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Event-triggered H-infinity finite-time consensus control for nonlinear second-order multi-agent systems with disturbances

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Abstract

The study considers the problem of finite-time event-triggered H-infinity consensus for second-order multi-agent systems (MASs) with intrinsic nonlinear dynamics and external bounded disturbances. Based on the designed triggering function, a distributed event-triggered control strategy is presented on the basis of the designed triggering function to ensure consensus in the system, which effectively reduces the data transmission. Then, sufficient conditions for the finite-time consensus with H-infinity performance level of the resulting event-triggered MAS are derived by utilizing the Lyapunov function and finite-time stability theory. Furthermore, Zeno behavior is proven to be excluded under the proposed event-triggered scheme. Finally, the validity of the proposed results is verified by numerical simulation.

Keywords: Multi-agent systems; Finite-time consensus; Event-triggered control; H-infinity consensus; Disturbances

1 Introduction

In recent years, multi-agent systems (MASs) have been widely applied in many practical fields, such as unmanned aerial vehicles (UAVs) [1, 2], collective control [3], sensor network [4], and multi-autonomous robot [5]. These systems have also attracted research interests from scholars in many fields. Consensus is an important and basic problem in MAS research. The consensus problem for MASs has been solved in many practical fields, such as UAV cruising and robotic arms. Therefore, achieving consensus among MASs is important and has become a major research area in this field in recent years [6–10].

At present, numerous advancements, such as asymptotic consensus, exponential asymptotic consensus, and finite-time consensus, have been made in MAS consensus control studies. In practical applications, MASs must achieve consensus within a limited amount of time. Therefore, the consensus convergence rate is a key performance index that should be considered by researchers. Finite-time consensus has many advantages over asymptotic consensus, such as higher accuracy, better robustness, and faster response times. Many researchers have been attracted to this field due to the good performances of finite-time implementations. Several interesting results have been obtained for different models. The

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finite-time consensus problem for a first-order MAS with a continuous time-varying interaction topology was studied in [11]. Zhang et al. [12] studied the finite-time control of first-order MASs. The above-mentioned works focused on the finite-time consensus problem of first-order MASs.

However, the model of the first-order MASs has some limitations in practical application. Some high-speed MASs accrue large errors when the first-order MAS model is used to describe the system for control. Thus, the description of the second-order system could better represent the essence of object motion change. In recent years, many scholars have studied consensus control problems of second-order MASs. For example, the author of [13] studied the finite-time consensus of second-order MASs, and the robust finite-time consensus problem in second-order nonlinear dynamic MASs was investigated in [14].

In the aforementioned studies, MAS consensus control mainly adopts continuous-time state feedback control. In our case, the agent controller is also in the field in some cases, that is, the agent must work in a limited energy environment, where the CPU frequency, memory capacity, and communication bandwidth in the control device are limited. These problems must be considered during the controller design to extend its service life and preserve communication capacity among agents. Therefore, changing the controller mode has become an important research topic. In response to the above-mentioned issue, an event-triggered control strategy has been proposed. Since its introduction, this method has become an important research area, has been widely applied in many fields, and research on MAS consensus based on event-triggered control has obtained some achievements. For general linear first-order MASs, the authors of [15–19] studied the event-triggered finite-time control algorithm that can adjust the expected convergence time. The author of [20] provided a self-triggering algorithm to ensure consensus in the system.

However, the aforementioned research results apply mainly to event-triggered first-order MAS finite-time consensus control; thus, some scholars are currently studying the consensus control of second-order MASs based on event-triggered strategies. For example, Qian [21] emphasized that the event-triggered control strategy is an effective way to reduce agent energy consumption that can significantly extend the operating life of MASs. Cao [22] proposed a distributed event-triggered control strategy to ensure consensus in MASs within a certain period of time. The authors of [23] investigated the distributed finite-time consensus problem for a class of second-order MASs under bounded perturbations and provided a continuous homogeneous finite-time consensus protocol based on nominal multiple agents. However, the multi-agent models studied in these works ignore nonlinear dynamics and disturbance factors.

The dynamics of agents are complex. Consensus control for nonlinear MASs has also been studied given that many systems contain nonlinear dynamics. For example, an event-triggered sliding mode controller was proposed in [24] to achieve finite-time consensus for a first-order nonlinear leader-following MAS. Chen [25] proposed a distributed protocol based on relative position information for second-order MASs with inherent nonlinear dynamics and communication delays. Accurately expressing the MAS model is difficult in many cases because the system may inherently be affected by uncertain factors, such as modeling errors and parameter fluctuations. Therefore, studying the MAS consensus control of systems with uncertain parameters is important. For example, Su and Huang [26] studied the consensus problem for leader-following MASs by viewing it as an adaptive stability problem for an explicit error system to achieve consensus under unknown

parameters. The author of [27] investigated the consensus control of MASs with uncertain model parameters.

MASs may also be subject to external disturbances in the field, such as while sending, transmitting, or receiving information. Therefore, nonlinear dynamics, parameter uncertainties, and external disturbances should be considered for practical MAS applications. However, the above-mentioned works consider only nonlinear MAS and fail to consider external disturbances.

A few research achievements have been made on the finite-time consensus of second-order MASs with disturbances. For example, Zhang [28] proposed a finite-time consensus problem for second-order MASs with external bounded disturbances. The author assumed that a disturbance could be represented by a bounded constant. However, this assumption limits the negative influence produced by the disturbance and has difficulty fulfilling the purpose of precise control in some cases. Therefore, H-infinity control methods that can effectively suppress the negative effects of disturbance have been proposed and have been successfully applied in some engineering fields [29]. So far, a lot of results have been published on this issue. For example, Jia and Huan [30, 31] studied first-order and higher-order MASs with robustness for the H-infinity consensus problem under external disturbances. Ban et al. [32] explored a first-order MAS with leader for the finite-time H-infinity consensus problem and introduced a nonlinear finite-time H-infinity tracking control protocol. These works focused on first-order systems, while the authors of [33] examined a class of second-order MASs with a distributed H-infinity composite spinning consensus problem. However, all these works ignored the finite-time consensus problem.

In recent years, a number of results on event-triggered control have been derived for second-order MASs with disturbance. However, to the best of our knowledge, the problem of event-triggered H-infinity finite-time consensus of nonlinear second-order MASs with disturbance is rarely studied, which is the main motivation of the study. Compared with some previous relevant works, the main contributions of this study are summarized as follows:

1. The problem of event-triggered strategy control for a class of second-order nonlinear MASs with external disturbances is addressed. To the best of the authors' knowledge, few results on this topic for such systems are available;
2. Under the proposed control protocol and distributed event-triggered strategy, sufficient conditions are derived such that the MAS under study not only can achieve consensus but also can meet the requirements of suitable performance. Moreover, an H-infinity optimal control algorithm that provides robust and dynamic characteristics for the second-order MASs is proposed. Distributed event-triggered control is proven to avoid Zeno behavior.

2 Problem description and preliminaries

This section presents the basic concepts of some algebraic graph theory and useful theorems.

