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On a fractional-order p-Laplacian boundary value problem at resonance on the half-line with two dimensional kernel

O.F. Imaga[1*] and S.A. Iyase[1]

*Correspondence:
imaga.ogbu@covenantuniversity.edu.ng
[1]Department of Mathematics, Covenant University, Ota, Ogun State, Nigeria

Abstract

In this work, we consider the solvability of a fractional-order *p*-Laplacian boundary value problem on the half-line where the fractional differential operator is nonlinear and has a kernel dimension equal to two. Due to the nonlinearity of the fractional differential operator, the Ge and Ren extension of Mawhin's coincidence degree theory is applied to obtain existence results for the boundary value problem at resonance. Two examples are used to validate the established results.

MSC: 70K30; 34B10; 34B15; 34A08

Keywords: Coincidence degree; Half-line; Integral boundary value problem; Fractional p-Laplacian; Resonance

1 Introduction

In this paper, we obtain existence results for the following fractional-order p-Laplacian boundary value problem at resonance on the half-line with integral boundary conditions:

$$\begin{cases} (\varphi_p(D^a_{0+}u(t)))' + v(t)w(t,u(t),D^a_{0+}u(t)) = 0, \quad t \in (0,+\infty), \\ \varphi_p(D^a_{0+}u(0)) = \int_0^{+\infty} v(t)\varphi_p(D^a_{0+}u(t))\,dt, \\ \varphi_p(D^a_{0+}u(+\infty)) = \int_0^{+\infty} v(t)\varphi_p(D^a_{0+}u(t))\,dt, \end{cases} \tag{1.1}$$

where D^a_{0+} is the Riemann–Liouville fractional derivative of order a, $0 < a \leq 1$, $w : [0,+\infty) \times \mathbb{R}^2 \to \mathbb{R}$ is a v-Carathéodory function, and $v(t) \in L^1[0,+\infty)$, $v(t) > 0$, with $\int_0^{+\infty} v(t)\,dt = 1$, $\varphi_p(r) = |r|^{p-2}r$, $p > 1$, $\varphi_p^{-1} = \varphi_q$.

Recently, fractional calculus has become popular because it is nonlocal and has many real life applications like in signal processing, viscoelasticity, bioengineering, and fluid dynamics [14]. Fractional-order derivatives have been found to handle models more accurately than integer-order ones because they have higher degree of freedom. Moreover, fractional-order derivatives contain a memory term which makes it suited to describe the memory and hereditary properties of various processes and materials [16].

Boundary value problems with *p*-Laplacian operator arise in the modeling of many natural phenomena like in unsteady flow of fluid through a semi-infinite porous medium.

They are also encountered in combustion theory, nonlinear elasticity, population biology, glaciology, non-Newtonian mechanics, plasma physics, and the study of drain flows (see [1, 10, 15]).

Boundary value problem (1.1) is said to be at resonance if the corresponding homogeneous problem

$$\begin{cases} (\varphi_p(D^a_{0+}u(t)))' = 0, \quad t \in (0, +\infty), \\ \varphi_p(D^a_{0+}u(0)) = \int_0^{+\infty} \nu(t)\varphi_p(D^a_{0+}u(t))\,dt, \qquad \varphi_p(D^a_{0+}u(+\infty)) = \int_0^{+\infty} \nu(t)\varphi_p(D^a_{0+}u(t))\,dt, \end{cases}$$

has nontrivial solutions.

p-Laplacian resonant boundary value problems can be expressed in the abstract form $Lu = Nu$, where L is a non vertible fractional-order differential operator. When $p = 2$, the differential operator L is linear, and Mawhin's coincidence degree theory [13] can be applied, for example, see [2, 3, 8, 9, 15]. However, when $p > 2$, Mawhin's coincidence degree can no longer be applied directly, hence the Ge and Ren extension of the coincidence degree [4] becomes an efficient tool for the investigation, see [6, 11, 17–19] and the references therein.

Imaga and Iyase [7] obtained existence results for the following p-Laplacian boundary value problem at resonance on the half-line:

$$\begin{cases} (\varphi_p(u'(t)))' + g(t, u(t), u'(t)) = 0, \quad t \in (0, +\infty), \\ \varphi_p(u'(0)) = \int_0^{+\infty} \nu(t)\varphi_p(u'(t))\,dt, \\ \varphi_p(u'(+\infty)) = \sum_{j=1}^{m} \beta_j \int_0^{\eta_j} \varphi_p(u'(t))\,dt, \end{cases}$$

where $g : [0, +\infty) \times \mathbb{R}^2 \to \mathbb{R}$ is an L^1-Carathéodory function, $0 < \eta_1 < \eta_2 < \cdots \leq \eta_m < +\infty$, $\beta_j \in \mathbb{R}$, $j = 1, 2, \ldots, m$, $\nu \in L^1[0, +\infty)$, $\nu(t) > 0$ on $[0, +\infty)$, and $\varphi_p(s) = |s|^{p-2}s$, $p > 1$.

In [15] the authors obtained existence conditions for the fractional-order p-Laplacian boundary value problem at resonance

$$\begin{cases} (D^a_{0+}\varphi_p(D^b_{0+}u(t)) = f(t, u(t), D^b_{0+}u(t)), \quad 0 \leq t \leq 1, \\ u(0) = 0 \quad D^b_{0+}u(0) = D^b_{0+}u(1), \end{cases}$$

where $0 < a, b \leq 1$, $1 < a + b \leq 2$, f is a continuous function, $p = 2$, D^a_{0+} and D^b_{0+} are Caputo fractional derivatives.

Chen et al. [2] obtained the existence of solution for the fractional-order p-Laplacian boundary value problem

$$\begin{cases} D^a_{0+}\varphi_p(D^b_{0+}u(t)) = f(t, u(t), D^b_{0+}u(t)), \quad 0 \leq t \leq 1, \\ D^b_{0+}u(0) = D^b_{0+}u(1) = 0, \end{cases}$$

where $1 < a \leq 2$, $p = 2$.

