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Analytical solitons for the space-time conformable differential equations using two efficient techniques

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Abstract

Exact solutions to nonlinear differential equations play an undeniable role in various branches of science. These solutions are often used as reliable tools in describing the various quantitative and qualitative features of nonlinear phenomena observed in many fields of mathematical physics and nonlinear sciences. In this paper, the generalized exponential rational function method and the extended sinh-Gordon equation expansion method are applied to obtain approximate analytical solutions to the space-time conformable coupled Cahn–Allen equation, the space-time conformable coupled Burgers equation, and the space-time conformable Fokas equation. Novel approximate exact solutions are obtained. The conformable derivative is considered to obtain the approximate analytical solutions under constraint conditions. Numerical simulations obtained by the proposed methods indicate that the approaches are very effective. Both techniques employed in this paper have the potential to be used in solving other models in mathematics and physics.

Keywords: Generalized exponential rational function method; Extended sinh-Gordon equation expansion method; Conformable derivative; Fokas equation; Burgers equation; Cahn–Allen equation

1 Introduction

Despite the recent extensive advances in the theory of differential equations, it can generally be said that it is still a complex task to determine an analytical solution for many ordinary and partial differential equations [1–9]. One of the events that led to the introduction of a wide range of new methods was the emergence and use of computers. So today it is almost impossible to use most of the existing techniques in solving differential equations, numerically or analytically, without the use of suitable computer software [10–19].

In recent years, the search for accurate solutions to differential equations has become a popular research topic. The natural result of this volume of attention has been the provision of efficient and powerful techniques. For example, the auxiliary equation method [20], the simplest equation method [21], the Hirota bilinear method [22], the homotopy analysis method [23], the Jacobi elliptic method [24, 25], the complex transform [26], the bilinear form approach [27], the G'/G expansion method [28], the $\exp(-\phi)$-expansion method [29], the generalized logistic equation method [30], the modified Kudryashov method, the extended tanh-coth method, the modified simple equation method and soliton ansatz method [31], the Hirota bilinear method [32], the modified form of an auxiliary equation approach [33]. Some more examples of differential equations and their applications can be followed in [34–52].

Khalil in [53] proposed an interesting definition of a derivative, namely the conformable derivative that generalizes the classical concept of derivative. This definition is well-behaved and obeys the Leibniz rule and the chain rule. Nonlinear conformable differential and integral equations have been the focus of many studies due to their applications in various applications in physics, biology, engineering, signal processing, control theory, finance, etc. [54–58]. More precisely, the extended Zakharov–Kuzetsov equation with conformable derivative using the generalized exponential rational function method was solved in [59]. In [60], a generalized type of conformable local fractal derivative (GCFD) was employed to investigate some nonlinear evolution equations. They also set up a general technique to find exact solutions for their under studied PDEs. In [61] the first integral method was employed to construct the solutions to the conformable Burgers equation, modified Burgers equation, and Burgers–Korteweg–de Vries equation. In [62], several wave solutions for Burgers' type equations in the sense of conformable fractional derivative have been obtained via the residual power series method. Moreover, in [63] the auxiliary equation method has been employed to solve (2 + 1)-dimensional time-fractional Zoomeron equation and the time-fractional third order modified KdV equation. Abundant solitary wave solutions to an extended nonlinear Schrödinger's equation with conformable derivative using an efficient integration method called the generalized exponential rational function method have been reported in [64]. Very recently, the conformable derivative and adequate fractional complex transform have been implemented to discuss the conformable higher-dimensional Ito equation [65].

In this paper, we apply both the generalized exponential rational function method and the extended sinh-Gordon equation expansion method for solving space-time conformable partial differential equations. Approximate analytical solutions for the coupled Cahn–Allen equation, coupled Burgers equation, and Fokas equation are obtained. Several exact solutions for them are successfully established. The solutions obtained by the methods indicate that they are easy to implement and effective. This article has been arranged as follows. In Sect. 2, we propose some mathematical definitions and prerequisites required later in the article. The section also illustrates general principles of the conformable derivative along with basic steps of techniques. In Sect. 3, three equations including the space-time conformable coupled Cahn–Allen equation, the space-time coupled Burgers equation, and the space-time conformable Fokas equation are examined, and the exact solution for them is determined using two techniques. This section also contains several numerical simulations of acquired solutions. Finally, the article ends with some conclusions.

2 Preliminaries and definitions

In this section, we review some of the necessary prerequisites that will be employed in the article.

2.1 The conformable derivative

Khalil proposed an interesting definition of derivative called conformable derivative [53]. This derivative can be considered to be a natural extension of the classical derivative. Furthermore, the conformable derivative satisfies all the properties of the standard calculus, for instance, the chain rule.

Definition 1 Let $f : [0,\infty) \to \mathbb{R}$, the conformable derivative of a function $f(t)$ of order α is defined as [53]

$$D_t^{\alpha} f(t) = \lim_{\epsilon\to 0} \frac{f(t+\epsilon t^{1-\alpha}) - f(t)}{\epsilon}, \quad \alpha \in (0,1], t > 0. \tag{1}$$

It should be noted that taking $\alpha = 1$ in this derivative yields the standard definition for derivative. Therefore, this method can be considered a natural generalization for the conventional derivative.

This new definition satisfies the following properties. Let $\alpha \in (0,1]$, f, g be α-differentiable at a point t, then

- $D_t^{\alpha}(af(t) + bg(t)) = aD_t^{\alpha}(f(t)) + bD_t^{\alpha}(g(t))$ for $a, b \in \mathbb{R}$.
- $D_t^{\alpha}(t^{\mu}) = \mu t^{\mu-\alpha}$ for $\mu \in \mathbb{R}$.
- $D_t^{\alpha}(fg) = f(t)D_t^{\alpha}(g(t)) + g(t)D_t^{\alpha}(f(t))$.
- $D_t^{\alpha}(\frac{f(t)}{g(t)}) = \frac{(t)D_t^{\alpha}(f(t)) - f(t)D_t^{\alpha}(g(t))}{g^2(t)}$.
- If $f(t)$ is a differentiable function (in standard sense), then we obtain $D_t^{\alpha}(f(t))(t) = t^{1-\sigma}\frac{df(t)}{dt}$ holds.

As stated in [64], many of the existing definitions for derivative do not meet some of these mentioned properties. Enjoying these features is one of the valuable and distinctive points for the conformable derivative.

2.2 The generalized exponential rational function method

In 2018, an integration method called the generalized exponential rational function method (GERFM) was introduced by Ghanbari et al. to solve the resonance nonlinear Schrödinger equation [66]. Following their work, the technique has been used successfully many times to handle other partial equations [67–82]. In this part, we outline the main steps of GERFM as follows.

1. Let us take the following problem with the conformable derivative:

$$\mathcal{L}\big(\psi, D_x^{\alpha}\{\psi\}, D_t^{\alpha}\{\phi\}, D_x^{2\alpha}\{\psi\}, \dots\big) = 0. \tag{2}$$

2. Using the transformations $\psi = \psi(\xi)$ and $\xi = \sigma\frac{x^{\alpha}}{\Gamma(\alpha)} - l\frac{t^{\alpha}}{\Gamma(\alpha)}$, we reduce the nonlinear partial differential equation to the following ordinary differential equation:

$$\mathcal{L}\big(\psi, \psi', \psi'', \dots\big) = 0, \tag{3}$$

where the values of σ and l will be found later.

3. Now, consider that Eq. (3) has the solution of the form

$$\psi(\xi) = A_0 + \sum_{k=1}^{M} A_k \Psi(\xi)^k + \sum_{k=1}^{M} B_k \Psi(\xi)^{-k}, \tag{4}$$

where

$$\Psi(\xi) = \frac{p_1 e^{q_1 \xi} + p_2 e^{q_2 \xi}}{p_3 e^{q_3 \xi} + p_4 e^{q_4 \xi}}. \tag{5}$$

The values of constants p_i, q_i $(1 \le i \le 4)$, A_0, A_k, and B_k $(1 \le k \le M)$ are determined in such a way that solution (4) always persuades Eq. (3). By considering the homogenous balance principle, the value of M is determined.

4. Putting Eq. (4) into Eq. (3) and collecting all terms, the left-hand side of Eq. (3) gives us an algebraic equation $P(Z_1, Z_2, Z_3, Z_4) = 0$ in terms of $Z_i = e^{q_i \xi}$ for $i = 1, \dots, 4$. Setting each coefficient of P to zero, a system of nonlinear equations in terms of p_i, q_i $(1 \le i \le 4)$ and σ, l, A_0, A_k and B_k $(1 \le k \le M)$ is constructed.
5. By solving the above system of equations using any symbolic computation software, the values of p_i, q_i $(1 \le i \le 4)$, A_0, A_k, and B_k $(1 \le k \le M)$ are determined, replacing these values in Eq. (4), we obtain the solutions of Eq. (2).

2.3 The extended sinh-Gordon equation expansion method

The extended sinh-Gordon equation expansion method (EShGEEM) is a robust method that may easily derive dark, bright, combined dark-bright, singular, combined singular soliton, and other trigonometric function solutions to nonlinear PDEs of an integer or noninteger order [83]. This technique has had many successful applications in solving various problems. For example, the authors of [84] used EShGEEM to study the conformable version of Biswas–Milovic equation with the Kerr law and parabolic law nonlinearity. Another application of EShGEEM can be found in [85], where they considered a nonlinear partial differential equation describing the wave propagation in nonlinear low-pass electrical transmission lines.

Following the works of [84, 85], we outline the main steps of EShGEEM as follows.

1. Let us take the following problem with the conformable derivative:

$$\mathcal{L}\big(\psi, D_x^{\alpha}\{\psi\}, D_t^{\alpha}\{\phi\}, D_x^{2\alpha}\{\psi\}, \dots\big) = 0. \tag{6}$$

Using the transformations $\Psi = \Psi(\xi)$ and $\xi = \sigma \frac{x^{\alpha}}{\Gamma(\alpha)} - l \frac{t^{\alpha}}{\Gamma(\alpha)}$, it is possible reduce the NPDE to the following ordinary differential equation:

$$\mathcal{L}\big(\Psi, \Psi', \Psi'', \dots\big) = 0, \tag{7}$$

where the values of σ and l will be found later, and the prime notation means the derivative of Ψ with respect to ξ.