2.1 Algebraic graph theory

The communication topology among agents in MAS can be modeled by graph theory, where each agent is a node and each communication path is an edge. Let $G = \{V, E, A\}$ be an undirected graph, where $V = \{1, 2, \dots, n\}$ is a set of vertices, $E \in V \times V$ is a set of edges,

and $A = [a_{ij}]_{N \times N}$ is a weighted adjacency matrix with weights $a_{ij} \geq 0$ for $\forall i, j \in V$. If a path exists between node i and node j , then edge $(i, j) \in E$ and $a_{ij} = a_{ji} > 0$. The elements $a_{ii} = 0$ for all $i \in V$ mean that no self-loops are present. If an edge exists between node j and node i , then node j is a neighbor of node i . The neighboring set of node i is $N_i = \{j | (i, j) \in E\}$. The Laplacian matrix L of graph G is denoted as $l_{ij} = -a_{ij}$ for $j \neq i$ and $l_{ii} = \sum_{j=1, j \neq i}^N a_{ij}$. Simultaneously, a pair of angular matrices can be defined as $D = \text{diag}\{d_1, d_2, \dots, d_N\}$ with $d_i = \text{deg}_{in}(v_i)$. Then, the Laplacian matrix for the undirected graph G can be defined as $L = D - A$.

Notation: The following notations will be used. Let R denote the real numbers set and R^n denote the n -dimensional real vector space. Given a vector $x = [x_1, x_2, \dots, x_n]^T \in R^n$, denote $\text{sig}(x)^\alpha = [\text{sign}(x_1)|x_1|^\alpha, \dots, \text{sign}(x_n)|x_n|^\alpha]^T$, where $\text{sign}(\cdot)$ is the signum function, and $|x_i| = [|x_1|, |x_2|, \dots, |x_n|]^T$.

2.2 Problem description

In a second-order MAS, n dynamic agents in continuous time share a common state space R with all agents. x_i represents the position state of agent i , and v_i represents the degree state of agent i . The dynamic behavior of the agent i can be described as follows:

$$\begin{cases} \dot{x}_i = v_i(t), \\ \dot{v}_i = u_i(t) + f(t, x_i(t), v_i(t)) + w_i(t), \end{cases} \quad i = 1, \dots, n, \tag{1}$$

where $x_i \in R^n$, $v_i \in R^n$ denote the position and velocity, respectively. $u_i(t) \in R^n$ is the control input of agent i . $f(t, x_i(t), v_i(t))$ represents the nonlinear dynamic function of the i th agent. $w_i(t)$ is the exogenous disturbance input that satisfies $\int_0^\infty w_i^T(t)w_i(t) dt < d$, $d \geq 0$, $i = 1, 2, \dots, n$.

Remark 1 In [16], second-order multi-agent consensus research was considered, but nonlinear functions and disturbance terms were ignored by the model. In [13], consensus under disturbance with MASs was studied, but the nonlinear dynamics of the system were ignored. In contrast, the model in the present study considers disturbance and nonlinear dynamics.

According to the relevant information, output control is defined as follows:

$$z_i(t) = [z_1(t), \dots, z_n(t)]^T, \tag{2}$$

and $z(t) = \frac{1}{\sqrt{n}}[(x_i(t) - x_j(t) + \gamma(x_i(t) - x_j(t)))]^\alpha$. The H-infinity performance indicators refer to $J(w) = \int_0^T (z^T(t)z(t) - \delta^2 w^T(t)w(t)) dt$, and δ is a positive number.

For system (1), we make the following two assumptions.

Assumption 1 The connection diagram of MAS (1) is undirected and connected.

Assumption 2 There exists a positive number μ , such that $\|f(t, x_i(t), v_i(t))\| < \mu$, $i = 1, 2, \dots, n$.

Definition 1 The finite-time consensus is achieved for second-order systems (1) if for any initial conditions we have $\kappa_1 > 0$, $\kappa_2 > 0$ and a finite time T such that $|x_i(t) - x_j(t)| < \kappa_1$ and $|v_i(t) - v_j(t)| < \kappa_2$ if $t \geq T$, where $i, j \in V$.

Definition 2 Nonlinear MAS (1) with the event-triggered state feedback controllers (4) is said to be FTB with a prescribed finite-time H-infinity performance level $\delta > 0$ if the following conditions hold:

1. MAS (1) with event-triggered state feedback controllers (4) is finite-time bounded.
2. Under zero-initial condition $\forall t \in [0, T]$, the controlled output $z(t)$ satisfies

$$\int_0^T z^T(t)z(t) dt < \delta^2 \int_0^T w^T(t)w(t) dt. \tag{3}$$

The following lemmas are used in this study.

Lemma 1 ([15])

$$\left(\sum_{i=1}^n |x_i|\right)^p \leq \sum_{i=1}^n |x_i|^p \leq n^{1-p} \left(\sum_{i=1}^n |x_i|\right)^p,$$

and $x_i \in R, 0 < p \leq 1$.

Lemma 2 ([12]) *Considering the system $\dot{x} = f(x)$ with $f(0) = 0, x(0) = x_0, x \in R^n$, if a positive definite continuous function $V(x) : U \rightarrow R$ exists, then we have real numbers $c > 0, \alpha \in (0, 1)$, and $d \geq 0$ such that*

$$\dot{V}(x) \leq -c(V(x))^\alpha + d.$$

Then $V(x)$ is finite-time bounded. If $d = 0$, then $V(x) \equiv 0$ for all $t \geq T$, where the settling time T is determined as

$$T \leq \frac{(V(0))^{1-\alpha}}{c(1-\alpha)}.$$

Lemma 3 ([16]) *If the undirected graph of MAS (1) is connected, then the Laplacian matrix L is symmetric. $\lambda_1 \leq \lambda_2 \leq \dots \leq \lambda_n$ are defined as the eigenvalues of matrix L . Then $\lambda_1 = 0$ and $\lambda_2 > 0$. The algebraic connectivity is defined as follows: if $1^T r = 0, r \neq 0$, then $a = \lambda_2 = \min \frac{r^T L r}{r^T r}, r^T L^2 r \geq a r^T L r$.*

Lemma 4 ([24]) *For $x \in R^+, y \in R^+$, and $0 < h < 1$, the inequality $(x + y)^h \leq x^h + y^h$ holds.*

Lemma 5 ([21]) *For any positive numbers c, d , and any real numbers a, b ,*

$$|a|^c |b|^d \leq \frac{c}{c+d} |a|^{c+d} + \frac{d}{c+d} |b|^{c+d}.$$

3 Main results

This section presents the design of an appropriate event-triggered protocol for a nonlinear second-order MAS with external disturbances. To ensure consensus in a finite time in the system, we design a new finite-time control protocol based on the event-triggered strategy:

$$u_i = -\beta \left[\sum_{j \in N_i} a_{ij} (x_i(t_k^i) - x_j(t_k^j)) \right] + \gamma \sum_{j \in N_i} a_{ij} (v_i(t_k^i) - v_j(t_k^j))$$

$$+ \text{sig} \left(\sum_{j \in N_i} a_{ij} (x_i(t_k^i) - x_j(t_k^j)) + \gamma \sum_{j \in N_i} a_{ij} (v_i(t_k^i) - v_j(t_k^j)) \right) \Big]^\alpha - \frac{1}{\gamma} v_i(t), \tag{4}$$

where $t \in [t_k, t_{k+1})$, $k = 0, 1, \dots$, and $0 < \alpha < 1$, $\beta > 0$, $\gamma > 0$.