In [19], the authors studied the following fractional-order boundary value problem:

$$\begin{cases} D^a_{0+}u(t) = f(t, u(t), D^{a-1}_{0+}u(t)), \quad t \in (0, +\infty) \\ u(0) = 0, \quad \lim_{t \to +\infty} D_{0+}u(t) = \beta u(\eta), \end{cases}$$

where $0 < a, b \leq 1$, $1 < a + b \leq 2$, $p = 2$, f is a continuous function, D^a_{0+} and D^b_{0+} are Caputo fractional derivatives.

Motivated by the above results, we study the solvability for the fractional-order p-Laplacian boundary value problem at resonance on the half-line with integral boundary conditions. We also note that p-Laplacian fractional boundary value problems with integral boundary conditions have not been given much attention in literature. In Sect. 2 of this work, the required lemmas, theorem, and definitions are given; Sect. 3 is dedicated to stating and proving the conditions for existence of solutions. An example is given in Sect. 4 to illustrate the results obtained.

2 Preliminaries

In this section, we give definitions, lemmas, and theorems that will be used in this work.

Definition 2.1 ([5]) Let $(U, \|\cdot\|_U)$ and $(Z, \|\cdot\|_Z)$ be any two Banach spaces and $M : U \cap \operatorname{dom} M \to Z$ be a continuous operator. If $M(X \cap \operatorname{dom} M)$ is a closed subset of Z and $\ker M = \{u \in U \cap \operatorname{dom} M : Mu = 0\}$ is linearly homeomorphic to $\mathbb{R}^n$, $n < \infty$, then M is quasi-linear.

Let $U_1 = \ker M$ and U_2 be the complement of U_1 in U such that $U = U_1 \oplus U_2$. Also, let $Z_1 \subset Z$ and Z_2 be the complement of Z_1 in Z such that $Z = Z_1 \oplus Z_2$. Similarly, let $E : U \to U$, $F : Z \to Z$ be continuous projectors and $\Omega \subset U$ be open and bounded with the origin $\theta \in \Omega$.

Definition 2.2 ([5]) Let $N_k : \overline{\Omega} \to Z$, $k \in [0, 1]$ be a continuous operator, and let $\bigvee_k = \{u \in \overline{\Omega} : Mu = N_k u\}$. We denote N_1 by N. N_k is said to be M-compact in $\overline{\Omega}$ if there exists a vector subspace Z_1 of Z with $\dim Z_1 = \dim U_1$ and a continuous and compact operator, $P : \overline{\Omega} \times [0, 1] \to U_2$ such that, for $k \in [0, 1]$,

(ρ_1) $(I - F)N_k(\overline{\Omega}) \subset \operatorname{Im} M \subset (I - F)Z$;

(ρ_2) $FN_k u = \theta$, $k \in (0, 1) \Leftrightarrow FNu = \theta$;

(ρ_3) $P(\cdot, k)$ is the zero operator and $P(\cdot, k)|_{\bigvee_k} = (I - E)|_{\bigvee_k}$;

(ρ_4) $M[E + P(\cdot, k)] = (I - F)N_k$.

Theorem 2.1 ([5]) *Let $(U, \|\cdot\|_U)$ and $(Z, \|\cdot\|_Z)$ be any two Banach spaces and $\Omega \subset U$ be bounded, open, and nonempty. Assume that $M : U \cap \operatorname{dom} M \to Z$ is a quasi-linear operator and $N_k : \overline{\Omega} \to Z$, $k \in [0, 1]$ is M-compact on $\overline{\Omega}$. Suppose that the following hold:*

(τ_1) $Mu \neq N_k u$, $\forall (u, k) \in \partial\Omega \times (0, 1)$,

(τ_2) $FNu \neq 0$, $\forall u \in \ker M \cap \partial\Omega$,

(τ_3) $\deg(JFN, \ker M \cap \Omega, 0) \neq 0$, *where $F : Z \to \operatorname{Im} F$ is a projector and $J : \operatorname{Im} F \to \ker M$ is a homeomorphism with $J(\theta) = \theta$.*

Then at least one solution exists for the abstract equation $Mu = Nu$ in $\operatorname{dom} M \cap \overline{\Omega}$, where $N = N_1$.

Definition 2.3 ([17]) A map $w : [0, +\infty) \times \mathbb{R}^2 \to \mathbb{R}$ is ν-Carathéodory if the following conditions are satisfied:

(i) for each $(d, e) \in \mathbb{R}^2$, the mapping $t \to w(t, d, e)$ is Lebesgue measurable;

(ii) for a.e. $t \in [0, \infty)$, the mapping $(d, e) \to w(t, d, e)$ is continuous on $\mathbb{R}^2$;

(iii) for each $k > 0$ and $\nu \in L^1[0, +\infty)$, there exists $\psi_k(t) : [0, +\infty) \to [0, +\infty)$ satisfying $\int_0^{+\infty} \nu(t)\psi_k(t)\, dt < +\infty$ such that, for a.e. $t \in [0, \infty)$ and every $(d, e) \in [-k, k]$, we have

$$\left|w(t,d,e)\right| \leq \psi_k(t).$$

Definition 2.4 ([1]) Let $K = \{u \in C[0,+\infty), \lim_{t\to+\infty} u(t) \text{ exist}\}$ and $T \subset K$. Then T is said to be relatively compact if all functions from T are bounded, equicontinuous on any compact subinterval of $[0,+\infty)$, and equiconvergent at ∞.

Definition 2.5 ([5]) Let $a > 0$, the Riemann–Liouville fractional-order integral of a function $z : (0,+\infty) \to \mathbb{R}$ is defined by

$$I^a_{0+}z(t) = \frac{1}{\Gamma(a)} \int_0^t (t-r)^{a-1} z(r)\,dr$$

provided that the right-hand side is pointwise defined on $(0,+\infty)$.

Definition 2.6 ([5]) Let $a > 0$, the Riemann–Liouville fractional-order derivative of a function $z : (0,+\infty) \to \mathbb{R}$ is defined by

$$D^a_{0+}z(t) = \frac{d^n}{dt^n} I^{n-a}_{0+} z(t) = \frac{1}{\Gamma(n-a)} \frac{d^n}{dt^n} \int_0^t (t-r)^{n-a-1} z(r)\,dr,$$

where $n = [a] + 1$, provided that the right-hand side is pointwise defined on $(0,+\infty)$.