2. Consider Eq. (7) has the solution of the form

$$\Psi(\theta) = A_0 + \sum_{j=1}^{M} \cosh^{j-1}(\theta)\big[B_j \sinh(\theta) + A_j \cosh(\theta)\big], \tag{8}$$

where A_0, A_j, B_j $(j = 1, 2, \dots, M)$ are constants to be determined later and θ is a function of ξ that satisfies the following ordinary differential equation:

$$\theta' = \sinh(\theta). \tag{9}$$

By considering the homogenous balance principle in (7), the value of M can be determined.

Equation (9) possesses the following solutions:

$$\sinh(\theta) = \pm \operatorname{csch}(\xi), \quad \text{or} \quad \sinh(\theta) = \pm i \operatorname{sech}(\xi) \tag{10}$$

and

$$\cosh(\theta) = -\coth(\xi), \quad \text{or} \quad \cosh(\theta) = -\tanh(\xi), \tag{11}$$

where $i = \sqrt{-1}$.

3. Substituting Eq. (8) along with Eqs. (10) and (11) into Eq. (7) and collecting all terms, we obtain a polynomial in terms of $\theta'^{l} \sinh^{i}(\theta) \cosh^{j}(\theta)$ for $l = 0, 1, i, j = 0, 1, 2, \ldots$. Setting each coefficient of such a polynomial equal to zero, a system of nonlinear equations in terms of σ, l, A_0, A_j, B_j $(1 \le k \le M)$ is generated.
4. Solving the above algebraic equations using any symbolic computation software, the values of σ, l and A_0, A_j, B_j $(1 \le j \le M)$ are determined.
5. Based on Eqs. (10) and (11), one can obtain the soliton solutions of Eq. (6) as follows:

$$\Psi(\xi) = A_0 + \sum_{j=1}^{M} \bigl(-\tanh(\xi)\bigr)^{j-1} \bigl[\pm i B_j \operatorname{sech}(\xi) - A_j \tanh(\xi)\bigr], \tag{12}$$

$$\Psi(\xi) = A_0 + \sum_{j=1}^{M} \bigl(-\coth(\xi)\bigr)^{j-1} \bigl[\pm B_j \operatorname{csch}(\xi) - A_j \coth(\xi)\bigr]. \tag{13}$$

3 Applications of techniques and the main results

In this section, to illustrate the applicability of the generalized exponential rational function method and the extended sinh-Gordon equation expansion method to solve nonlinear conformable partial differential equations, three examples are considered.

3.1 The space-time conformable coupled Cahn–Allen equation

Consider the space-time conformable Cahn–Allen equation [86]

$$D_t^{\alpha} u - u_{xx} + u^3 - u = 0. \tag{14}$$

Using the transformation

$$u(x,t) = \mathcal{U}(\xi), \quad \xi = c\left(x - \frac{\nu t^{\alpha}}{\Gamma(\alpha)}\right), \tag{15}$$

where c and ν are two nonzero constants.

Utilizing the wave transformation (15) converts Eq. (14) into the following NODE:

$$-c\nu \mathcal{U}' - c^2 \mathcal{U}'' - \mathcal{U} + \mathcal{U}^3 = 0. \tag{16}$$

Using the balance principle on the terms $\mathcal{U}^3$ and $\mathcal{U}''$ in Eq. (16), we have $M + 2 = 3M$, so $M = 1$.

Application of GERFM for (14)

Using Eq. (5) together with $M = 1$, we have

$$\mathcal{U}(\xi) = A_0 + A_1\Psi(\xi) + \frac{B_1}{\Psi(\xi)}. \tag{17}$$

Proceeding as outlined in the second section, we acquire the following sets of solutions to Eq. (14).

Set 1: One obtains $r = [-1, 0, 1, 1]$ and $s = [1, 0, 1, 0]$, so Eq. (5) turns into

$$\Psi(\xi) = -\frac{1}{1 + e^{\xi}}. \tag{18}$$

Case 1: We obtain

$$c = \frac{\sqrt{2}}{2}, \qquad \nu = \frac{3\sqrt{2}}{2}, \qquad A_0 = 0, \qquad A_1 = -1, \qquad B_1 = 0.$$

Putting values in Eqs. (17) and (18) yields the following solution:

$$\mathcal{U}(\xi) = \frac{1}{1 + e^{\xi}}.$$

Consequently, we get the solution of Eq. (14) as

$$u_1(x,t) = \frac{1}{1 + \mathrm{e}^{\frac{\sqrt{2}}{2}(x - \frac{3\sqrt{2}t^{\alpha}}{3\Gamma(\alpha)})}}. \tag{19}$$

Figure 1 depicts the dynamic behavior of solution $u_1(x,t)$ presented in (19).
Case 2: We obtain

$$c = \frac{\sqrt{2}}{2}, \qquad \nu = -\frac{3\sqrt{2}}{2}, \qquad A_0 = 1, \qquad A_1 = 1, \qquad B_1 = 0.$$

Putting values in Eqs. (17) and (18) yields the following solution:

$$\mathcal{U}(\xi) = \frac{e^{\xi}}{1 + e^{\xi}}.$$

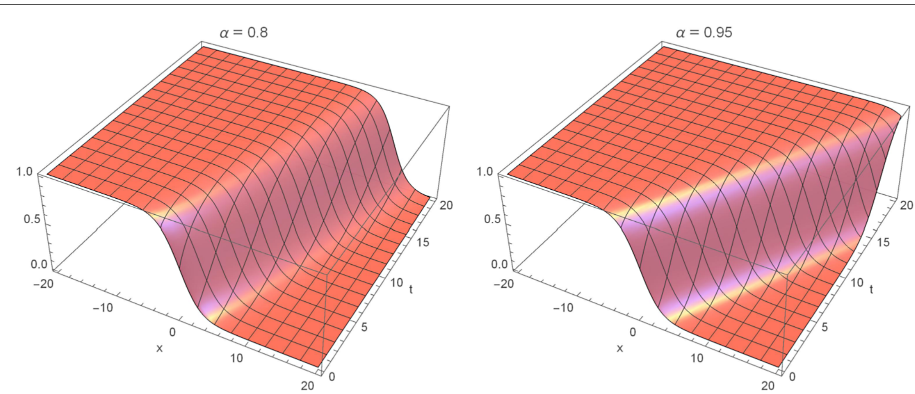

Figure 1 Dynamic behavior of solution $u_1(x,t)$ given by (19) for $\alpha = 0.8$ (left) and $\alpha = 0.95$ (right)

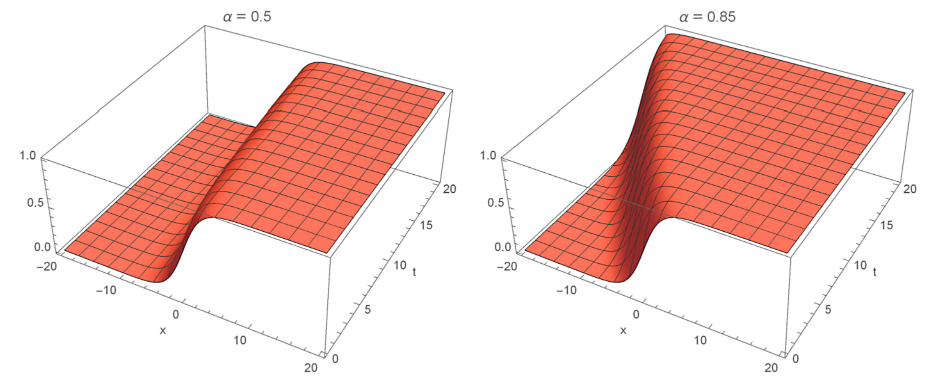

Figure 2 Dynamic behavior of solution $u_2(x,t)$ given by (20) for $\alpha = 0.5$ (left) and $\alpha = 0.85$ (right)

Consequently, we get the solution of Eq. (14) as

$$u_2(x,t) = \frac{e^{\frac{\sqrt{2}}{2}(x+\frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)})}}{1+e^{\frac{\sqrt{2}}{2}(x+\frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)})}}. \tag{20}$$

Figure 2 depicts the dynamic behavior of solution $u_2(x,t)$ presented in (19).

Set 2: One obtains $r = [-3,-1,1,1]$ and $s = [1,-1,-1,1]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{-2\cosh(\xi) - \sinh(\xi)}{\cosh(\xi)}. \tag{21}$$

We obtain

$$c = \frac{\sqrt{2}}{4}, \qquad \nu = -\frac{3\sqrt{2}}{2}, \qquad A_0 = -\frac{3}{2}, \qquad A_1 = 0, \qquad B_1 = -\frac{3}{2}.$$

Putting values in Eqs. (17) and (21) yields the following solution:

$$\mathcal{U}(\xi) = \frac{-3\cosh(\xi) - 3\sinh(\xi)}{4\cosh(\xi) + 2\sinh(\xi)}.$$

Consequently, we get the solution of Eq. (14) as

$$u_3(x,t) = -\frac{3\cosh(\frac{\sqrt{2}}{4}(x+\frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)})) + 3\sinh(\frac{\sqrt{2}}{4}(x+\frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)}))}{4\cosh(\frac{\sqrt{2}}{4}(x+\frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)})) + 2\sinh(\frac{\sqrt{2}}{4}(x+\frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)}))}. \tag{22}$$

Figure 3 depicts the dynamic behavior of solution $u_3(x,t)$ presented in (22).