In the interval $t \in [t_k^i, t_{k+1}^i)$, the state combination of agent i is given as follows:

$$q_i = \sum_{j \in N_i} a_{ij} (x_i(t) - x_j(t)), \tag{5}$$

$$p_i = \sum_{j \in N_i} a_{ij} (v_i(t) - v_j(t)). \tag{6}$$

The measurement error is

$$e_{x_i}(t) = x_i(t_k^i) - x_i(t), \tag{7}$$

$$e_{v_i}(t) = v_i(t_k^i) - v_i(t). \tag{8}$$

The combined measurement error is

$$E_{x_i}(t) = \sum_{j \in N_i} a_{ij} (e_{x_i}(t) - e_{x_j}(t)), \tag{9}$$

$$E_{v_i}(t) = \sum_{j \in N_i} a_{ij} (e_{v_i}(t) - e_{v_j}(t)). \tag{10}$$

MAS (1) can be rewritten as

$$\begin{cases} \dot{x}_i = v_i(t), \\ \dot{v}_i = -\beta \left[\sum_{j \in N_i} a_{ij} (x_i(t_k^i) - x_j(t_k^j)) + \gamma \sum_{j \in N_i} a_{ij} (v_i(t_k^i) - v_j(t_k^j)) \right. \\ \quad \left. + \text{sig} \left(\sum_{j \in N_i} a_{ij} (x_i(t_k^i) - x_j(t_k^j)) + \gamma \sum_{j \in N_i} a_{ij} (v_i(t_k^i) - v_j(t_k^j)) \right) \right]^\alpha \\ \quad - \frac{1}{\gamma} v_i(t) + f(t, x_i(t), v_i(t)) + w_i(t). \end{cases} \tag{11}$$

Finite-time consensus protocol (4) can be converted to

$$\begin{aligned} u_i &= -\beta \left[\sum_{j \in N_i} a_{ij} (x_i(t_k^i) - x_j(t_k^j)) + \gamma \sum_{j \in N_i} a_{ij} (v_i(t_k^i) - v_j(t_k^j)) \right. \\ &\quad \left. + \text{sig} \left(\sum_{j \in N_i} a_{ij} (x_i(t_k^i) - x_j(t_k^j)) + \gamma \sum_{j \in N_i} a_{ij} (v_i(t_k^i) - v_j(t_k^j)) \right) \right]^\alpha - \frac{1}{\gamma} v_i(t) \\ &= -\beta \left[\sum_{j \in N_i} a_{ij} [(x_i(t) + e_{x_i}(t) - x_j(t) - e_{x_j}(t)) + \gamma (v_i(t) + e_{v_i}(t) \right. \\ &\quad \left. - v_j(t) - e_{v_j}(t))] + \text{sig} \left(\sum_{j \in N_i} a_{ij} (x_i(t) + e_{x_i}(t) - x_j(t) - e_{x_j}(t) \right. \right. \\ &\quad \left. \left. + \gamma (v_i(t) + e_{v_i}(t) - v_j(t) - e_{v_j}(t))) \right) \right]^\alpha - \frac{1}{\gamma} v_i(t) \\ &= -\beta [q_i(t) + E_{x_i}(t) + \gamma (p_i(t) + E_{v_i}(t))] \end{aligned}$$

$$+ \operatorname{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))^\alpha - \frac{1}{\gamma} v_i(t). \tag{12}$$

The event triggering function of multi-agent i is set as

$$h_i(t) = \|L\| |e_{xi} + \gamma e_{vi}(t)| - \sigma |q_i(t_k^i) + \gamma p_i(t_k^i)|, \tag{13}$$

where $\sigma > 0$, and $\|L\|$ denotes the 2-norm of Laplacian matrix L . Then, the triggering condition is defined as

$$t_{k+1}^i = \inf\{t > t_k^i, h_i(t) > 0\}. \tag{14}$$

For system (1), the Lyapunov method is used to study the finite-time consensus under event-triggered control when $w_i(t) = 0$.

Theorem 1 *Let the assumption be that Assumption 1 is satisfied and the undirected graph of MAS (1) is connected. With the event-triggered control algorithm (4) and the triggering function (13), if suitable positive scalars β and γ exist, then the finite-time consensus problem can be solved when the following conditions are satisfied:*

$$0 < \eta = \gamma \left(2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho - \frac{\mu}{2} \right) (1 + \alpha)^{\frac{2\alpha}{1+\alpha}} \beta^{\frac{2}{1+\alpha}},$$

where $0 < \sigma < 1$, $\gamma > 0$, $\beta > 0$, $0 < \alpha < 1$, and $\eta > 0$.

The finite-time T can be estimated using the following inequalities:

$$T \leq \frac{V(0)^{1-\alpha}}{\eta(1-\alpha)},$$

where $V(0) = \sum_{i=1}^n \frac{\beta}{1+\alpha} |q_i(0) + \gamma p_i(0)|^{1+\alpha}$.

Proof A Lyapunov function is established for MAS (1) as follows:

$$V(t) = \sum_{i=1}^n \frac{\beta}{1+\alpha} |q_i(t) + \gamma p_i(t)|^{1+\alpha}. \tag{15}$$

The derivative of $V(t)$ is

$$\begin{aligned} \dot{V}(t) &= \sum_{i=1}^n \beta \operatorname{sig}(q_i(t) + \gamma p_i(t))^\alpha (\dot{q}_i(t) + \gamma \dot{p}_i(t)) \\ &= \sum_{i=1}^n \beta \operatorname{sig}(q_i(t) + \gamma p_i(t))^\alpha \left(p_i(t) + \gamma L_i \left(-\beta [q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t))] \right. \right. \\ &\quad \left. \left. + \operatorname{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))^\alpha - \frac{1}{\gamma} v_i(t) + f(t, x_i(t), v_i(t)) \right) \right) \\ &= \sum_{i=1}^n \beta \operatorname{sig}(q_i(t) + \gamma p_i(t))^\alpha \left(L_i v(t) + \gamma \left(-\beta L_i [q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t))] \right) \right) \end{aligned}$$

$$\begin{aligned}
 & + \text{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))^\alpha - \frac{1}{\gamma} v_i(t) + f(t, x_i(t), v_i(t)) \Big) \\
 & = \sum_{i=1}^n \beta \text{sig}(q_i(t) + \gamma p_i(t))^\alpha \gamma (-\beta L_i [q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)) \\
 & \quad + \text{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))]^\alpha + f(t, x_i(t), v_i(t))) \\
 & = - \sum_{i=1}^n \sum_{j \in N_i} \beta \text{sig}(q_i(t) + \gamma p_i(t))^\alpha \gamma l_{ij} \beta L_i [q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)) \\
 & \quad + \text{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))]^\alpha + f(t, x_i(t), v_i(t)). \tag{16}
 \end{aligned}$$

Denote

$$\begin{aligned}
 L_i &= [l_{i1}, l_{i2}, \dots, l_{in}], & e_{xi} &= [e_{x1}, e_{x2}, \dots, e_{xn}]^T, & e_{vi} &= [e_{v1}, e_{v2}, \dots, e_{vn}]^T, \\
 q_i &= [q_1, q_2, \dots, q_n]^T, & p_i &= [p_1, p_2, \dots, p_n]^T, & E_{xi} &= [E_{x1}, E_{x2}, \dots, E_{xn}]^T, \\
 E_{vi} &= [E_{v1}, E_{v2}, \dots, E_{vn}]^T.
 \end{aligned}$$

From Lemma 4

$$\begin{aligned}
 & [q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)) + \text{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))]^\alpha \\
 & \leq (q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))^\alpha + \text{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))^\alpha. \tag{17}
 \end{aligned}$$

Because of

$$\text{sig}(q_i(t) + \gamma p_i(t))^\alpha \leq |q_i(t) + \gamma p_i(t)|^\alpha, \tag{18}$$

we can obtain

$$\text{sig}(q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)))^\alpha \leq |q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t))|^\alpha. \tag{19}$$

For $i \neq j$, $l_{ij} < 0$, which means

$$\begin{aligned}
 \dot{V} & \leq - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta [|q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t))| \\
 & \quad + |q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t))|]^\alpha \\
 & \quad - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta f(t, x_i(t), v_i(t)) \\
 & \leq - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta 2^\alpha |q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t))|^\alpha \\
 & \quad - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta f(t, x_i(t), v_i(t)), \tag{20}
 \end{aligned}$$

define $\Phi_i(t) = q_i(t) + E_{xi}(t) + \gamma(p_i(t) + E_{vi}(t)) = q_i(t_k^i) + \gamma p_i(t_k^i)$, $\Phi_i = [\Phi_1^T, \Phi_2^T, \dots, \Phi_n^T]^T$.