Lemma 2.1 ([12]) *Let $a \in (0,+\infty)$, then the general solution of the fractional differential equation*

$$D^a_{0+}w(t) = 0$$

is $w(t) = b_1 t^{a-1} + b_2 t^{a-2} + \cdots + b_n t^{a-n}$, where $b_j \in \mathbb{R}$, $j = 1,2\ldots,n$, and $n = [a] + 1$ is the smallest integer greater than or equal to a.

Lemma 2.2 ([12]) *Let $a \in (0,+\infty)$ and $i = 1,2,\ldots,n$. Assume that $w \in L^1[0,+\infty)$ with a fractional integration of order $n-a$ which belongs to $AC^n[0,+\infty)$, then*

$$\left(I^a_{0+}D^a_{0+}w\right)(t) = w(t) - \sum_{i=1}^{n} \frac{((I^{n-a}_{0+}u)(t))^{(n-i)}|_{t=0}}{\Gamma(a-i+1)} t^{a-i}$$

holds almost everywhere on $[0,+\infty)$.

Lemma 2.3 ([12]) *Let $a > 0$, $\rho > -1$, $t > 0$, then*

$$I^a_{0+}t^\rho = \frac{\Gamma(\rho+1)}{\Gamma(\rho+1+a)} t^{a+\rho}, \qquad D^a_{0+}t^\rho = \frac{\Gamma(\rho+1)}{\Gamma(\rho+1-a)},$$

in particular $D^a_{0+}t^{a-m} = 0$, $m = 1,2,\ldots,n$, where $n = [a] + 1$.

Lemma 2.4 ([12]) *Let $a > b > 0$. If $w(t) \in (0,+\infty)$, then*

$$D^a_{0+}I^a_{0+}w(t) = w(t), \qquad D^b_{0+}I^a_{0+}w(t) = I^{a-b}_{0+}w(t).$$

Remark 2.1 ([4]) We will use the following properties of φ_p. For $d, e \geq 0$,

(i) $\varphi_p(d+e) \leq \varphi_p(d) + \varphi_p(e)$, if $1 < p < 2$;

(ii) $\varphi_p(d+e) \leq 2^{p-2}(\varphi_p(d) + \varphi_p(e))$, if $p \geq 2$.

Let

$$U = \left\{ u : D^a_{0+}u(t) \in C\big([0,+\infty),\mathbb{R}\big), \lim_{t\to+\infty} \frac{|u(t)|}{1+t^a}, \text{ and } \lim_{t\to+\infty} \big|D^a_{0+}u(t)\big| \text{ exists} \right\}$$

with the norm $\|u\| = \max\{\|u\|_0, \|D^a_{0+}u(t)\|_\infty\}$ defined on U, where

$$\|u\|_0 = \sup_{t\in[0,+\infty)} \frac{|u(t)|}{1+t^a}, \qquad \|u\|_\infty = \sup_{t\in[0,+\infty)} \big|D^a_{0+}u\big|.$$

Let $Z = \{z : [0,+\infty) \to \mathbb{R} : \int_0^{+\infty} v(t)|z(t)|\,dt < +\infty\}$ with the norm $\|z\|_Z = \int_0^{+\infty} v(s)|z(s)|\,ds$. Then the spaces $(U, \|\cdot\|)$ and $(Z, \|\cdot\|_Z)$ by the standard argument are Banach Spaces.

We define $M : \operatorname{dom} U \to Z$ as $Mu = \frac{1}{v(t)}(\varphi_p(D^a_{0+}u))'$ and $N_k : U \to Z$ as $= -kw(t, u(t), D^a_{0+}u(t))$, $k \in [0,1]$, where

$$\operatorname{dom} L = \left\{ u \in U : \varphi_p\big(D^a_{0+}u(t)\big) \in AC[0,+\infty), \varphi_p\big(D^a_{0+}u(0)\big) = \int_0^{+\infty} v(t)\varphi_p\big(D^a_{0+}u(t)\big)\,dt, \right.$$
$$\left. \lim_{t\to+\infty} \varphi_p\big(D^a_{0+}u(t)\big) = \int_0^{+\infty} v(t)\varphi_p\big(D^a_{0+}u(t)\big)\,dt \right\}.$$

Then boundary value problem (1.1) in an abstract form is $Mu = N_k u$.

Throughout this paper, we assume that

$$D = \big(Q_1 e^{-t} \cdot Q_2 t e^{-t}\big) - \big(Q_2 e^{-t} \cdot Q_1 t e^{-t}\big) := (d_{11} \cdot d_{22}) - (d_{21} \cdot d_{12}) \neq 0,$$

where

$$Q_1 z = \int_0^{+\infty} v(t) \int_t^{+\infty} v(r)z(r)\,dr\,dt$$

and

$$Q_2 z = \int_0^{+\infty} v(t) \int_t^{+\infty} v(r)z(r)\,dr\,dt - \int_0^{+\infty} v(r)z(r)\,dr.$$

Lemma 2.5 $\operatorname{Im} M = \{z \in Z : Q_1 z = Q_2 z = 0\}$ *and* M *is a quasi-linear operator.*

Proof By simple calculation, we can see that $\ker L = \{b_1 t^{a-1} + b_2 t^a : b_1, b_2 \in \mathbb{R}, t \in [0,+\infty)\}$. Suppose $z \in \operatorname{Im} M$, then there exists $u \in \operatorname{dom} M$ such that $(\varphi_p(D^a_{0+}u(t)))' = -v(t)z(t)$. Therefore

$$\varphi_p\big(D^a_{0+}u(t)\big) = \varphi_p\big(D^a_{0+}u(+\infty)\big) + \int_t^{+\infty} v(r)z(r)\,dr. \tag{2.1}$$

As $t \to +\infty$, then

$$Q_1 z = \int_0^{\infty} v(t) \int_t^{+\infty} v(r)z(r)\,dr\,dt = 0, \tag{2.2}$$

while, when $t = 0$, we have

$$Q_2z = \int_0^{+\infty} v(t) \int_t^{+\infty} v(r)z(r)\,dr - \int_0^{+\infty} v(r)z(r)\,dr = 0. \tag{2.3}$$