Set 3: One obtains $r = [1-i, 1+i, 1, 1]$ and $s = [-i, i, -i, i]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{-\sin(\xi) + \cos(\xi)}{\cos(\xi)}. \tag{23}$$

We obtain

$$c = \frac{\sqrt{2}}{4}, \qquad \nu = \frac{3\sqrt{2}}{2}, \qquad A_0 = -\frac{1}{2} - \frac{i}{2}, \qquad A_1 = \frac{i}{2}, \qquad B_1 = 0.$$

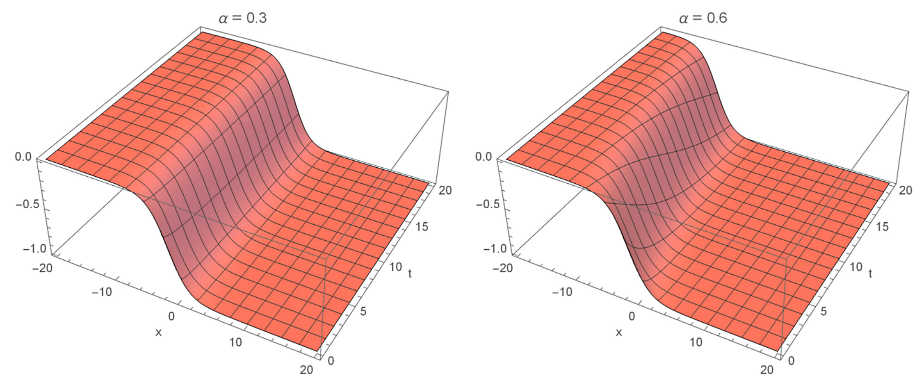

Figure 3 Dynamic behavior of solution $u_3(x,t)$ given by (22) for $\alpha = 0.5$ (left) and $\alpha = 0.85$ (right)

Putting values in Eqs. (17) and (23) yields the following solution:

$$\mathcal{U}(\xi) = \frac{-\cosh(\xi) + \sinh(\xi)}{2\cosh(\xi)}.$$

Consequently, we get the solution of Eq. (14) as

$$u_4(x,t) = -\frac{\cosh(\frac{\sqrt{2}}{4}(x - \frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)})) - \sinh(\frac{\sqrt{2}}{4}(x - \frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)}))}{2\cosh(\frac{\sqrt{2}}{4}(x - \frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)}))}. \tag{24}$$

Set 4: One obtains $r = [1,1,1,1]$ and $s = [1,-1,1,-1]$, so Eq. (5) turns to

$$\Psi(\xi) = -\frac{\cosh(\xi)}{\sinh(\xi)}. \tag{25}$$

We obtain

$$c = \frac{\sqrt{2}}{8}, \qquad \nu = \frac{3\sqrt{2}}{2}, \qquad A_0 = -\frac{1}{2}, \qquad A_1 = -\frac{1}{4}, \qquad B_1 = -\frac{1}{4}.$$

Putting values in Eqs. (17) and (25) yields the following solution:

$$\mathcal{U}(\xi) = \frac{(\coth(\xi) - 1)^2}{4\coth(\xi)}.$$

Consequently, we get the solution of Eq. (14) as

$$u_5(x,t) = \frac{(\coth(\frac{\sqrt{2}}{8}(x - \frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)})) - 1)^2}{4\coth(\frac{\sqrt{2}}{8}(x - \frac{3\sqrt{2}t^\alpha}{2\Gamma(\alpha)}))}. \tag{26}$$

Figure 4 depicts the dynamic behavior of solution $u_5(x,t)$ presented in (26).

Set 5: One obtains $r = [3,2,1,1]$ and $s = [1,0,1,0]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{3e^\xi + 2}{e^\xi + 1}. \tag{27}$$

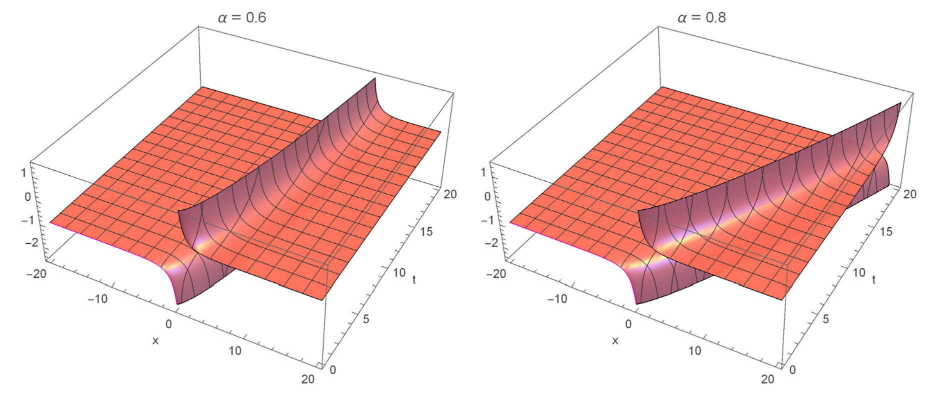

Figure 4 Dynamic behavior of solution $u_5(x,t)$ given by (26) for $\alpha = 0.6$ (left) and $\alpha = 0.8$ (right)

We obtain

$$c = \frac{\sqrt{2}}{2}, \qquad v = -\frac{3\sqrt{2}}{2}, \qquad A_0 = -3, \qquad A_1 = 0, \qquad B_1 = 6.$$

Putting values in Eqs. (17) and (27) yields the following solution:

$$\mathcal{U}(\xi) = -\frac{3e^{\xi}}{3e^{\xi} + 2}.$$

Consequently, we get the solution of Eq. (14) as

$$u_6(x,t) = -\frac{3e^{(\frac{\sqrt{2}}{2}(x+\frac{3\sqrt{2}t^{\alpha}}{2\Gamma(\alpha)}))}}{3e^{(\frac{\sqrt{2}}{2}(x+\frac{3\sqrt{2}t^{\alpha}}{2\Gamma(\alpha)}))} + 2}. \tag{28}$$

Application of EShGEEM for (14)

According to what was discussed above, we obtain $M = 1$. Taking $M = 1$ into account in Eqs. (8), (12), and (13), we respectively obtain

$$\mathcal{U}(\theta) = A_0 + B_1 \sinh(\theta) + A_1 \cosh(\theta) \tag{29}$$

and

$$\begin{aligned} \mathcal{U}_1(\xi) &= A_0 \pm iB_1 \operatorname{sech}(\xi) - A_1 \tanh(\xi), \\ \mathcal{U}_2(\xi) &= A_0 \pm B_1 \operatorname{csch}(\xi) - A_1 \coth(\xi). \end{aligned} \tag{30}$$

Inserting Eq. (29) into Eq. (16) gives a polynomial in powers of hyperbolic functions. Summing each coefficient of the hyperbolic functions of the same power and equating each summation to zero, we get a group of over-determined nonlinear algebraic equations. For each set, if we substitute the values of the parameters into any of Eqs. (30), the solutions to Eq. (14) are constructed as follows.

Set 1:

$$c = 1/4\sqrt{2}, \qquad v = -3/2\sqrt{2}, \qquad A_0 = -1/2, \qquad A_1 = 1/2, \qquad B_1 = 0.$$

Using these values, the following solution for (16) is obtained:

$$\begin{aligned} \mathcal{U}_1(\xi) &= -\frac{\cosh(\xi)+\sinh(\xi)}{2\cosh(\xi)}, \\ \mathcal{U}_2(\xi) &= -\frac{\cosh(\xi)+\sinh(\xi)}{2\sinh(\xi)}. \end{aligned} \tag{31}$$

Consequently, we get the solution of Eq. (14) as

$$\begin{aligned} u_7(x,t) &= -\frac{\cosh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})+\sinh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}{2\cosh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}, \\ u_8(x,t) &= -\frac{\cosh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})+\sinh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}{2\sinh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}. \end{aligned} \tag{32}$$

Set 2:

$$c = 1/4\sqrt{2}, \qquad \nu = 3/2\sqrt{2}, \qquad A_0 = 1/2, \qquad A_1 = 1/2, \qquad B_1 = 0.$$

Using these values, the following solution for (16) is obtained:

$$\begin{aligned} \mathcal{U}_1(\xi) &= \frac{\cosh(\xi)-\sinh(\xi)}{2\cosh(\xi)}, \\ \mathcal{U}_2(\xi) &= -\frac{\cosh(\xi)-\sinh(\xi)}{2\sinh(\xi)}. \end{aligned} \tag{33}$$

Consequently, we get the solution of Eq. (14) as

$$\begin{aligned} u_9(x,t) &= \frac{\cosh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})-\sinh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}{2\cosh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}, \\ u_{10}(x,t) &= -\frac{\cosh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})-\sinh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}{2\sinh(\frac{\sqrt{2}x\Gamma(\alpha)+3t^\alpha}{4\Gamma(\alpha)})}. \end{aligned} \tag{34}$$

Figure 5 depicts the dynamic behavior of solution $u_{10}(x,t)$ presented in (34).