Thus, according to the event-triggered function

$$\begin{aligned}
 |E_x(t) + \gamma E_v(t)| &\leq \|L\| |e_x(t) + \gamma e_v(t)| \\
 &= \sqrt{\|L\|^2 \sum_{i=1}^n |e_{ix}(t) + \gamma e_{iv}(t)|^2} \\
 &\leq \sqrt{\sigma^2 \sum_{i=1}^n |q_i(t_k^i) + \gamma p_i(t_k^i)|^2},
 \end{aligned} \tag{21}$$

we have

$$\begin{aligned}
 |\Phi_i(t)| &\leq |q_i(t) + \gamma p_i(t)| + |E_{xi}(t) + \gamma E_{vi}(t)| \\
 &\leq |q_i(t) + \gamma p_i(t)| + \sigma |\Phi_i(t)| \\
 &\leq \frac{1}{1-\sigma} |q_i(t) + \gamma p_i(t)|.
 \end{aligned} \tag{22}$$

According to Assumption 2, we can find that

$$\begin{aligned}
 \dot{V}(t) &\leq - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta 2^\alpha \left(\frac{1}{1-\sigma} |q_i(t) + \gamma p_i(t)| \right)^\alpha \\
 &\quad - \sum_{i=1}^N \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta \mu \\
 &\leq -\beta \gamma (|q(t) + \gamma p(t)|^\alpha)^T L \beta 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha (|q(t) + \gamma p(t)|^\alpha) \\
 &\quad - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta \mu.
 \end{aligned} \tag{23}$$

It is given by Lemma 3

$$\frac{(\beta |q_i(t) + \gamma p_i(t)|^\alpha)^T L (\beta |q_i(t) + \gamma p_i(t)|^\alpha)}{(\beta |q_i(t) + \gamma p_i(t)|^\alpha)^T (\beta |q_i(t) + \gamma p_i(t)|^\alpha)} \geq \min_{\varepsilon \in \Omega} \varepsilon^T M \varepsilon \triangleq \rho > 0,$$

and $c_i = |\sum_{j \in N_i} l_{ij}|$, $c_{\max} = \max_{i \in V} c_i$, $c_{\min} = \min_{i \in V} c_i$

$$\begin{aligned}
 \dot{V}(t) &\leq -\gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho \sum_{i=1}^n \beta^2 |q_i(t) + \gamma p_i(t)|^{2\alpha} + \gamma \mu \sum_{i=1}^n \beta^2 |q_i(t) + \gamma p_i(t)|^\alpha c_i \\
 &\leq -\gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho (1+\alpha)^{\frac{2\alpha}{1+\alpha}} \beta^{\frac{2}{1+\alpha}} \left(\sum_{i=1}^n \frac{\beta}{1+\alpha} |q_i(t) + \gamma p_i(t)|^{1+\alpha} \right)^{\frac{2\alpha}{1+\alpha}} \\
 &\quad + \gamma \mu \sum_{i=1}^n \left(\frac{\beta^2 |q_i(t) + \gamma p_i(t)|^{2\alpha}}{2} + \frac{c_i^2}{2} \right) \\
 &\leq -\gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho (1+\alpha)^{\frac{2\alpha}{1+\alpha}} \beta^{\frac{2}{1+\alpha}} \left(\sum_{i=1}^n \frac{\beta}{1+\alpha} |q_i(t) + \gamma p_i(t)|^{1+\alpha} \right)^{\frac{2\alpha}{1+\alpha}}
 \end{aligned}$$

$$\begin{aligned}
 & + \gamma \mu \sum_{i=1}^n \frac{\beta^2 |q_i(t) + \gamma p_i(t)|^{2\alpha}}{2} + \frac{\gamma \mu n c_{\max}^2}{2} \\
 & \leq -\gamma \left(2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho - \frac{\mu}{2} \right) (1 + \alpha)^{\frac{2\alpha}{1+\alpha}} \beta^{\frac{2}{1+\alpha}} \left(\sum_{i=1}^n \frac{\beta}{1 + \alpha} |q_i(t) + \gamma p_i(t)|^{1+\alpha} \right)^{\frac{2\alpha}{1+\alpha}} \\
 & \quad + \frac{\gamma \mu n c_{\max}^2}{2}, \tag{24}
 \end{aligned}$$

where $\eta = \gamma 2^\alpha \left(\frac{1}{1-\sigma}\right)^\alpha \rho (1 + \alpha)^{\frac{2\alpha}{1+\alpha}} \beta^{\frac{2}{1+\alpha}}$, and $\eta > 0$. Then we have

$$\dot{V}(t) \leq -\eta V(t)^{\frac{2\alpha}{1+\alpha}} + \frac{\gamma \mu n c_{\max}^2}{2} \tag{25}$$

and

$$T \leq \frac{V(0)^{1-\alpha}}{\eta(1-\alpha)}. \tag{26}$$

According to Definition 1, the second-order MAS (1) with control protocol (4) and event-triggered condition (13), the system can achieve finite-time consensus, and $w_i(t) = 0$.

When $w_i(t) \neq 0$, we will prove that MAS (1) has an H-infinity performance. □

Theorem 2 *Let Assumption 1 be satisfied and the undirected graph of MAS (1) be connected. With the event-triggered control algorithm (4) and the triggering function (13), if suitable positive scalars β and γ exist, then the finite-time H-infinity tacking consensus problem can be solved when the following conditions are satisfied:*

$$\begin{cases} \frac{1}{n} - \gamma 2^\alpha \left(\frac{1}{1-\sigma}\right)^\alpha < 0, \\ \frac{\beta^4 \gamma^2 c_{\min}}{4\delta^2} - \beta^2 \gamma \mu < 0. \end{cases} \tag{27}$$

Proof In view of the proof of Theorem 1, we have

$$\begin{aligned}
 \dot{V}(t) & \leq -\eta V(t)^{\frac{2\alpha}{1+\alpha}} - \sum_{i=1}^n \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma L_i \beta w_i(t) \\
 & \leq -\eta V(t)^{\frac{2\alpha}{1+\alpha}} - \sum_{i=1}^n \beta^2 \gamma \rho |q_i(t) + \gamma p_i(t)|^\alpha w_i(t). \tag{28}
 \end{aligned}$$

And

$$\begin{aligned}
 \sum_{i=1}^n z^T(t)z(t) & = \sum_{i=1}^n \frac{1}{n} [(x_i(t) - x_j(t) + \gamma(v_i(t) - v_j(t)))]^{2\alpha} \\
 & = \sum_{i=1}^n \frac{1}{n} [(q_i(t) + \gamma p_i(t))]^{2\alpha} \\
 & \leq \sum_{i=1}^n \frac{1}{n} |q_i(t) + \gamma p_i(t)|^{2\alpha}. \tag{29}
 \end{aligned}$$