From (2.1), we obtain

$$u(t) = b_1 t^{a-1} + I_{0+}^a \varphi_q \left(\varphi_p\big(D_{0+}^a u(+\infty)\big) + \int_t^{+\infty} v(r)z(r)\,dr \right). \tag{2.4}$$

Conversely, if (2.2) and (2.3) hold for $u \in \operatorname{dom} M$ and u is as defined in (2.4), then $\varphi_p(D_{0+}^a u(t)) = \varphi_p(D_{0+}^a u(+\infty)) + \int_t^{+\infty} v(r)z(r)\,dr$. Since $\int_0^{+\infty} v(t)\,dt = 1$, we have

$$\begin{aligned}
\varphi_p\big(D_{0+}^a u(+\infty)\big) &= \varphi_p\big(D_{0+}^a u(+\infty)\big) \\
&= \int_0^{+\infty} v(t)\varphi_p\big(D_{0+}^a u(+\infty)\big)\,dt + \int_0^{\infty} v(t) \int_t^{+\infty} v(r)z(r)\,dr\,dt \\
&= \int_0^{+\infty} v(t)\varphi_p\big(D_{0+}^a u(t)\big)\,dt
\end{aligned}$$

and

$$\begin{aligned}
\varphi_p\big(D_{0+}^a u(0)\big) &= \varphi_p\big(D_{0+}^a u(+\infty)\big) + \int_0^{+\infty} v(r)z(r)\,dr \\
&= \int_0^{+\infty} v(t)\varphi_p\big(D_{0+}^a u(+\infty)\big)\,dt + \int_0^{+\infty} v(r)z(r)\,dr \\
&\quad + \int_0^{+\infty} v(t) \int_t^{+\infty} v(r)z(r)\,dr - \int_0^{+\infty} v(r)z(r)\,dr \\
&= \int_0^{+\infty} v(t)\varphi_p\big(D_{0+}^a u(t)\big)\,dt.
\end{aligned}$$

Hence,

$$\operatorname{Im} M = \{z \in Z : Q_1 z = Q_2 z\}.$$

For $u \in \operatorname{dom} M$, it is clearly seen that $\dim \ker L = 2$ and $\operatorname{Im} M$, a subset of Z is closed. Hence, M is a quasi-linear operator. □

We define the projector $E : U \to U_1$ as

$$Eu(t) = \frac{D_{0+}^a u(0)}{\Gamma(a)} t^{a-1} + \frac{D_{0+}^a u(+\infty)}{\Gamma(a+1)} t^a$$

and $F : Z \to Z_1$ as

$$Fz = D_1 z(t) \cdot t^{a-1} + D_2 z(t) \cdot t^a,$$

where $D_1 z = \frac{d_{22}F_1z - d_{21}F_2z}{D} e^{-t}$ and $D_2 z = \frac{-d_{12}F_1z + d_{11}F_2z}{D} e^{-t}$. F can easily be shown to be a semi-projector.

Lemma 2.6 *Let $P : U \times [0,1] \to U_2$ be defined as*

$$P(u,k) = I_{0+}^{a}\varphi_q\left(\varphi_p\big(D_{0+}^{a}u(+\infty)\big) + \int_t^{+\infty} \nu(r)(I-F)N_k u(r)\,dr\right) - \frac{D_{0+}^{a}u(+\infty)}{\Gamma(a+1)}t^a.$$

If w is ν-Caratheodory, then P is M-compact.

Proof Let $\Omega \subset U$, then, for $u \in \overline{\Omega}$, $\|u\| < j$ with $j > 0$. Since w is a ν-Caratheodory function, then, for a.e $t \in [0,+\infty)$ and $k \in [0,1]$, there exists $\psi_j : [0,+\infty) \to [0,+\infty)$ such that $\int_0^{+\infty} \nu(t)\psi_j(t)\,dt < +\infty$, $|w(t,u(t),D_{0+}^{a}u(t))| \leq \psi_j(t)$, and

$$\int_0^{+\infty} \nu(r)\big|(I-F)N_k u(r)\big|\,dr \leq \|\psi_j\|_Z + \|FNu\|_Z.$$

Hence, for $u \in \overline{\Omega}$,

$$\begin{aligned}
\big\|P(u,k)\big\|_0 &= \sup_{t\in[0,+\infty)} \frac{|P(u,k)(t)|}{1+t^a} \\
&\leq \sup_{t\in[0,+\infty)} \left[\frac{I_{0+}^{a}\varphi_q(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z)}{1+t^a} + \frac{t^a}{(1+t^a)\Gamma(a+1)}j\right] \\
&= \sup_{t\in[0,+\infty)} \frac{1}{1+t^a}\left[\varphi_q\big(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z\big)\int_0^t \frac{(t-r)^{a-1}}{\Gamma(a)}\,dr + \frac{jt^a}{\Gamma(a+1)}\right] \\
&\leq \frac{1}{\Gamma(a+1)}\big(\varphi_q\big(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z\big) + j\big) \\
&\leq \varphi_q\big(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z\big) + j
\end{aligned}$$

and

$$\begin{aligned}
\big\|D_{0+}^{a}P(u,k)\big\|_\infty &= \sup_{t\in[0,+\infty)} \big|D_{0+}^{a}P(u,k)u(t)\big| \\
&\leq \sup_{t\in[0,+\infty)} \varphi_q\big(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z\big) + j \\
&\leq \varphi_q\big(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z\big) + j.
\end{aligned}$$