Set 3:

$$c = 1/2\sqrt{2}, \qquad \nu = 3/2\sqrt{2}, \qquad A_0 = 1/2, \qquad A_1 = 1/2, \qquad B_1 = 1/2.$$

Using these values, the following solution for (16) is obtained:

$$\begin{aligned} \mathcal{U}_1(\xi) &= \frac{i+\cosh(\xi)-\sinh(\xi)}{2\cosh(\xi)}, \\ \mathcal{U}_2(\xi) &= \frac{-\cosh(\xi)+\sinh(\xi)+1}{2\sinh(\xi)}. \end{aligned} \tag{35}$$

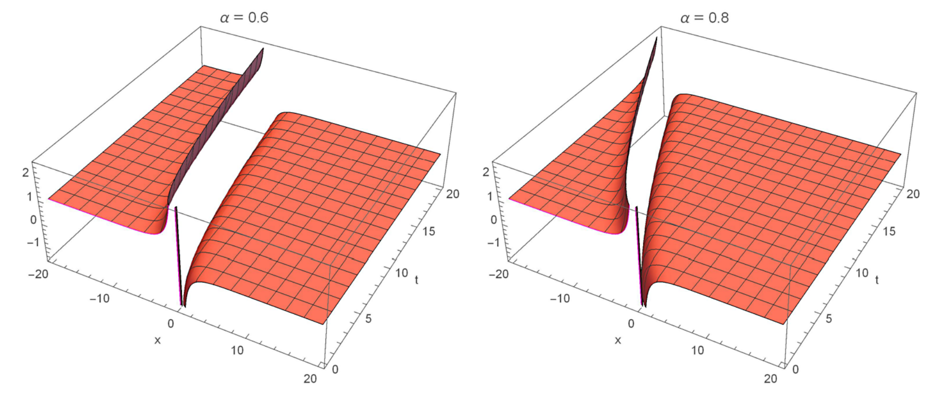

Figure 5 Dynamic behavior of solution $u_{10}(x,t)$ given by (34) for $\alpha = 0.6$ (left) and $\alpha = 0.8$ (right)

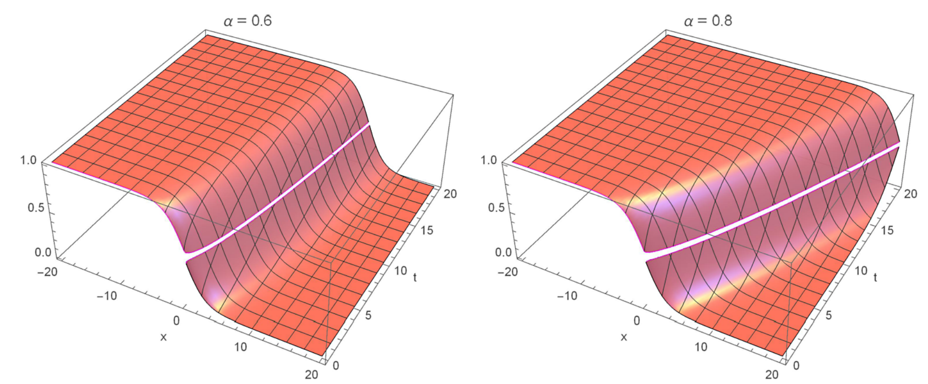

Figure 6 Dynamic behavior of solution $u_{12}(x,t)$ given by (36) for $\alpha = 0.6$ (left) and $\alpha = 0.8$ (right)

Consequently, we get the solution of Eq. (14) as

$$\begin{aligned} u_{11}(x,t) &= \frac{i + \cosh(\frac{\sqrt{2}x\Gamma(\alpha)-3t^{\alpha}}{2\Gamma(\alpha)}) - \sinh(\frac{\sqrt{2}x\Gamma(\alpha)-3t^{\alpha}}{2\Gamma(\alpha)})}{2\cosh(\frac{\sqrt{2}x\Gamma(\alpha)-3t^{\alpha}}{2\Gamma(\alpha)})}, \\ u_{12}(x,t) &= \frac{-\cosh(\frac{\sqrt{2}x\Gamma(\alpha)-3t^{\alpha}}{2\Gamma(\alpha)}) + \sinh(\frac{\sqrt{2}x\Gamma(\alpha)-3t^{\alpha}}{2\Gamma(\alpha)}) + 1}{2\sinh(\frac{\sqrt{2}x\Gamma(\alpha)-3t^{\alpha}}{2\Gamma(\alpha)})}. \end{aligned} \tag{36}$$

Figure 6 depicts the dynamic behavior of solution $u_5(x,t)$ presented in (36).

3.2 The space-time coupled Burgers equation

Consider the space-time conformable coupled Burgers equations [87]

$$\begin{aligned} &D_t^{\alpha}u - D_x^{2\alpha}u + 2uD_x^{\alpha}u + pD_x^{\alpha}(uv) = 0, \\ &D_t^{\alpha}v - D_x^{2\alpha}v + 2vD_x^{\alpha}v + qD_x^{\alpha}(uv) = 0. \end{aligned} \tag{37}$$

Using the transformation

$$u(x,t)=\mathcal{U}(\xi),\qquad v(x,t)=\mathcal{V}(\xi),\quad \xi=\frac{x^{\alpha}}{\Gamma(\alpha)}+\frac{ct^{\alpha}}{\Gamma(\alpha)}, \tag{38}$$

where c is a nonzero constant.

Utilizing the wave transformation (38) converts Eq. (37) into the following NODE:

$$\begin{aligned}&c\mathcal{U}'-\mathcal{U}''+2\mathcal{U}\mathcal{U}'+p(\mathcal{U}\mathcal{V})'=0,\\&c\mathcal{V}'-\mathcal{V}''+2\mathcal{V}\mathcal{V}'+q(\mathcal{U}\mathcal{V})'=0.\end{aligned} \tag{39}$$

Using the balance principle on the terms $\mathcal{U}\mathcal{U}'$ and $\mathcal{U}''$ in Eq. (39), we have $M+2=M+M+1$, so $M=1$.

Application of GERFM for (37)

Using Eq. (5) together with $M=1$, we have

$$\begin{aligned}&\mathcal{U}(\xi)=A_0+A_1\Psi(\xi)+\frac{B_1}{\Psi(\xi)},\\&\mathcal{V}(\xi)=A'_0+A'_1\Psi(\xi)+\frac{B'_1}{\Psi(\xi)}.\end{aligned} \tag{40}$$

Proceeding as outlined in the second section, we acquire the following sets of solutions to Eq. (37).

Set 1: One obtains $r=[1,1,-1,1]$ and $s=[1,-1,1,-1]$, so Eq. (5) turns into

$$\Psi(\xi)=-\frac{\cosh(\xi)}{\sinh(\xi)}. \tag{41}$$

We obtain

$$\begin{aligned}&c=-\frac{2A_0(pq-1)}{p-1},\qquad A_0=A_0,\qquad A_1=\frac{p-1}{pq-1},\qquad B_1=B_1,\\&A'_0=\frac{A_0(q-1)}{p-1},\qquad A'_1=\frac{q-1}{pq-1},\qquad B'_1=B'_1.\end{aligned}$$

Putting values in Eqs. (40) and (41) yields the following solution:

$$\begin{aligned}&\mathcal{U}(\xi)=\frac{pqA_0-\coth(\xi)p+\coth(\xi)-A_0}{pq-1},\\&\mathcal{V}(\xi)=\frac{pqA_0-\coth(\xi)q+\coth(\xi)-A_0}{pq-1}.\end{aligned} \tag{42}$$

Consequently, we get the solution of Eq. (37) as

$$\begin{aligned}&u_1(x,t)=\frac{pqA_0-\coth(\xi)p+\coth(\xi)-A_0}{pq-1},\\&v_1(x,t)=\frac{pqA_0-\coth(\xi)q+\coth(\xi)-A_0}{pq-1}.\end{aligned} \tag{43}$$

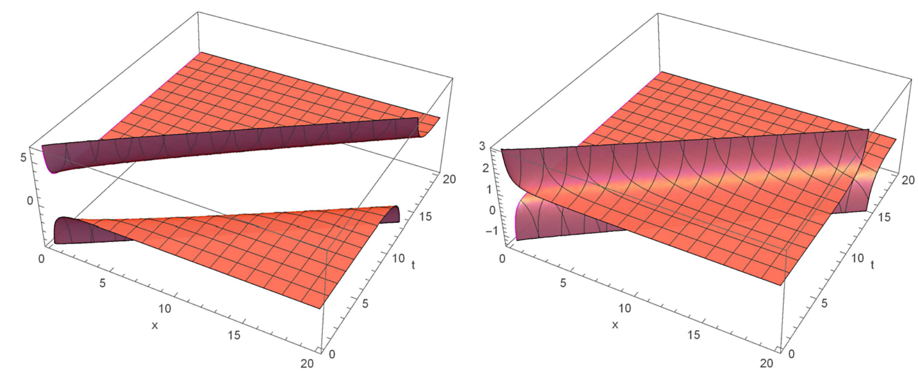

Figure 7 Dynamic behavior of solution $u_1(x,t)$ (left) and $v_1(x,t)$ (right) given by (43) for $p = 0.5$, $q = 1.2$, $A_0 = 0.8$, and $\alpha = 0.8$

Figure 7 depicts the dynamic behavior of solution $u_1(x,t)$, $v_1(x,t)$ presented in (43).

Set 2: One obtains $r = [-2-i, -2+i, 1, 1]$ and $s = [i, -i, i, -i]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{-2\cos(\xi) + \sin(\xi)}{\cos(\xi)}. \tag{44}$$

We obtain

$$c = \frac{-2pA_0q + 4p + 2A_0 - 4}{p-1}, \qquad A_0 = A_0, \qquad A_1 = \frac{p-1}{pq-1}, \qquad B_1 = B_1,$$
$$A'_0 = \frac{A_0(q-1)}{p-1}, \qquad A'_1 = \frac{q-1}{pq-1}, \qquad B'_1 = B'_1.$$

Putting values in Eqs. (40) and (44) yields the following solution:

$$\begin{aligned} \mathcal{U}(\xi) &= \frac{(pqA_0 - 2p - A_0 + 2)\cos(\xi) + \sin(\xi)(p-1)}{(pq-1)\cos(\xi)}, \\ \mathcal{V}(\xi) &= \frac{(pqA_0 - 2q - A_0 + 2)\cos(\xi) + \sin(\xi)(q-1)}{(pq-1)\cos(\xi)}. \end{aligned} \tag{45}$$

Consequently, we get the solution of Eq. (37) as

$$\begin{aligned} u_2(x,t) &= \frac{(pqA_0 - 2p - A_0 + 2)\cos(\xi) + \sin(\xi)(p-1)}{(pq-1)\cos(\xi)}, \\ v_2(x,t) &= \frac{(pqA_0 - 2q - A_0 + 2)\cos(\xi) + \sin(\xi)(q-1)}{(pq-1)\cos(\xi)}. \end{aligned} \tag{46}$$

Figure 8 depicts the dynamic behavior of solution $u_2(x,t)$, $v_2(x,t)$ presented in (46).