Then we can obtain

$$\begin{aligned}
 & \int_0^T [z^T(t)z(t) - \delta^2 w^T(t)w(t) + \dot{V}(t)] dt \\
 &= \int_0^T \left[\sum_{i=1}^n \frac{1}{n} |q_i(t) + \gamma p_i(t)|^{2\alpha} - \delta^2 w^T(t)w(t) \right. \\
 &\quad \left. - \gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho \sum_{i=1}^n \beta^2 |q_i(t) + \gamma p_i(t)|^{2\alpha} \right. \\
 &\quad \left. - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta \mu - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta w_i(t) \right] dt \\
 &= \int_0^T \left[\left(\frac{1}{n} - \gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho \beta^2 \right) \sum_{i=1}^n |q_i(t) + \gamma p_i(t)|^{2\alpha} - \delta^2 w^T(t)w(t) \right. \\
 &\quad \left. - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta \mu - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta w_i(t) \right] dt \\
 &\leq \int_0^T \left[\left(\frac{1}{n} - \gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho \beta^2 \right) \sum_{i=1}^n |q_i(t) + \gamma p_i(t)|^{2\alpha} - \delta^2 \sum_{i=1}^n (w_i(t))^2 \right. \\
 &\quad \left. - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta \mu - \sum_{i=1}^n \sum_{j \in N_i} \beta |q_i(t) + \gamma p_i(t)|^\alpha \gamma l_{ij} \beta w_i(t) \right] dt \\
 &\leq \int_0^T \left[\left(\frac{1}{n} - \gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho \beta^2 \right) \sum_{i=1}^n |q_i(t) + \gamma p_i(t)|^{2\alpha} \right. \\
 &\quad \left. - \left[\delta \sum_{i=1}^n (w_i(t)) - \frac{\beta^2 \gamma c_{\min}}{2\delta} \sum_{i=1}^n |q_i(t) + \gamma p_i(t)|^{\frac{\alpha}{2}} \right]^2 \right. \\
 &\quad \left. + \left(\frac{\beta^4 \gamma^2 c_{\min}}{4\delta^2} - \beta^2 \gamma \mu \right) \sum_{i=1}^n \sum_{j \in N_i} l_{ij} |q_i(t) + \gamma p_i(t)|^\alpha \right] dt \tag{30}
 \end{aligned}$$

According to condition (27), we have

$$\int_0^T z^T(t)z(t) - \delta^2 w^T(t)w(t) + V(t) - V(0) dt < 0. \tag{31}$$

By $V(t) \geq 0, V(0) = 0$, we have $\int_0^T z^T(t)z(t) - \delta^2 w^T(t)w(t) dt < 0, \|z(t)\|_2^2 < \delta^2 \|w(t)\|_2^2$. Thus, from Definition 2, the multi-agent system (1) has an H-infinity performance level δ , and the proof of this theorem is completed. \square

Remark 2 Theorem 2 shows that, when the design parameter δ is closer to the optimal value δ_{opt} [34] of the H-infinity norm, the H-infinity control performance is better and the anti-interference is stronger. Our subsequent simulation results also verify this conclusion. The conclusion of Theorem 2 also indicates that, when the controller ensures consensus in the MAS, the selected control gains β and γ are larger and the performance of H-infinite control is better. However, when the control gain is greater, the cost of control is higher.

Corollary 1 *The following MAS is considered:*

$$\begin{cases} \dot{x}_i = v_i(t), \\ \dot{v}_i = u_i(t), \end{cases} \quad i = 1, \dots, n, \tag{32}$$

with the event-triggered control algorithm (4) and the triggering function (13), if there exist suitable positive scalars β and γ , the finite-time consensus problem can be solved when the following conditions are satisfied:

$$0 < \gamma 2^\alpha \left(\frac{1}{1-\sigma} \right)^\alpha \rho (1+\alpha)^{\frac{2\alpha}{1+\alpha}} \beta^{\frac{2}{1+\alpha}},$$

where $\gamma > 0, \beta > 0, 0 < \alpha < 1, 0 < \sigma < 1$.

In addition, the finite time T can be estimated using the following inequality:

$$T \leq \frac{V(0)^{1-\alpha}}{\eta(1-\alpha)},$$

where $V(0) = \sum_{i=1}^n \frac{\beta}{1+\alpha} |q_i(0) + \gamma p_i(0)|^{1+\alpha}$.

Remark 3 In [24], the consensus control problem was studied for a nonlinear second-order MAS based on the event-triggered mechanism. The current study additionally considers external disturbances. The author of [28] investigated the finite-time consensus of second-order MASs with disturbances. However, the disturbances were limited to positive numbers, which brings difficulty in achieving accurate control. In [31], a first-order MAS was studied with disturbance for finite-time H-infinity consensus, but this study ignored event-triggered strategy control.

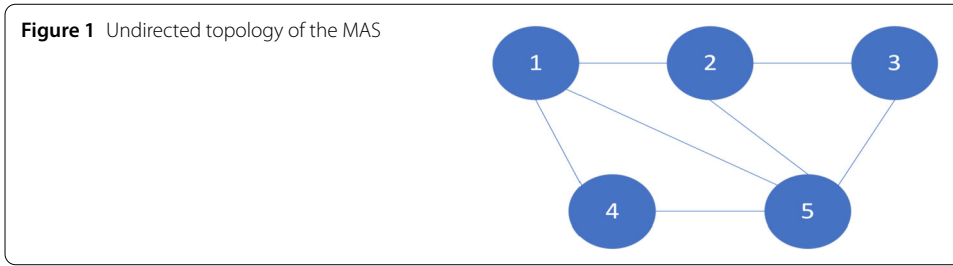
Theorem 3 *MAS (1) with an event-triggered function (13) and control strategy (4) is considered. A positive lower bound T_{\min} of the event-triggered execution interval is given as follows:*

$$T_{\min} = \frac{\sigma |q_i(t_k^i) + \gamma p_i(t_k^i)|}{\|L\|(|v_i| + \gamma(|\beta(q_i(t_k^i) + p_i(t_k^i)) + \text{sig}(q_i(t_k^i) + p_i(t_k^i))| + \mu + w))}.$$

Thus, each agent i can avoid Zeno behavior before consensus is achieved.

Proof At $t = t_k^i$, the controller of agent i updates its control output. Thus, the measurement error is set to 0, that is, $|e_{ix}(t_k^i)| = 0, |e_{iv}(t_k^i)| = 0$. During the interval $[t_k^i, t_{k+1}^i)$, we have

$$\begin{aligned} & \frac{d}{dt} |e_{ix}(t) + \gamma e_{iv}(t)| \\ & \leq \left| \frac{d}{dt} e_{ix}(t) + \gamma e_{iv}(t) \right| \\ & = |v_i(t) + \gamma \dot{v}_i(t)| \\ & \leq |v_i(t)| + \gamma (|\beta(q_i(t_k^i) + p_i(t_k^i)) + \text{sig}(q_i(t_k^i) + p_i(t_k^i))| + \mu + d), \tag{33} \\ & |e_{ix}(t) + \gamma e_{iv}(t)| \end{aligned}$$



$$\begin{aligned}
 &= \int_{t_k^i}^t \frac{d}{dt} |e_{ix}(t) + \gamma e_{iv}(t)| ds \\
 &\leq \int_{t_k^i}^t (|v_i| + \gamma (|\beta(q_i(t_k^i) + p_i(t_k^i)) + \text{sig}(q_i(t_k^i) + p_i(t_k^i))| + \mu + d)) ds \\
 &= (|v_i| + \gamma (|\beta(q_i(t_k^i) + p_i(t_k^i)) + \text{sig}(q_i(t_k^i) + p_i(t_k^i))| + \mu + d))(t - t_k^i). \tag{34}
 \end{aligned}$$