Hence $P(u,k)\overline{\Omega}$ is uniformly bounded in U. We will next show the equicontinuity of $P(u,k)$ on any compact interval of $[0,+\infty)$. Let $G > 0$, $t_1, t_2 \in [0,G]$, $u \in \overline{\Omega}$, and $k \in [0,1]$, we obtain

$$\begin{aligned}
&\left|\frac{P(u,k)(t_1)}{1+t_1^a} - \frac{P(u,k)(t_2)}{1+t_2^a}\right| \\
&\quad\leq \left[\frac{\varphi_q(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z) + j}{\Gamma(a)}\left[\int_0^{t_1}\left|\frac{(t_1-r)^{a-1}}{1+t_1^a} - \frac{(t_2-r)^{a-1}}{1+t_2^a}\right|dr + \frac{1}{a}\frac{(t_2-t_1)^a}{1+t_2^a}\right]\right. \\
&\quad\quad\left. + \frac{j}{\Gamma(a+1)}\left|\frac{t_2}{1+t_2^a} - \frac{t_1}{1+t_1^a}\right|\right] \to 0 \quad \text{as } t_1 \to t_2
\end{aligned}$$

and

$$\begin{aligned}
&\left|D^a_{0+}P(u,k)(t_1) - D^a_{0+}P(u,k)(t_2)\right| \\
&\quad = \left|\varphi_q\left(\varphi_p\left(D^a_{0+}u(+\infty)\right) + \int_{t_1}^{t_2} v(r)(I-F)N_k u(r)\,dr + \int_{t_2}^{+\infty} v(r)(I-F)N_k u(r)\,dr\right)\right. \\
&\qquad \left. - \varphi_q\left(\varphi_p\left(D^a_{0+}u(+\infty)\right) + \int_{t_2}^{+\infty} v(r)(I-F)N_k u(r)\,dr\right)\right| \to 0 \quad \text{as } t_1 \to t_2.
\end{aligned}$$

Thus, $P(u,k)\overline{\Omega}$ is equicontinuous on $[0,G]$. We now establish that $P(u,k)\overline{\Omega}$ is equiconvergent at $+\infty$. Let $u \in \overline{\Omega}$, since

$$\left|\varphi_p\left(D^a_{0+}u(+\infty)\right) + \int_t^{+\infty} v(r)(I-F)N_k u(r)\,dr\right| \leq \varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z,$$

then $\varphi_q(u)$ is uniformly continuous on $[-y,y]$ where $y = \varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z$. Thus, given any $\epsilon > 0$, and for all $u \in \overline{\Omega}$, there exists $l > 0$ such that if $r \geq l$, then

$$\left|\varphi_q\left(\varphi_p\left(D^a_{0+}u(+\infty)\right) + \int_r^{+\infty} v(x)(F-I)N_k u(x)\,dx\right)\right| < \epsilon.$$

Conversely, since $\lim_{t\to+\infty} \frac{t^{a-1}}{1+t^a} = 0$, $\lim_{t\to+\infty} \frac{t^a}{1+t^a} = 0$, and $\lim_{t\to+\infty} \frac{1}{1+t^a} = 0$, then for same $\epsilon > 0$ above, there exists $l_1 > l > 0$ such that, for any $t_1, t_2 \geq x_1$ and $r \in [0,l]$, we have

$$\begin{aligned}
&\left|\frac{(t_1-r)^{a-1}}{1+t_1^a} - \frac{(t_2-r)^{a-1}}{1+t_2^a}\right| \leq \frac{t_1^{a-1}}{1+t_1^a} + \frac{t_2^{a-1}}{1+t_2^a} < \epsilon, \\
&\left|\frac{t_1^a}{1+t_1^a} - \frac{t_2^a}{1+t_2^a}\right| \leq \frac{t_1^a}{1+t_1^a} + \frac{t_2^a}{1+t_2^a} < \epsilon
\end{aligned}$$

and

$$\left|\frac{1}{1+t_1^a} - \frac{1}{1+t_2^a}\right| \leq \frac{1}{1+t_1^a} + \frac{1}{1+t_2^a} < \epsilon.$$

Hence, for $t_1, t_2 \geq l$, we have

$$\begin{aligned}
&\left|\frac{P(u,k)(t_1)}{1+t_1^a} - \frac{P(u,k)(t_2)}{1+t_2^a}\right| \\
&\quad = \left|\frac{I^a_{0+}\varphi_q(\varphi_p(D^a_{0+}u(+\infty)) + \int_{t_1}^{+\infty} v(r)(I-F)N_k u(r)\,dr)}{1+t_1^a} - \frac{D^a_{0+}u(+\infty)t^a_{t_1}}{(1+t_1^a)\Gamma(a+1)}\right. \\
&\qquad \left. - \frac{I^a_{0+}\varphi_q(\varphi_p(D^a_{0+}u(+\infty)) + \int_{t_2}^{+\infty} v(r)(I-F)N_k u(r)\,dr)}{1+t_2^a} + \frac{D^a_{0+}u(+\infty)t^a_{t_2}}{(1+t_2^a)\Gamma(a+1)}\right| \\
&\quad \leq \frac{\varphi_q(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z)}{\Gamma(a)}\left[\int_0^{l_1}\left|\frac{(t_1-r)^{a-1}}{1+t_1^a} - \frac{(t_2-r)^{a-1}}{1+t_2^a}\right|dr\right. \\
&\qquad \left. + \int_{l_1}^{t_1} \frac{(t_1-r)^{a-1}}{1+t_1^a}\,dr + \int_{l_1}^{t_2} \frac{(t_2-r)^{a-1}}{1+t_s^a}\,dr\right] + \frac{j}{\Gamma(a+1)}\left|\frac{t_1^a}{1+t_1^a} - \frac{t_2^a}{1+t_2^a}\right| \\
&\quad \leq \frac{\varphi_q(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z)}{\Gamma(a)} l_1\epsilon + \frac{1}{\Gamma(a+1)}\left[2\varphi_q\left(\varphi_p(j) + \|\psi_j\|_Z + \|FNu\|_Z\right) + j\right]\epsilon
\end{aligned}$$

and

$$
\begin{aligned}
&|D^{a}_{0+}P(u,k)(t_1) - |D^{a}_{0+}P(u,k)(t_2)| \\
&= \left|\varphi_q\left(\varphi_p\left(D^{a}_{0+}u(+\infty)\right) + \int_{t_1}^{+\infty} v(r)(I-F)N_k u(r)\,dr\right)\right. \\
&\quad \left. - \varphi_q\left(\varphi_p\left(D^{a}_{0+}u(+\infty)\right) + \int_{t_2}^{+\infty} v(r)(I-F)N_k u(r)\,dr\right)\right| < 2\epsilon. \qquad \square
\end{aligned}
$$

Thus, $P(u,k)\overline{\Omega}$ is equiconvergent at $+\infty$. Therefore, by Lemma 2.4, $P:\overline{\Omega}\times[0,1]\to U_2$ is relatively compact.

