set $\Omega = \{u \in \mathbb{C}(\mathbb{T},\mathbb{R}) : u(t) = \nu F(u(t)); 0 < \nu < 1\}$ is bounded. Let $u \in \Omega$, Thus, for each $t \in \mathbb{T}$, we have

$$\begin{aligned}
\left|u(t)\right| &\le \nu\left|F\big(u(t)\big)\right| \\
&\le \nu\left(\mu + \frac{M_3}{1-\beta}\frac{T^{2\alpha}}{\alpha^2}\right) \\
&\le \mu + \frac{M_3}{1-\beta}\frac{T^{2\alpha}}{\alpha^2}.
\end{aligned}$$

By Schauder's fixed point theorem, we can conclude that F has a fixed point, which represents a solution for problem (1)–(2).

4 Conclusion

In the last decade, fractional thermistor problems have attracted the interest of several researchers with flourishing applications. As an additional stone in this subject, existence and uniqueness results for a fractional nonlocal thermistor problem in the conformable sense are examined. The proof of the main result is based on the employment of Schauder's fixed point and Arzelà–Ascoli theorems.

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