When the event is triggered, we have

$$|e_{xi} + \gamma e_{vi}(t)| > \frac{\sigma}{\|L\|} |q_i(t_k^i) + \gamma p_i(t_k^i)|. \tag{35}$$

Theorem 1 indicates that $|q_i(t_k^i) + \gamma p_i(t_k^i)| > 0$ before the system trajectory reaches consensus. Thus, we have

$$(t_{k+1}^i - t_k^i) > \frac{\sigma |q_i(t_k^i) + \gamma p_i(t_k^i)|}{\|L\|(|v_i| + \gamma (|\beta(q_i(t_k^i) + p_i(t_k^i)) + \text{sig}(q_i(t_k^i) + p_i(t_k^i))| + \mu + d))} > 0. \tag{36}$$

We can conclude that $t_{k+1}^i - t_k^i > 0$ before consensus is achieved. In turn $t_{k+2}^i - t_{k+1}^i > 0$. Thus, the Zeno behavior can be excluded. □

Remark 4 At the current moment t_k^i , given that the agents have not yet achieved consensus, $q_i(t_k^i)$ and $p_i(t_k^i)$ are not equal to 0. Obviously, they are certain constants, and the next triggering time t_k^i is determined by constants such as $q_i(t_k^i)$ and $p_i(t_k^i)$. Theorem 3 proves that the size of $(t_{k+1}^i - t_k^i)$ satisfies formula (35). Obviously, the right-hand side of formula (35) is a certain constant. Therefore, Zeno behavior can be avoided.

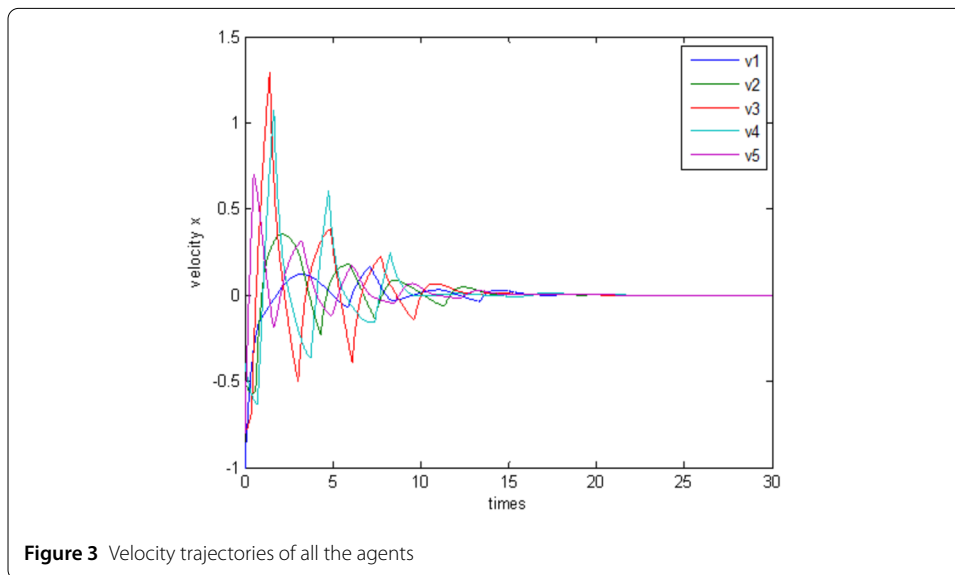
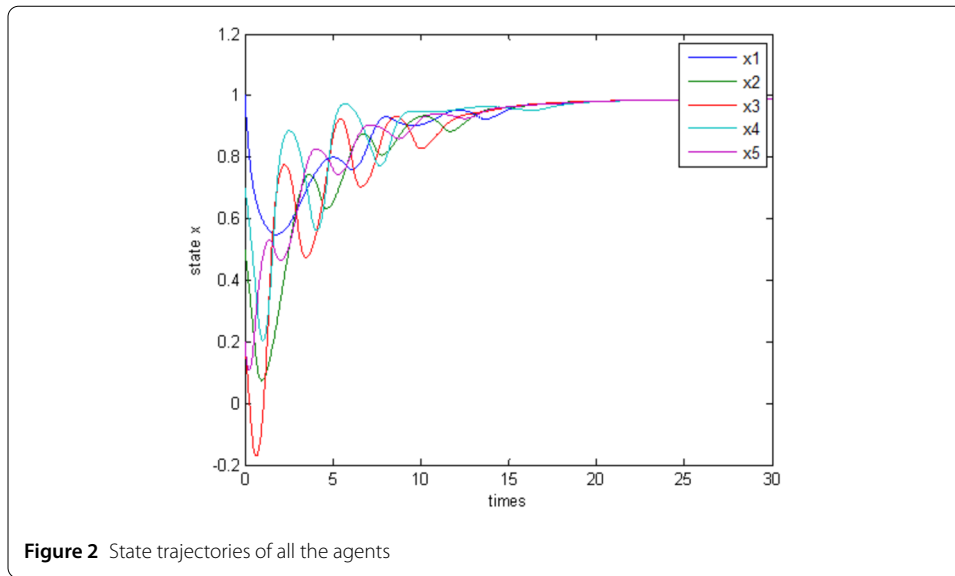
4 Numerical simulation

This section presents a numerical example to verify the theoretical results. Figure 1 shows the undirected connection topology of a MAS with five nodes.