Let $u \in \bigvee_k = \{u \in \overline{\Omega} : Mu = N_k u\}$, then $\frac{1}{v(t)}(\varphi_p(D^{a}_{0+}u(t)))' = -N_k u(t) \in \operatorname{Im} M = \ker F$. We will now prove that (ρ_3) and (ρ_4) of Definition 2.2 hold. Clearly, (ρ_3) holds since $P(u,0) = 0$ and

$$
\begin{aligned}
P(u,k)(t) &= I^{a}_{0+}\varphi_q\left(\varphi_p\left(D^{a}_{0+}u(+\infty)\right) + \int_t^{+\infty} v(r)N_k u(r)\,dr\right) - \frac{D^{a}_{0+}u(+\infty)}{\Gamma(a+1)}t^{a} \\
&= I^{a}_{0+}\varphi_q\left(\varphi_p\left(D^{a}_{0+}u(+\infty)\right) + \int_t^{+\infty} -\left(\varphi_p\left(D^{a}_{0+}u(r)\right)\right)'\,dr\right) - \frac{D^{a}_{0+}u(+\infty)}{\Gamma(a+1)}t^{a} \\
&= I^{a}_{0+}D^{a}_{0+}u(t) - \frac{D^{a}_{0+}u(+\infty)}{\Gamma(a+1)}t^{a}.
\end{aligned}
$$

Hence, from Lemma 2.2 we have

$$
P(u,k)(t) = u(t) - \left(\frac{D^{a}_{0+}u(0)}{\Gamma(a)}t^{a-1} + \frac{D^{a}_{0+}u(+\infty)}{\Gamma(a+1)}t^{a}\right) = \left[(I-E)u\right](t).
$$

Also, for $u \in \overline{\Omega}$,

$$
\begin{aligned}
&M\left[Eu + P(u,k)\right](t) \\
&= M\left[\frac{D^{a}_{0+}u(0)}{\Gamma(a)}t^{a-1} + I^{a}_{0+}\varphi_q\left(\varphi_p\left(D^{a}_{0+}u(+\infty)\right) + \int_t^{+\infty} v(r)(I-F)N_k u(r)\,dr\right)\right] \\
&= \frac{1}{v(t)}\left(\varphi_p\left(D^{a}_{0+}u(+\infty)\right) + \int_t^{+\infty} v(t)(I-F)N_k u(r)\,dr\right)' \\
&= (I-F)N_k u(r)\,dr.
\end{aligned}
$$

Hence, N_k is M-compact on $\overline{\Omega}$.

3 Main results

Theorem 3.1 *Suppose that $D \neq 0$, w is v-Caratheodory, and the following hold:*

(i) *There exist functions $m_1(t) > 0$, $m_2(t) > 0$, $m_3(t) > 0$ in Z such that*

$$
\left|w(t,u,v)\right| \le m_1(t) + m_2(t)\frac{|u|^{p-1}}{(1+t^{a})^{p-1}} + m_3(t)|v|^{p-1}, \quad \forall t \in [0,+\infty), (u,v) \in \mathbb{R}^2;
$$

(ii) *There exists constant $B_1 > 0$ such that $|D^{a}_{0+}u(t_0)| > B_1$ for every $t_0 \in [0,+\infty)$, then either $F_1 Nu \neq 0$ or $F_2 Nu \neq 0$;*

(iii) *There exists $C_1 > 0$ such that, if $|b_1| > C_1$ or $|b_2| > C_1$, then either*

$$b_1 F_1 N\big(b_1 t^{a-1} + b_2 t^a\big) + b_2 F_2 N\big(b_1 t^{a-1} + b_2 t^a\big) < 0 \tag{3.1}$$

or

$$b_1 F_1 N\big(b_1 t^{a-1} + b_2 t^a\big) + b_2 F_2 N\big(b_1 t^{a-1} + b_2 t^a\big) > 0, \tag{3.2}$$

where $b_1, b_2 \in \mathbb{R}$, $|b_1| + |b_2| > C_1$ and $t \in [0, +\infty)$. Then boundary value problem (1.1) has at least one solution in U if

$$2^{2q-4}\big(\|m_2\|_Z^{q-1} + \Gamma(a+1)\|m_3\|_Z^{q-1}\big) < 1, \quad \text{if } 1 < p < 2, \quad \text{or}$$
$$\|m_2\|_Z^{q-1} + \Gamma(a+1)\|m_3\|_Z^{q-1} < 1, \quad \text{if } p \geq 2.$$

Before proving Theorem 3.1, we will state and prove two lemmas.

Lemma 3.1 *Let $\Omega_1 = \{u \in \operatorname{dom} M \ker M : Mu = N_k u, k \in [0,1]\}$. Then Ω_1 is bounded in U if* (i) *and* (ii) *of Theorem 3.1 hold.*

Proof Let $u \in \Omega_1$, then $Mu = N_k u$ and $FN_k u = 0$. Hence, from Lemma 2.5, we have

$$\big|u(t)\big| = \left|b_1 t^{a-1} + \frac{1}{\Gamma(a)}\int_0^t (t-r)^{a-1} D_{0+}^a u(r)\,dr\right|, \quad b_1 \in \mathbb{R}.$$

Therefore,

$$\begin{aligned}\|u\|_0 &\leq \sup_{t\in[0,+\infty)} \frac{t^{a-1}}{1+t^a}|b_1| + \sup_{t\in[0,+\infty)} \frac{t^a}{1+t^a}\frac{1}{\Gamma(a+1)}\big\|D_{0+}^a u\big\|_\infty \\ &\leq |b_1| + \frac{1}{\Gamma(a+1)}\big\|D_{0+}^a u\big\|_\infty.\end{aligned}$$