Set 3: One obtains $r = [1, 0, 1, 1]$ and $s = [1, 0, 1, 0]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{e^{\xi}}{1 + e^{\xi}}. \tag{47}$$

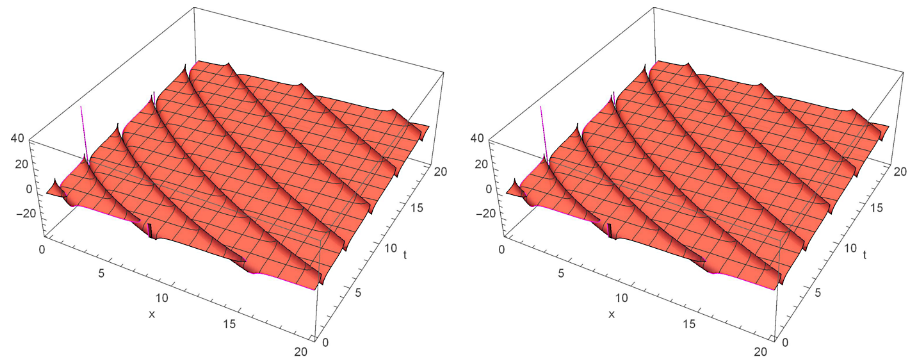

Figure 8 Dynamic behavior of solution $u_2(x,t)$ (left) and $v_2(x,t)$ (right) given by (46) for $p = 0.2$, $q = 0.2$, $A_0 = 1$, and $\alpha = 0.8$

We obtain

$$c = \frac{-2pA'_0q + q + 2A'_0 - 1}{q-1}, \qquad A_0 = \frac{(p-1)A'_0}{q-1}, \qquad A_1 = \frac{-p+1}{pq-1}, \qquad B_1 = B_1,$$
$$A'_0 = A'_0, \qquad A'_1 = \frac{-q+1}{pq-1}, \qquad B'_1 = B'_1.$$

Putting values in Eqs. (40) and (47) yields the following solution:

$$\begin{aligned} \mathcal{U}(\xi) &= \frac{(p-1)((pqA'_0 - q - A'_0 + 1)e^{\xi} + A'_0(pq-1))}{(q-1)(pq-1)(1+e^{\xi})}, \\ \mathcal{V}(\xi) &= \frac{(q-1)((pqA'_0 - p - A'_0 + 1)e^{\xi} + A'_0(pq-1))}{(p-1)(pq-1)(1+e^{\xi})}. \end{aligned} \tag{48}$$

Consequently, we get the solution of Eq. (37) as

$$\begin{aligned} u_3(x,t) &= \frac{(p-1)((pqA'_0 - q - A'_0 + 1)e^{\xi} + A'_0(pq-1))}{(q-1)(pq-1)(1+e^{\xi})}, \\ v_3(x,t) &= \frac{(q-1)((pqA'_0 - p - A'_0 + 1)e^{\xi} + A'_0(pq-1))}{(p-1)(pq-1)(1+e^{\xi})}. \end{aligned} \tag{49}$$

Figure 9 depicts the dynamic behavior of solution $u_3(x,t)$, $v_3(x,t)$ presented in (49).

Application of EShGEEM for (37)

The initial assumption of the solution structure of (39) is taken to be:

$$\begin{aligned} \mathcal{U}(\theta) &= A_0 + B_1 \sinh(\theta) + A_1 \cosh(\theta), \\ \mathcal{V}(\theta) &= A'_0 + B'_1 \sinh(\theta) + A'_1 \cosh(\theta), \end{aligned} \tag{50}$$

$$\begin{aligned} \mathcal{U}_1(\xi) &= A_0 \pm iB_1 \operatorname{sech}(\xi) - A_1 \tanh(\xi), \\ \mathcal{V}_1(\xi) &= A'_0 \pm iB'_1 \operatorname{sech}(\xi) - A'_1 \tanh(\xi), \end{aligned} \tag{51}$$

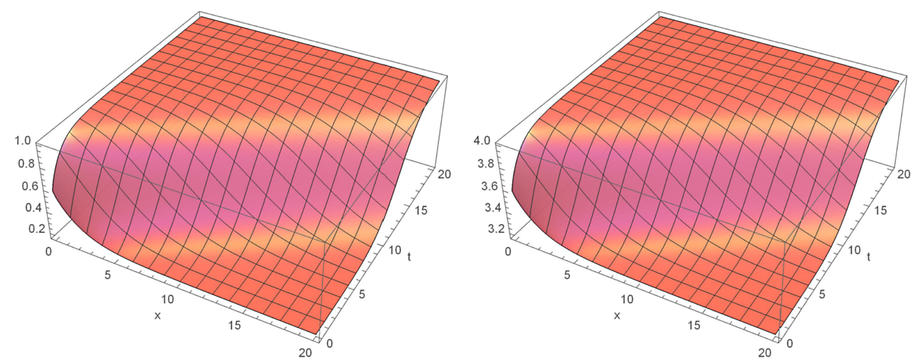

Figure 9 Dynamic behavior of solution $u_3(x,t)$ (left) and $v_3(x,t)$ (right) given by (49) for $p = 0.2$, $q = 0.2$, $A'_0 = 1$, and $\alpha = 0.8$

and

$$\begin{aligned}
\mathcal{U}_2(\xi) &= A_0 \pm B_1 \operatorname{csch}(\xi) - A_1 \coth(\xi),\\
\mathcal{V}_2(\xi) &= A'_0 \pm B'_1 \operatorname{csch}(\xi) - A'_1 \coth(\xi).
\end{aligned} \tag{52}$$

Applying the extended EShGEEM with the help of Eqs. (50)–(52), the following new exact soliton solutions of the space-time conformable coupled Burgers equations (37) are obtained.

Set 1:

$$\begin{aligned}
&c = -\frac{2A_0(pq-1)}{p-1}, \qquad A_0 = A_0, \qquad A_1 = \frac{p-1}{2pq-2}, \qquad B_1 = \frac{p-1}{2pq-2},\\
&A'_0 = \frac{A_0(q-1)}{p-1}, \qquad A'_1 = \frac{q-1}{2pq-2}, \qquad B'_1 = \frac{q-1}{2pq-2}.
\end{aligned}$$

Using these values, the following solution for (16) is obtained:

$$\begin{aligned}
\mathcal{U}_1(\xi) &= \frac{(2pqA_0 - 2A_0)\cosh(\xi) + (i - \sinh(\xi))(p-1)}{2(pq-1)\cosh(\xi)},\\
\mathcal{V}_1(\xi) &= \frac{(q-1)((2pqA_0 - 2A_0)\cosh(\xi) + (i - \sinh(\xi))(p-1))}{2(pq-1)(p-1)\cosh(\xi)},
\end{aligned} \tag{53}$$

and

$$\begin{aligned}
\mathcal{U}_2(\xi) &= \frac{(2pqA_0 - 2A_0)\sinh(\xi) - (\cosh(\xi)-1)(p-1)}{2(pq-1)\sinh(\xi)},\\
\mathcal{V}_2(\xi) &= \frac{(q-1)(A_0(pq-1)\sinh(\xi) - 1/2(\cosh(\xi)-1)(p-1))}{(pq-1)(p-1)\sinh(\xi)}.
\end{aligned} \tag{54}$$

Consequently, we get the solution of Eq. (14) as

$$\begin{aligned}
u_4(x,t) &= \frac{(2pqA_0 - 2A_0)\cosh(\xi) + (i - \sinh(\xi))(p-1)}{2(pq-1)\cosh(\xi)},\\
v_4(x,t) &= \frac{(q-1)((2pqA_0 - 2A_0)\cosh(\xi) + (i - \sinh(\xi))(p-1))}{2(pq-1)(p-1)\cosh(\xi)},
\end{aligned} \tag{55}$$

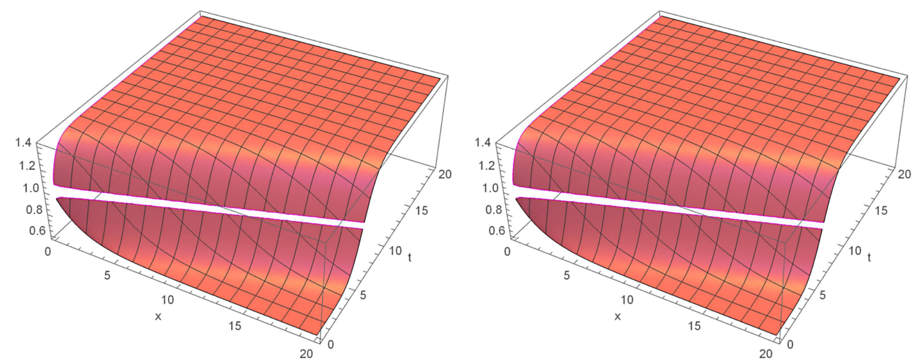

Figure 10 Dynamic behavior of solution $u_5(x,t)$ (left) and $v_5(x,t)$ (right) given by (56) for $p = 0.2$, $q = 0.2$, $A_0 = 1$, and $\alpha = 0.8$

and

$$\begin{aligned} u_5(x,t) &= \frac{(2pqA_0 - 2A_0)\sinh(\xi) - (\cosh(\xi) - 1)(p-1)}{2(pq-1)\sinh(\xi)}, \\ v_5(x,t) &= \frac{(q-1)(A_0(pq-1)\sinh(\xi) - 1/2(\cosh(\xi)-1)(p-1))}{(pq-1)(p-1)\sinh(\xi)}, \end{aligned} \tag{56}$$

where $\xi = \frac{1}{\Gamma(\alpha)}(x^\alpha + \frac{2A_0(pq-1)}{p-1}t^\alpha)$.

Figure 10 depicts the dynamic behavior of solution $u_5(x,t)$, $v_5(x,t)$ presented in (56).