The Laplacian matrix L is

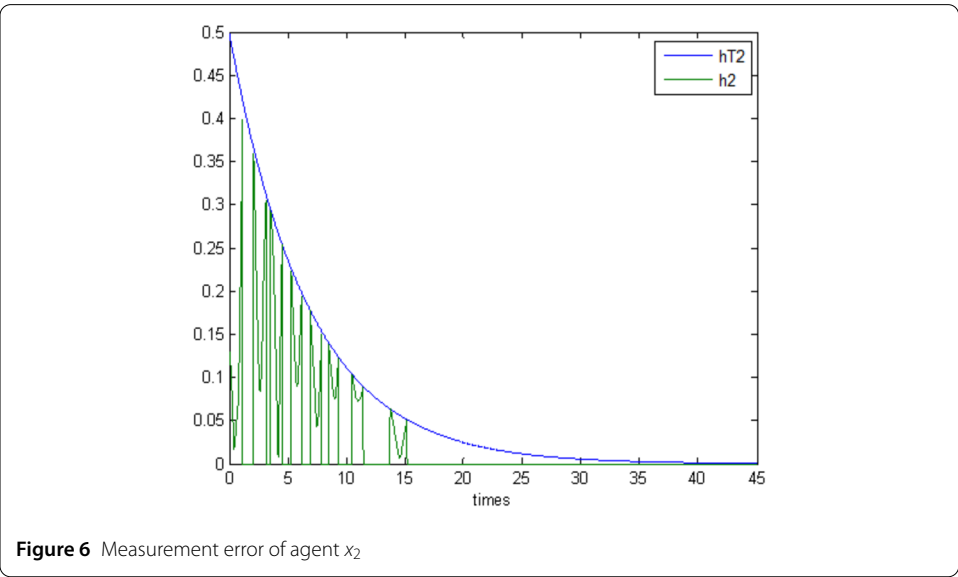
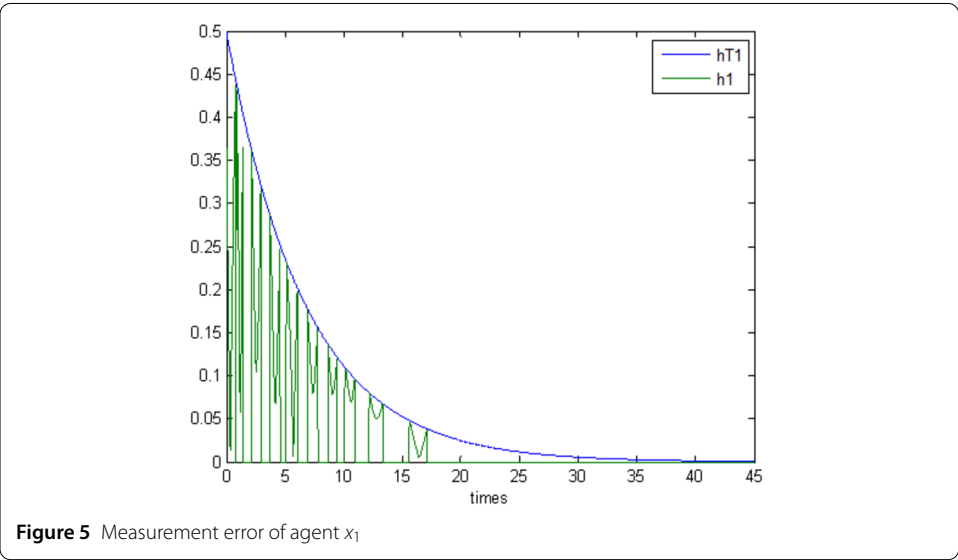
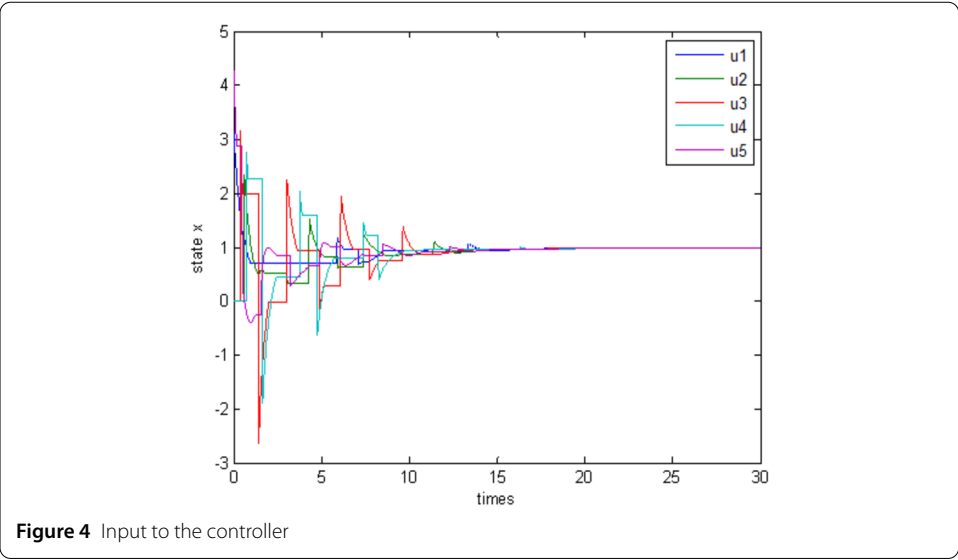
$$L = \begin{bmatrix} 3 & -1 & 0 & -1 & -1 \\ -1 & 3 & -1 & 0 & -1 \\ 0 & -1 & 2 & 0 & -1 \\ -1 & 0 & 0 & 2 & -1 \\ -1 & -1 & -1 & -1 & 4 \end{bmatrix}.$$

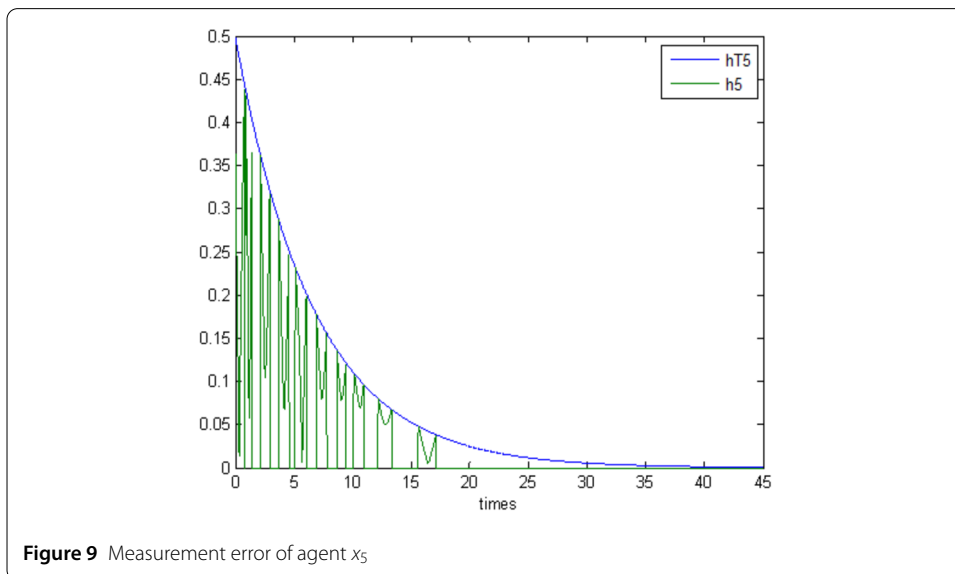
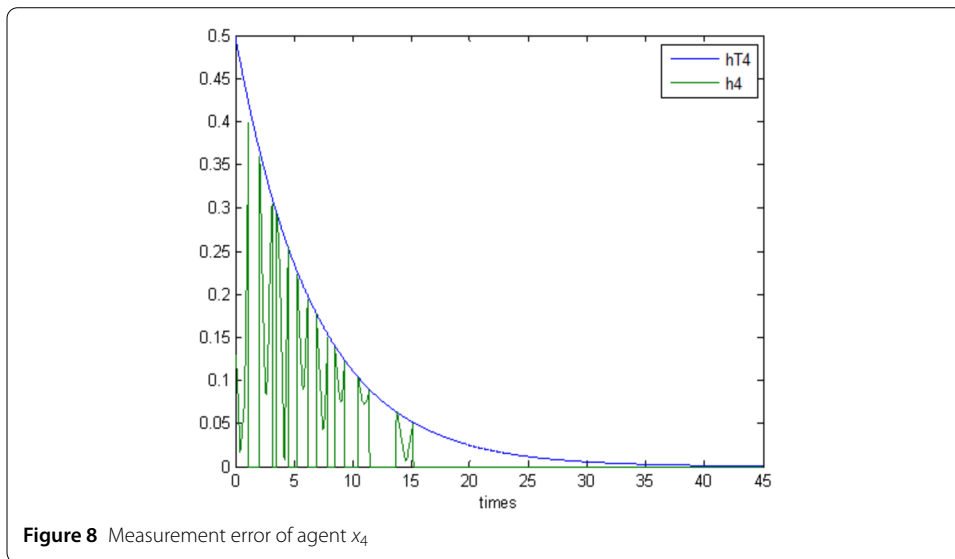
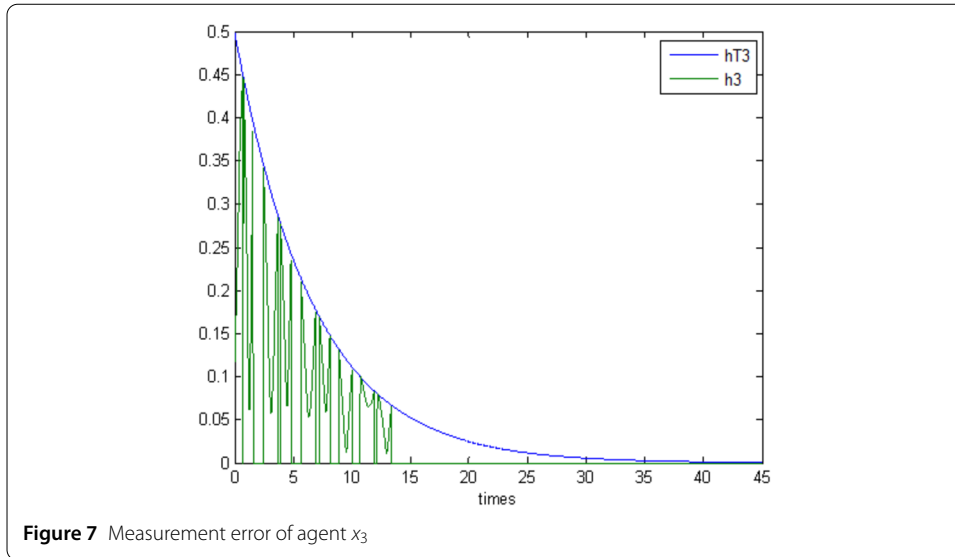
We set $\alpha = 0.1, \beta = 0.5, \gamma = 0.3, \sigma = 0.5$, and $D = \text{diag}([3 \ 3 \ 2 \ 2 \ 4])$. To satisfy the conditions of Theorem 1, we set the initial state of system (1) to $x_1(0) = [1, -1]^T, x_2(0) = [0.5, -0.5]^T$,