Thus, from (ii) of Theorem 3.1, there exist constants $t_0 \in [0, \infty)$ such that $|D_{0+}^a u(t_1)| < B_1$. Also, by (i) of Theorem 3.1 and from $\frac{1}{v(t)}(\varphi_p(D_{0+}^a u(t)))' = -kw(t, u(t), D_{0+}^a u(t))$, we have

$$\begin{aligned}\big|\varphi_p\big(D_{0+}^a u(t)\big)\big| &= \left|\varphi_p\big(D_{0+}^a u(t_0)\big) - \int_{t_0}^t kv(r)w\big(r, u(r), D_{0+}^a u(r)\big)\,dr\right| \\ &\leq \big|\varphi_p\big(D_{0+}^a u(t_0)\big)\big| + \left|\int_{t_0}^t kv(r)w\big(r, u(r), D_{0+}^a u(r)\big)\,dr\right| \\ &\leq \varphi_p(B_1) + \int_0^{+\infty} v(r)\left[m_1(r) + m_2(r)\frac{|u|^{p-1}}{(1+t^a)^{p-1}} + m_3(r)\big|D_{0+}^a u\big|^{p-1}\right]dr \\ &\leq \varphi_p(B_1) + \|m_1\|_Z + \|m_2\|_Z\varphi_p\big(\|u\|_0\big) + \|m_3\|_Z\varphi_p\big(\big\|D_{0+}^a\big\|_\infty\big) \\ &\leq \varphi_p(B_1) + \|m_1\|_Z + \|m_2\|_Z\varphi_p\left(|b_1| + \frac{1}{\Gamma(a+1)}\big\|D_{0+}^a u\big\|_\infty\right) \\ &\quad + \|m_3\|_Z\varphi_p\big(\big\|D_{0+}^a\big\|_\infty\big).\end{aligned}$$

Then

$$\varphi_p\left(\left\|D^a_{0+}u\right\|_\infty\right) \le \varphi_p(B_1) + \|m_1\|_Z + \|m_2\|_Z\varphi_p\left(|b_1| + \frac{1}{\Gamma(a+1)}\left\|D^a_{0+}u\right\|_\infty\right) + \|m_3\|_Z\varphi_p\left(\left\|D^a_{0+}\right\|_\infty\right).$$

If $1 < p < 2$, then

$$\left\|D^a_{0+}u\right\|_\infty \le 2^{2q-4}\left(B_1 + \|m_1\|_Z^{q-1} + \|m_2\|_Z^{q-1}\left(|b_1| + \frac{1}{\Gamma(a+1)}\left\|D^a_{0+}u\right\|_\infty\right) + \|m_3\|_Z^{q-1}\left\|D^a_{0+}\right\|_\infty\right),$$

and hence,

$$\left\|D^a_{0+}u\right\|_\infty \le \frac{2^{2q-4}\Gamma(a+1)(B_1 + \|m_1\|_Z^{q-1} + \|m_2\|_Z^{q-1}|b_1|)}{1 - 2^{2q-4}(\|m_2\|_Z^{q-1} - \Gamma(a+1)\|m_3\|_Z^{q-1})}.$$

□

Similarly, if $p \ge 2$, then

$$\left\|D^a_{0+}u\right\|_\infty \le B_1 + \|m_1\|_Z^{q-1} + \|m_2\|_Z^{q-1}\left(|b_1| + \frac{1}{\Gamma(a+1)}\left\|D^a_{0+}u\right\|_\infty\right) + \|m_3\|_Z^{q-1}\left\|D^a_{0+}\right\|_\infty.$$

Thus

$$\left\|D^a_{0+}u\right\|_\infty \le \frac{\Gamma(a+1)(B_1 + \|m_1\|_Z^{q-1} + \|m_2\|_Z^{q-1}|b_1|)}{1 - \|m_2\|_Z^{q-1} - \Gamma(a+1)\|m_3\|_Z^{q-1}}.$$

Hence,

$$\|u\| = \max\left\{\|u\|_0, \left\|D^a_{0+}u\right\|_\infty\right\} \le |b_1| + \left\|D^a_{0+}u\right\|_\infty.$$

Therefore Ω_1 is bounded in U_1.

Lemma 3.2 *If* (iii) *of Theorem* 3.1 *holds, then* $\Omega_2 = \{u \in \ker M : Nu \in \operatorname{Im} M\}$ *is bounded.*

Proof Let $u \in \Omega_2$, then $u = b_1t^{a-1} + b_2t^a$, where $b_1, b_2 \in \mathbb{R}$. Since $Nu \in \operatorname{Im} M$, then $F_1Nu = F_2Nu = 0$. From (iii) of Theorem 3.1, we have $|b_1| < C_1$ and $|b_2| < C_1$, then

$$\begin{aligned}
\|u\| &= \max\left\{\|u\|_0, \left\|D^a_{0+}u\right\|_\infty\right\} \\
&= \max\left\{\sup_{t\in[0,+\infty)} \frac{t^{a-1}}{1+t^a}|b_1| + \sup_{t\in[0,+\infty)} \frac{t^a}{1+t^a}|b_2|, \sup_{t\in[0,+\infty)} \left|D^a_{0+}\left(b_1t^{a-1} + b_2t^a\right)\right|\right\} \\
&= \max\left\{|b_1| + |b_2|, \frac{\Gamma(a+1)}{\Gamma(1)}|b_2|\right\} \\
&\le |b_1| + |b_2| + \Gamma(a+1)|b_2|.
\end{aligned}$$

Therefore, Ω_2 is bounded. □

Proof of Theorem 3.1 We have previously proved that M is quasi-linear and N_k is M-compact on $\overline{\Omega}$. Also, from Lemma 3.1 and Lemma 3.2 we proved that (τ_1) and (τ_1) of Theorem 2.1 hold. Finally, we will prove that (τ_3) of Theorem 2.1 also holds. Let $J : \operatorname{Im} F \to \ker M$ be defined as

$$J\big(b_1 t^{a-1} + b_2 t^a\big) = \frac{1}{D}\big(\big(d_{22}|b_1| - d_{21}|b_2|\big)t^{a-1} + \big(-d_{12}|b_1| + d_{11}|b_2|\big)t^a\big)e^{-t}.$$