Set 2:

$$\begin{aligned} &c = -\frac{2A_0(pq-1)}{p-1}, \qquad A_0 = A_0, \qquad A_1 = \frac{p-1}{pq-1}, \qquad B_1 = 0, \\ &A'_0 = \frac{A_0(q-1)}{p-1}, \qquad A'_1 = \frac{q-1}{pq-1}, \qquad B'_1 = 0. \end{aligned}$$

Using these values, the following solution for (16) is obtained:

$$\begin{aligned} \mathcal{U}_1(\xi) &= \frac{A_0(pq-1)\cosh(\xi) - (p-1)\sinh(\xi)}{(pq-1)\cosh(\xi)}, \\ \mathcal{V}_1(\xi) &= \frac{(A_0(pq-1)\cosh(\xi) - (p-1)\sinh(\xi))(q-1)}{(pq-1)(p-1)\cosh(\xi)}, \end{aligned} \tag{57}$$

and

$$\begin{aligned} \mathcal{U}_2(\xi) &= \frac{A_0(pq-1)\sinh(\xi) - \cosh(\xi)(p-1)}{(pq-1)\sinh(\xi)}, \\ \mathcal{V}_2(\xi) &= \frac{(q-1)(A_0(pq-1)\sinh(\xi) - \cosh(\xi)(p-1))}{(pq-1)(p-1)\sinh(\xi)}. \end{aligned} \tag{58}$$

Consequently, we get the solution of Eq. (14) as

$$\begin{aligned} u_6(x,t) &= \frac{A_0(pq-1)\cosh(\xi) - (p-1)\sinh(\xi)}{(pq-1)\cosh(\xi)}, \\ v_6(x,t) &= \frac{(A_0(pq-1)\cosh(\xi) - (p-1)\sinh(\xi))(q-1)}{(pq-1)(p-1)\cosh(\xi)}, \end{aligned} \tag{59}$$

and

$$u_7(x,t) = \frac{A_0(pq-1)\sinh(\xi) - \cosh(\xi)(p-1)}{(pq-1)\sinh(\xi)},$$
$$v_7(x,t) = \frac{(q-1)(A_0(pq-1)\sinh(\xi) - \cosh(\xi)(p-1))}{(pq-1)(p-1)\sinh(\xi)}, \tag{60}$$

where $\xi = \frac{1}{\Gamma(\alpha)}(x^\alpha + \frac{2A_0(pq-1)}{p-1}t^\alpha)$.

Set 3:

$$c = -\frac{2A_0(pq-1)}{p-1}, \qquad A_0 = A_0, \qquad A_1 = \frac{p-1}{2pq-2}, \qquad B_1 = \frac{-p+1}{2pq-2},$$
$$A'_0 = \frac{A_0(q-1)}{p-1}, \qquad A'_1 = \frac{q-1}{2pq-2}, \qquad B'_1 = \frac{-q+1}{2pq-2}.$$

Using these values, the following solution for (16) is obtained:

$$\mathcal{U}_1(\xi) = \frac{(2pqA_0 - 2A_0)\cosh(\xi) - (i+\sinh(\xi))(p-1)}{2(pq-1)\cosh(\xi)},$$
$$\mathcal{V}_1(\xi) = -\frac{(q-1)((-2pqA_0 + 2A_0)\cosh(\xi) + (i+\sinh(\xi))(p-1))}{2(pq-1)(p-1)\cosh(\xi)}, \tag{61}$$

and

$$\mathcal{U}_2(\xi) = \frac{(2pqA_0 - 2A_0)\sinh(\xi) - (\cosh(\xi)+1)(p-1)}{2(pq-1)\sinh(\xi)},$$
$$\mathcal{V}_2(\xi) = \frac{(A_0(pq-1)\sinh(\xi) - 1/2(\cosh(\xi)+1)(p-1))(q-1)}{(pq-1)(p-1)\sinh(\xi)}. \tag{62}$$

Consequently, we get the solution of Eq. (14) as

$$u_8(x,t) = \frac{(2pqA_0 - 2A_0)\cosh(\xi) - (i+\sinh(\xi))(p-1)}{2(pq-1)\cosh(\xi)},$$
$$v_8(x,t) = -\frac{(q-1)((-2pqA_0 + 2A_0)\cosh(\xi) + (i+\sinh(\xi))(p-1))}{2(pq-1)(p-1)\cosh(\xi)}, \tag{63}$$

and

$$u_9(x,t) = \frac{(2pqA_0 - 2A_0)\sinh(\xi) - (\cosh(\xi)+1)(p-1)}{2(pq-1)\sinh(\xi)},$$
$$v_9(x,t) = \frac{(A_0(pq-1)\sinh(\xi) - 1/2(\cosh(\xi)+1)(p-1))(q-1)}{(pq-1)(p-1)\sinh(\xi)}, \tag{64}$$

where $\xi = \frac{1}{\Gamma(\alpha)}(x^\alpha + \frac{2A_0(pq-1)}{p-1}t^\alpha)$.

Figure 11 depicts the dynamic behavior of solution $u_9(x,t)$, $v_9(x,t)$ presented in (64).

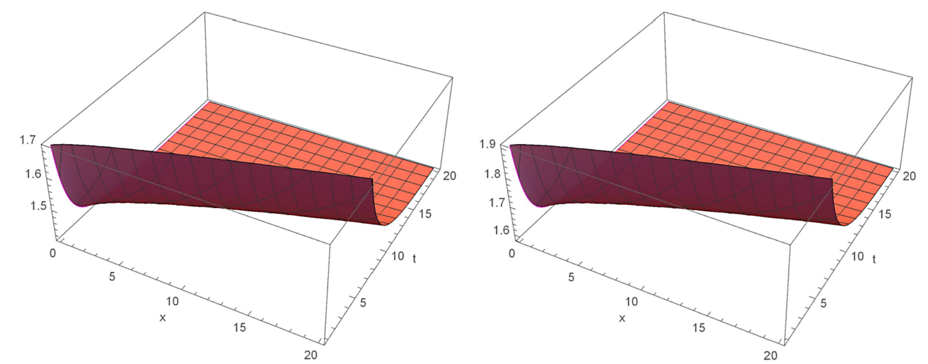

Figure 11 Dynamic behavior of solution $u_9(x,t)$ (left) and $v_9(x,t)$ (right) given by (56) for $p = 0.2$, $q = 0.1$, $A_0 = 1$, and $\alpha = 0.6$

3.3 The space-time conformable Fokas equation

Consider the space-time conformable Fokas equation [88]

$$4\frac{\partial^{2\alpha}u}{\partial t^{\alpha}\partial x_1^{\alpha}} - \frac{\partial^{4\alpha}u}{\partial x_1^{3\alpha}\partial x_2^{\alpha}} + \frac{\partial^{4\alpha}u}{\partial x_2^{3\alpha}\partial x_1^{\alpha}} + 12\frac{\partial^{\alpha}u}{\partial x_1^{\alpha}}\frac{\partial^{\alpha}u}{\partial x_2^{\alpha}} + 12u\frac{\partial^{2\alpha}u}{\partial x_1^{\alpha}\partial x_2^{\alpha}} - 6\frac{\partial^{2\alpha}u}{\partial y_1^{\alpha}\partial y_2^{\alpha}} = 0, \quad 0 < \alpha \leq 1. \tag{65}$$

Let us introduce the wave transformation as

$$u(x_1,x_2,y_1,y_2,t) = \mathcal{U}(\xi), \quad \xi = \frac{ct^{\alpha}}{\Gamma(\alpha)} + \frac{k_1x_1^{\alpha}}{\Gamma(\alpha)} + \frac{k_2x_2^{\alpha}}{\Gamma(\alpha)} + \frac{l_1y_1^{\alpha}}{\Gamma(\alpha)} + \frac{l_2y_2^{\alpha}}{\Gamma(\alpha)}, \tag{66}$$

where c, k_1, k_2, l_1, l_2 are nonzero constants.

Utilizing Eq. (66) converts Eq. (65) into the following NODE:

$$4ck_1\mathcal{U}'' - k_1^3k_2\mathcal{U}'''' + k_2^3k_1\mathcal{U}'''' + 12k_1k_2(\mathcal{U}')^2 + 12k_1k_2\mathcal{U}\mathcal{U}'' - 6l_1l_2\mathcal{U}'' = 0. \tag{67}$$

If we apply the balance principle on the terms $\mathcal{U}\mathcal{U}'$ and $\mathcal{U}''''$ in Eq. (67), we have $2M = M+2$, so $M = 2$.

Application of GERFM for (65)

Using Eq. (5) together with $M = 2$, we have

$$\mathcal{U}(\xi) = A_0 + A_1\Psi(\xi) + A_2\Psi^2(\xi) + \frac{B_1}{\Psi(\xi)} + \frac{B_2}{\Psi^2(\xi)}. \tag{68}$$

Proceeding as outlined in the second section, we acquire the following sets of solutions to Eq. (65).

Set 1: One obtains $r = [-1,3,1,-1]$ and $s = [1,-1,1,-1]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{\cosh(\xi) - 2\sinh(\xi)}{\sinh(\xi)}. \tag{69}$$

Case 1: We obtain

$$c = \frac{20k_1^3 k_2 - 20k_1 {k_2}^3 - 6k_1 k_2 A_0 + 3l_1 l_2}{2k_1},$$
$$A_0 = A_0, \qquad A_1 = 4{k_1}^2 - 4{k_2}^2, \qquad A_2 = {k_1}^2 - {k_2}^2, \qquad B_1 = 0, \qquad B_2 = 0,$$
$$k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.$$

Putting values in Eqs. (68) and (69) yields the following solution:

$$\mathcal{U}(\xi) = \frac{(-3{k_1}^2 + 3{k_2}^2 + A_0)\cosh^2(\xi) + 4{k_1}^2 - 4{k_2}^2 - A_0}{\sinh^2(\xi)}.$$

Consequently, we get the solution of Eq. (65) as

$$u_1(x_1, x_2, y_1, y_2, t) = \frac{(-3{k_1}^2 + 3{k_2}^2 + A_0)\cosh^2(\xi) + 4{k_1}^2 - 4{k_2}^2 - A_0}{\sinh^2(\xi)}, \tag{70}$$

where $\xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1 x_1^\alpha}{\Gamma(\alpha)} + \frac{k_2 x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1 y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2 y_2^\alpha}{\Gamma(\alpha)}$.