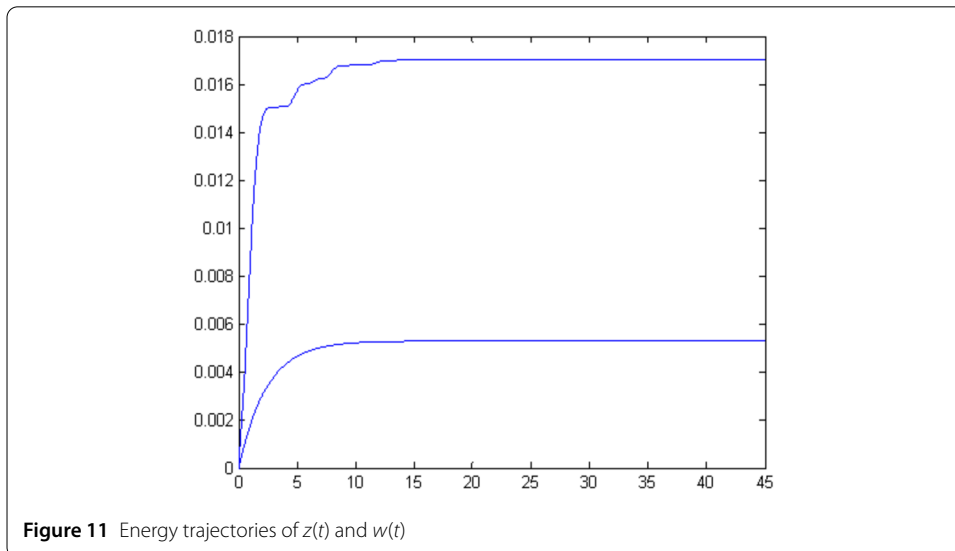
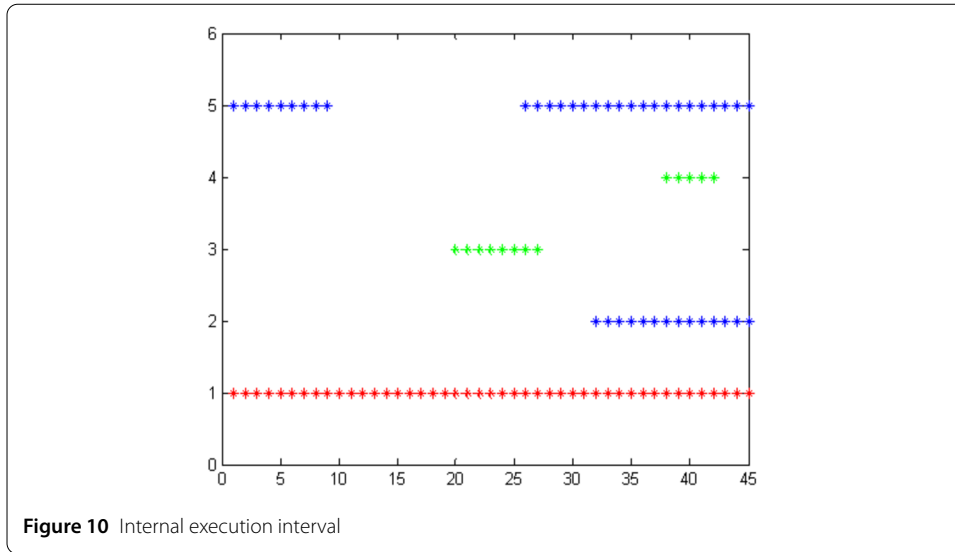


$x_3(0) = [0.2, -0.8]^T$, $x_4(0) = [0.7, -0.4]^T$, and $x_5(0) = [0.2, -0.8]^T$. The initial value of velocity for the five agents is set to 0. The simulation results of the system are given below.

Figures 2 and 3 represent the state and velocity trajectory diagrams of all the agents under controller (4). The figures show that all agents can achieve consensus under the event-triggered control strategy. As shown in Fig. 2, each agent's controller updates at its own event time only and remains unchanged during the triggering interval. Figure 4 shows the input to the distributed controller. Figures 5–9 show the measurement error of agents x_1 , x_2 , x_3 , x_4 , and x_5 under the event-triggered control strategy in finite time. This figure indicates that the system error converges to 0 in finite time. Thus, the system can achieve consensus in finite time. Figure 10 shows the event triggering interval of each agent under the event-triggered strategy (13). Figure 11 shows that the energy of the output signal $z(t)$ is smaller than that of the external disturbance $w(t)$. The results of the numerical simulation







verify the validity of the conclusion. The designed controller and algorithm can ensure consensus in a MAS in finite time.

5 Conclusion

Event-triggered finite-time H-infinity consensus has been studied for second-order multi-agent nonlinear systems with external disturbances. An event-triggered strategy has been introduced to save communication resources. The data can be sampled by the system only when the event-triggered condition is satisfied. A sufficient condition on finite-time consensus has been obtained by employing the Lyapunov method and analysis technology. A theoretical analysis proves that the designed finite-time controller can suppress the influence of disturbances on the system and satisfy the robust H-infinity performance. The analysis also proves that the system has good anti-interference performance under external disturbance. Moreover, the Zeno behavior can be avoided given that a positive lower bound of the event-triggered execution interval is ensured. The validity of the proposed

method has also been verified by numerical simulation. In future research, we will guarantee cost finite-time consensus of second-order uncertain MAS based on distributed event-triggered strategy.

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Availability of data and materials

The data used to support the findings of this study are available from the corresponding author upon request.

Competing interests

The authors declare that they have no competing interests.

Authors' contributions

Both authors contributed equally to the writing of this paper. Furthermore, both authors also read carefully and approved the final manuscript.

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