If (3.1) holds for any $u \in \operatorname{dom} \partial\Omega \cap \ker M$, where $u = b_1 t^{a-1} + b_2 t^a \neq 0$. We define the homeomorphism by

$$H(u,k) = -ku + (1-k)JFNU, \quad k \in [0,1]. \qquad \square$$

Then $H(u,1) = -u \neq 0$ and $H(u,0) = JFNu \neq 0$ since $Nu \notin \operatorname{Im} M$. For $k \in (0,1)$ and by way of contradiction, we assume $H(u,k) = 0$, then

$$\begin{aligned}
&d_{22}\big(-k|b_1| + (1-k)F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)\big) \\
&\quad - d_{21}\big(-k|b_2| + (1-k)F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)\big) = 0, \\
&-d_{12}\big(-k|b_1| + (1-k)F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)\big) \\
&\quad + d_{11}\big(-k|b_2| + (1-k)F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)\big) = 0.
\end{aligned}$$

Since $D \neq 0$, we have

$$\begin{aligned}
&k|b_1| = (1-k)F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)), \\
&k|b_2| = (1-k)F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)).
\end{aligned}$$

Hence,

$$|b_1| + |b_2| = \frac{1-k}{k}\big(F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)\big) + F_2 N\big(b_1 t^{a-1} + b_2 t^a\big))) < 0,$$

which contradicts $|b_1| + |b_2| \geq 0$. If (3.2) holds, then we define

$$H(u,k) = -ku - (1-k)JFNU, \quad k \in [0,1].$$

Then

$$|b_1| + |b_2| = -\frac{1-k}{k}\big(F_1 N\big(b_1 t^{a-1} + b_2 t^a\big)\big) + F_2 N\big(b_1 t^{a-1} + b_2 t^a\big))) < 0,$$

which is also a contradiction. Hence, by the homotopy property of Brouwer degree, we have

$$\begin{aligned}
\deg(JFN, \Omega \cap M, 0) &= \deg\big(H(\cdot,1), \Omega \cap \ker M, 0\big) \\
&= \deg\big(H(\cdot,0), \Omega \cap \ker M, 0\big) \\
&= \deg(I, \Omega \cap \ker M, 0) \neq 0.
\end{aligned}$$

Therefore, at least one solution of (1.1) exists in $\overline{\Omega}$.

4 Example

Example 4.1 Consider the following boundary value problem:

$$\begin{cases} (\varphi_{1/2}(D_{0+}^{1/2}u(t)))' + 4e^{-4t}(\frac{7}{2} + \frac{5\sin^{-1}\sqrt{|u(t)|}}{3\sqrt{1+t^{1/2}}} + 3\varphi_{1/2}(D_{0+}^{1/2}u(t))) = 0, \\ \quad t \in [0, +\infty) \\ \varphi_{1/2}(D_{0+}^{1/2}u(0)) = 4\int_0^{+\infty} e^{-4t}\varphi_{1/2}(D_{0+}^{1/2}u(t))\,dt, \\ \varphi_{1/2}(D_{0+}^{1/2}u(+\infty)) = 4\int_0^{+\infty} e^{-4t}\varphi_{1/2}(D_{0+}^{1/2}u(t))\,dt, \end{cases} \tag{4.1}$$

where $p = q = \frac{1}{2}$, $\int_0^{+\infty} 4e^{-t}\,dt = 1$, $D = (\frac{16}{45})(-\frac{68}{625}) - (-\frac{4}{9})(\frac{32}{625}) = -0.016 \neq 0$. $m_2 = \frac{5}{3}$, $m_3 = 3$, then $2^{2q-4}(\|m_2\|_Z^{q-1} + \Gamma(a+1)\|m_3\|_Z^{q-1}) = \frac{1}{8}(\sqrt{\frac{3}{5}} + \frac{\sqrt{\pi}}{2}\frac{1}{\sqrt{3}}) = 0.161 < 1$.

Conditions (i), (ii), and (iii) of Theorem 3.1 can also be shown to hold. Hence, there exists at least one solution of (4.1).

Example 4.2 Consider the following boundary value problem:

$$\begin{cases} (\varphi_3(D_{0+}^{\frac{1}{3}}u(t)))' + e^{-t}(\frac{1}{1+2t^2} + \frac{\sin t|u(t)|^2}{7(1+t^{1/3})^2} + \frac{1}{15}\varphi_3(D_{0+}^{\frac{1}{3}}u(t))) = 0, \quad t \in [0, +\infty), \\ \varphi_3(D_{0+}^{\frac{1}{3}}u(0)) = \int_0^{+\infty} e^{-t}\varphi_3(D_{0+}^{\frac{1}{3}}u(t))\,dt, \\ \varphi_3(D_{0+}^{\frac{1}{3}}u(+\infty)) = \int_0^{+\infty} e^{-t}\varphi_3(D_{0+}^{\frac{1}{3}}u(t))\,dt, \end{cases} \tag{4.2}$$

where $p = 3$, $q = \frac{3}{2}$, $\int_0^{+\infty} e^{-t}\,dt = 1$, $D = (\frac{1}{6})(-\frac{1}{6}) - (-\frac{1}{3})(\frac{1}{12}) = -0.0694 \neq 0$. $m_2 = \frac{\sin t}{7}$, $m_3 = \frac{1}{15}$. Then $\|m_2\|_Z^{q-1} + \Gamma(a+1)\|m_3\|_Z^{q-1}) = \sqrt{\frac{1}{7}} + (0.8923)\sqrt{\frac{1}{15}} = 0.0.6085 < 1$.

Conditions (i), (ii), and (iii) of Theorem 3.1 also hold. Hence, there exists at least one solution of (4.2).

5 Conclusion

Fractional differential equations are an efficient tool for describing the memory of different substances and have become popular recently. In order to further enrich this subject area, this work considers existence results for fractional-order p-Laplacian boundary value problem on the half-line at resonance where the differential operator is nonlinear and has a kernel dimension equal to two. The proof of the main result is based on the Ge and Ren coincidence degree theory, and the results obtained are new and extend some current results to the two-dimensional kernel. Examples were given to demonstrate the practicability and validity of our main results.

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Authors' contributions

OF conceived the idea. SA supervised the work. All authors discussed and contributed to the final manuscript. All authors read and approved the final manuscript.

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