Case 2: We obtain

$$c = -\frac{3}{\sqrt{9{k_2}^2 + B_2}}\left(k_2\left(A_0 - \frac{10B_2}{27}\right)\sqrt{9{k_2}^2 + B_2} - 3/2l_1 l_2\right),$$
$$A_0 = A_0, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = 4/3B_2, \qquad B_2 = B_2,$$
$$k_1 = 1/3\sqrt{9{k_2}^2 + B_2}, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.$$

Putting values in Eqs. (68) and (69) yields the following solution:

$$\mathcal{U}(\xi) = \frac{(27A_0 - 9B_2)\cosh^4(\xi) + (-72A_0 + 29B_2)\cosh^2(\xi) + 4B_2\sinh(\xi)\cosh(\xi) + 48A_0 - 20B_2}{3(3\cosh^2(\xi) - 4)^2}.$$

Consequently, we get the solution of Eq. (65) as

$$\begin{aligned} &u_2(x_1, x_2, y_1, y_2, t) \\ &\quad = \frac{(27A_0 - 9B_2)\cosh^4(\xi) + (-72A_0 + 29B_2)\cosh^2(\xi) + 4B_2\sinh(\xi)\cosh(\xi) + 48A_0 - 20B_2}{3(3\cosh^2(\xi) - 4)^2}. \end{aligned} \tag{71}$$

Set 2: One obtains $r = [1, 1, 1, -1]$ and $s = [1, -1, 1, -1]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}. \tag{72}$$

Case 1: We obtain

$$c = \frac{-4k_1^3 k_2 + 4k_1 {k_2}^3 - 6k_1 k_2 A_0 + 3l_1 l_2}{2k_1},$$

$$A_0 = A_0, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = 0, \quad B_2 = {k_1}^2 - {k_2}^2,$$
$$k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.$$

Putting values in Eqs. (68) and (69) yields the following solution:

$$\mathcal{U}(\xi) = \frac{({k_1}^2 - {k_2}^2 + A_0)\cosh^2(\xi) - {k_1}^2 + {k_2}^2}{\cosh^2(\xi)}.$$

Consequently, we get the solution of Eq. (65) as

$$u_3(x_1, x_2, y_1, y_2, t) = \frac{({k_1}^2 - {k_2}^2 + A_0)\cosh^2(\xi) - {k_1}^2 + {k_2}^2}{\cosh^2(\xi)}. \tag{73}$$

Case 2: We obtain

$$c = \frac{-4k_1^3 k_2 + 4k_1 {k_2}^3 - 6k_1 k_2 A_0 + 3l_1 l_2}{2k_1},$$
$$A_0 = A_0, \quad A_1 = 0, \quad A_2 = {k_1}^2 - {k_2}^2, \quad B_1 = 0, \quad B_2 = {k_1}^2 - {k_2}^2,$$
$$k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.$$

Putting values in Eqs. (68) and (69) yields the following solution:

$$\mathcal{U}(\xi) = \frac{(2{k_1}^2 - 2{k_2}^2 + A_0)\cosh^4(\xi) + (-2{k_1}^2 + 2{k_2}^2 - A_0)\cosh^2(\xi) + {k_1}^2 - {k_2}^2}{\sinh^2(\xi)\cosh^2(\xi)}.$$

Consequently, we get the solution of Eq. (65) as

$$\begin{aligned} &u_4(x_1, x_2, y_1, y_2, t) \\ &\quad = \frac{(2{k_1}^2 - 2{k_2}^2 + A_0)\cosh^4(\xi) + (-2{k_1}^2 + 2{k_2}^2 - A_0)\cosh^2(\xi) + {k_1}^2 - {k_2}^2}{\sinh^2(\xi)\cosh^2(\xi)}. \end{aligned} \tag{74}$$

Set 2: One obtains $r = [1, 1, 1, -1]$ and $s = [1, -1, 1, -1]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{\cosh(\xi)}{\sinh(\xi)}. \tag{75}$$

Case 1: We obtain

$$c = \frac{-4k_1^3 k_2 + 4k_1 {k_2}^3 - 6k_1 k_2 A_0 + 3l_1 l_2}{2k_1},$$
$$A_0 = A_0, \quad A_1 = 0, \quad A_2 = 0, \quad B_1 = 0, \quad B_2 = {k_1}^2 - {k_2}^2,$$
$$k_1 = k_1, \quad k_2 = k_2, \quad l_1 = l_1, \quad l_2 = l_2.$$

Putting values in Eqs. (68) and (69) yields the following solution:

$$\mathcal{U}(\xi) = \frac{({k_1}^2 - {k_2}^2 + A_0)\cosh^2(\xi) - {k_1}^2 + {k_2}^2}{\cosh^2(\xi)}.$$

Consequently, we get the solution of Eq. (65) as

$$u_5(x_1,x_2,y_1,y_2,t) = \frac{({k_1}^2 - {k_2}^2 + A_0)\cosh^2(\xi) - {k_1}^2 + {k_2}^2}{\cosh^2(\xi)}. \tag{76}$$

Set 3: One obtains $r = [-1,0,1,1]$ and $s = [0,0,1,1]$, so Eq. (5) turns into

$$\Psi(\xi) = -\frac{1}{1+e^{\xi}}. \tag{77}$$

We obtain

$$\begin{aligned}
&c = \frac{k_1^3k_2 - k_1k_2^3 - 12k_1k_2A_0 + 6l_1l_2}{4k_1},\\
&A_0 = A_0, \qquad A_1 = {k_1}^2 - {k_2}^2, \qquad A_2 = {k_1}^2 - {k_2}^2, \qquad B_1 = 0, \qquad B_2 = 0,\\
&k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.
\end{aligned}$$

Putting values in Eqs. (68) and (77) yields the following solution:

$$\mathcal{U}(\xi) = \frac{e^{2\xi}A_0 + (-{k_1}^2 + {k_2}^2 + 2A_0)e^{\xi} + A_0}{(1+e^{\xi})^2}.$$

Consequently, we get the solution of Eq. (65) as

$$u_6(x_1,x_2,y_1,y_2,t) = \frac{e^{2\xi}A_0 + (-{k_1}^2 + {k_2}^2 + 2A_0)e^{\xi} + A_0}{(1+e^{\xi})^2}. \tag{78}$$

Set 4: One obtains $r = [-3,-2,1,1]$ and $s = [0,1,0,1]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{-3 - 2e^{\xi}}{1+e^{\xi}}. \tag{79}$$

We obtain

$$\begin{aligned}
&c = -3\frac{1}{\sqrt{36{k_2}^2 + B_2}}\left(k_2\left(A_0 - \frac{73B_2}{432}\right)\sqrt{36{k_2}^2 + B_2} - 3l_1l_2\right),\\
&A_0 = A_0, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = 5/6B_2, \qquad B_2 = B_2,\\
&k_1 = 1/6\sqrt{36{k_2}^2 + B_2}, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.
\end{aligned}$$

Putting values in Eqs. (68) and (79) yields the following solution:

$$\mathcal{U}(\xi) = \frac{(24A_0 - 4B_2)e^{2\xi} + (72A_0 - 13B_2)e^{\xi} + 54A_0 - 9B_2}{6(3+2e^{\xi})^2}.$$

Consequently, we get the solution of Eq. (65) as

$$u_7(x_1,x_2,y_1,y_2,t) = \frac{(24A_0 - 4B_2)e^{2\xi} + (72A_0 - 13B_2)e^{\xi} + 54A_0 - 9B_2}{6(3+2e^{\xi})^2}. \tag{80}$$

Set 5: One obtains $r = [-2 - i, 2 - i, 1, -1]$ and $s = [-i, i, -i, i]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{\cos(\xi) + 2\sin(\xi)}{\sin(\xi)}. \tag{81}$$

We obtain

$$\begin{aligned} &c = \frac{28k_1^3k_2 - 28k_1{k_2}^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1}, \\ &A_0 = A_0, \qquad A_1 = -4{k_1}^2 + 4{k_2}^2, \qquad A_2 = {k_1}^2 - {k_2}^2, \qquad B_1 = 0, \qquad B_2 = 0, \\ &k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2. \end{aligned}$$

Putting values in Eqs. (68) and (81) yields the following solution:

$$\mathcal{U}(\xi) = \frac{(5{k_1}^2 - 5{k_2}^2 - A_0)\cos^2(\xi) - 4{k_1}^2 + 4{k_2}^2 + A_0}{\sin^2(\xi)}.$$

Consequently, we get the solution of Eq. (65) as

$$u_8(x_1, x_2, y_1, y_2, t) = \frac{(5{k_1}^2 - 5{k_2}^2 - A_0)\cos^2(\xi) - 4{k_1}^2 + 4{k_2}^2 + A_0}{\sin^2(\xi)}. \tag{82}$$

Set 6: One obtains $r = [1 - i, -1 - i, 1, -1]$ and $s = [-i, i, -i, i]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{\cos(\xi) + \sin(\xi)}{\sin(\xi)}. \tag{83}$$

We obtain

$$\begin{aligned} &c = \frac{10k_1^3k_2 - 10k_1{k_2}^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1}, \\ &A_0 = A_0, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = -4{k_1}^2 + 4{k_2}^2, \qquad B_2 = 4{k_1}^2 - 4{k_2}^2, \\ &k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2. \end{aligned}$$

Putting values in Eqs. (68) and (83) yields the following solution:

$$\mathcal{U}(\xi) = \frac{2\sin(\xi)(-2{k_1}^2 + 2{k_2}^2 + A_0)\cos(\xi) + A_0}{2\cos(\xi)\sin(\xi) + 1}.$$

Consequently, we get the solution of Eq. (65) as

$$u_9(x_1, x_2, y_1, y_2, t) = \frac{2\sin(\xi)(-2{k_1}^2 + 2{k_2}^2 + A_0)\cos(\xi) + A_0}{2\cos(\xi)\sin(\xi) + 1}. \tag{84}$$

Set 7: One obtains $r = [2, 0, 1, -1]$ and $s = [1, 0, 1, -1]$, so Eq. (5) turns into

$$\Psi(\xi) = \frac{\cosh(\xi) + \sinh(\xi)}{\sinh(\xi)}. \tag{85}$$

We obtain

$$c = \frac{2k_1^3k_2 - 2k_1k_2{}^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},$$
$$A_0 = A_0, \qquad A_1 = -2k_1{}^2 + 2k_2{}^2, \qquad A_2 = k_1{}^2 - k_2{}^2, \qquad B_1 = 0, \qquad B_2 = 0,$$
$$k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.$$

Putting values in Eqs. (68) and (85) yields the following solution:

$$\mathcal{U}(\xi) = \frac{\cosh^2(\xi)A_0 + k_1{}^2 - k_2{}^2 - A_0}{\sinh^2(\xi)}.$$

Consequently, we get the solution of Eq. (65) as

$$u_{10}(x_1, x_2, y_1, y_2, t) = \frac{\cosh^2(\xi)A_0 + k_1{}^2 - k_2{}^2 - A_0}{\sinh^2(\xi)}. \tag{86}$$

Set 8: One obtains $r = [i, -i, 1, 1]$ and $s = [i, -i, i, -i]$, so Eq. (5) turns into

$$\Psi(\xi) = -\frac{\sin(\xi)}{\cos(\xi)}. \tag{87}$$

We obtain

$$c = \frac{4k_1^3k_2 - 4k_1k_2^3 - 6k_1k_2A_0 + 3l_1l_2}{2k_1},$$
$$A_0 = A_0, \qquad A_1 = 0, \qquad A_2 = 0, \qquad B_1 = 0, \qquad B_2 = k_1^2 - k_2^2,$$
$$k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.$$

Putting values in Eqs. (68) and (87) yields the following solution:

$$\mathcal{U}(\xi) = \frac{(k_1{}^2 - k_2{}^2 - A_0)\cos^2(\xi) + A_0}{\sin^2(\xi)}.$$

Consequently, we get the solution of Eq. (65) as

$$u_{11}(x_1, x_2, y_1, y_2, t) = \frac{(k_1{}^2 - k_2{}^2 - A_0)\cos^2(\xi) + A_0}{\sin^2(\xi)}. \tag{88}$$

Application of EShGEEM for (65)

Firstly, we assume that the solution of Eq. (67) takes the following form:

$$\mathcal{U}(\theta) = A_0 + B_1\sinh(\xi) + A_1\cosh(\xi) + \cosh(\xi)\big(B_2\sinh(\xi) + A_2\cosh(\xi)\big). \tag{89}$$

and

$$\begin{aligned}\mathcal{U}_1(\xi) &= A_0 + iB_1\operatorname{sech}(\xi) - A_1\tanh(\xi) - \tanh(\xi)(iB_2\operatorname{sech}(\xi) - A_2\tanh(\xi)),\\ \mathcal{U}_2(\xi) &= A_0 + B_1\operatorname{csch}(\xi) - A_1\coth(\xi) - \coth(\xi)(B_2\operatorname{csch}(\xi) - A_2\coth(\xi)).\end{aligned} \tag{90}$$

Now, the extended EShGEEM with the help of Eqs. (89)–(90) can introduce the following new exact soliton solutions of the space-time conformable Fokas equation given by (65).

Set 1:

$$c = c, \qquad A_0 = \frac{-4k_1^3k_2 - 2ck_1 + 3l_1l_2}{6k_1k_2}, \qquad A_1 = 0, \qquad A_2 = k_1^2,$$
$$B_1 = B_2 = 0, \qquad k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.$$

Using these values, the following solution for (67) is obtained:

$$\begin{aligned} \mathcal{U}_1(\xi) &= \frac{(2k_1^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 - 6{k_1}^3k_2}{6k_1k_2(\cosh(\xi))^2}, \\ \mathcal{U}_2(\xi) &= \frac{(2{k_1}^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 + 4{k_1}^3k_2 + 2ck_1 - 3l_1l_2}{6k_1k_2(\sinh(\xi))^2}. \end{aligned} \tag{91}$$

Consequently, we get the solution of Eq. (65) as

$$\begin{aligned} u_{12}(x_1,x_2,y_1,y_2,t) &= \frac{(2{k_1}^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 - 6{k_1}^3k_2}{6k_1k_2(\cosh(\xi))^2}, \\ u_{13}(x_1,x_2,y_1,y_2,t) &= \frac{(2{k_1}^3k_2 - 2ck_1 + 3l_1l_2)(\cosh(\xi))^2 + 4{k_1}^3k_2 + 2ck_1 - 3l_1l_2}{6k_1k_2(\sinh(\xi))^2}, \end{aligned} \tag{92}$$

where

$$\xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1x_1^\alpha}{\Gamma(\alpha)} + \frac{k_2x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2y_2^\alpha}{\Gamma(\alpha)}.$$

Set 2:

$$c = c, \qquad A_0 = \frac{-5{k_1}^3k_2 - 4ck_1 + 6l_1l_2}{12k_1k_2}, \qquad A_1 = 0, \qquad A_2 = B_2 = k_1^2/2, \qquad B_1 = 0,$$
$$k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.$$

Using these values, the following solution for (67) is obtained:

$$\begin{aligned} \mathcal{U}_1(\xi) &= \frac{({k_1}^3k_2 - 4ck_1 + 6l_1l_2)(\cosh(\xi))^2 - 6(i\sinh(\xi) + 1)k_2{k_1}^3}{12k_1k_2(\cosh(\xi))^2}, \\ \mathcal{U}_2(\xi) &= \frac{({k_1}^3k_2 - 4ck_1 + 6l_1l_2)\cosh(\xi) - 512{k_1}^3k_2 - 4ck_1 + 6l_1l_2}{k_1k_2(\cosh(\xi) + 1)}. \end{aligned} \tag{93}$$

Consequently, we get the solution of Eq. (65) as

$$\begin{aligned} u_{12}(x_1,x_2,y_1,y_2,t) &= \frac{({k_1}^3k_2 - 4ck_1 + 6l_1l_2)(\cosh(\xi))^2 - 6(i\sinh(\xi) + 1)k_2{k_1}^3}{12k_1k_2(\cosh(\xi))^2}, \\ u_{13}(x_1,x_2,y_1,y_2,t) &= \frac{({k_1}^3k_2 - 4ck_1 + 6l_1l_2)\cosh(\xi) - 512{k_1}^3k_2 - 4ck_1 + 6l_1l_2}{k_1k_2(\cosh(\xi) + 1)}, \end{aligned} \tag{94}$$

where

$$\xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1 x_1^\alpha}{\Gamma(\alpha)} + \frac{k_2 x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1 y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2 y_2^\alpha}{\Gamma(\alpha)}.$$

Set 3:

$$c = c, \qquad A_0 = \frac{-5{k_1}^3 k_2 - 4ck_1 + 6l_1 l_2}{12k_1 k_2}, \qquad A_1 = 0, \qquad A_2 = -B_2 = k_1^2/2, \qquad B_1 = 0,$$
$$k_1 = k_1, \qquad k_2 = k_2, \qquad l_1 = l_1, \qquad l_2 = l_2.$$

Using these values, the following solution for (67) is obtained:

$$\begin{aligned} \mathcal{U}_1(\xi) &= \frac{(k_1^3 k_2 - 4ck_1 + 6l_1 l_2)(\cosh(\xi))^2 + 6(i\sinh(\xi) - 1)k_2 k_1^3}{12k_1 k_2(\cosh(\xi))^2}, \\ \mathcal{U}_2(\xi) &= \frac{(k_1^3 k_2 - 4ck_1 + 6l_1 l_2)\cosh(\xi) + 5k_1^3 k_2 + 4ck_1 - 6l_1 l_2}{12k_1 k_2(\cosh(\xi) - 1)}. \end{aligned} \tag{95}$$

Consequently, we get the solution of Eq. (65) as

$$\begin{aligned} u_{14}(x_1, x_2, y_1, y_2, t) &= \frac{(k_1^3 k_2 - 4ck_1 + 6l_1 l_2)(\cosh(\xi))^2 + 6(i\sinh(\xi) - 1)k_2 k_1^3}{12k_1 k_2(\cosh(\xi))^2}, \\ u_{15}(x_1, x_2, y_1, y_2, t) &= \frac{(k_1^3 k_2 - 4ck_1 + 6l_1 l_2)\cosh(\xi) + 5k_1^3 k_2 + 4ck_1 - 6l_1 l_2}{12k_1 k_2(\cosh(\xi) - 1)}, \end{aligned} \tag{96}$$

where

$$\xi = \frac{ct^\alpha}{\Gamma(\alpha)} + \frac{k_1 x_1^\alpha}{\Gamma(\alpha)} + \frac{k_2 x_2^\alpha}{\Gamma(\alpha)} + \frac{l_1 y_1^\alpha}{\Gamma(\alpha)} + \frac{l_2 y_2^\alpha}{\Gamma(\alpha)}.$$

The correctness of all the solutions obtained in the paper has been examined by placing them directly in the main equation, and it has been found that they satisfy the main equation.

4 Conclusion

Pursuing new concepts in mathematics provides a promising framework for describing many complex phenomena and structures in the real world. Many of these structures cannot be described by the existing classical definitions. This is an incentive for researchers to explore new definitions in differential calculus. In this paper, based on the generalized exponential rational function method and the extended sinh-Gordon equation expansion method, we have obtained several new exact solutions of the space-time conformable coupled Cahn–Allen equation, coupled Burgers equation, and Fokas equation. Both schemes are easy to implement in computer programs and take small memory. On the other hand, they require less computational cost compared to other techniques. Numerical results clearly indicate the reliability and efficiency of the proposed method. To the best of our knowledge, the solutions obtained for these nonlinear equations considering the GERFM and EShGEEM are new and have not been reported in the literature. It is important to note that a wide range of solutions, such as exponential, triangular, dark, and light solitons, periodic solutes, for the equations considered in this paper are determined by two methods that have not been previously explored in previous references. Since the techniques are direct, powerful, and efficient, they can be efficiently used to find the exact solutions of different nonlinear differential equations in several branches of nonlinear sciences.

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