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Dynamical analysis of a stochastic three-species predator–prey system with distributed delays

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Abstract

A stochastic two-prey one-predator system with distributed delays is proposed in this paper. Firstly, applying the linear chain technique, we transform the predator–prey system with distributed delays to an equivalent system with no delays. Then, by use of the comparison method and the inequality technique, we investigate the stability in mean and extinction of species. Further, by constructing some suitable functionals, using M-matrix theory and three important lemmas, we establish sufficient conditions assuring the existence of distribution and the attractivity of solutions. Finally, some numerical simulations are given to illustrate the main results.

MSC: Primary 34A37; secondary 92B05

Keywords: Distributed delay; Stability in mean; Distribution; Simulation

1 Introduction

Predator–prey system is very popular in the world. In order to reveal the dynamical relationship between predator and prey, a lot of predator–prey systems have been widely investigated and many good results have been obtained in the last decades, which has long been one of the hot topics in ecology [1–3]. Since two-species ecological models cannot describe the natural phenomena accurately and many vital behaviors can only be exhibited by systems with three or more species, for example, in the natural world, the predator often feeds on some competing prey, and hence, a three- or multi-species population system attracts more and more attention [4–7].

On the other hand, all species are inevitably affected by environmental noise. To better describe ecological phenomena, the white noise is introduced into a predator–prey model to reveal richer and more complex dynamics [8–15]. There are many kinds of stochastic perturbation. Considering the stochastic influence on the intrinsic growth rates of populations, we have $a_i \to a_i + \xi_i \, d\omega(t)$, where $\omega(t)$ is a standard Brownian motion defined on a complete probability space $(\Omega, \mathcal{F}, \mathcal{P})$ with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$, ξ^2 is the intensity of white noise. For example, Liu [6] proposed the following three-species predator–prey model:

$$\begin{cases} dN_1(t) = N_1(t)(a_1 - d_{11}N_1(t) - d_{12}N_2(t) - d_{13}N_3(t))\,dt + \xi_1 N_1(t)\,d\omega(t), \\ dN_2(t) = N_2(t)(a_2 - d_{21}N_1(t) - d_{22}N_2(t) - d_{23}N_3(t))\,dt + \xi_2 N_2(t)\,d\omega(t), \\ dN_3(t) = N_3(t)(a_3 - d_{31}N_1(t) - d_{32}N_2(t) - d_{33}N_3(t))\,dt + \xi_3 N_3(t)\,d\omega(t), \end{cases}$$

where $N_1(t)$ and $N_2(t)$ are the population sizes of prey species, $N_3(t)$ is the population size of predator species, $a_i > 0$ $(i = 1, 2)$ are the intrinsic rates of increase, $a_3 < 0$ is the intrinsic rate of decrease, $d_{12} > 0$ and $d_{21} > 0$ are the parameters representing competitive effects between two prey, $d_{13} > 0$ and $d_{23} > 0$ are the coefficients of decrease of prey species due to predation, $d_{31} < 0$ and $d_{32} < 0$ are the predation rate of predator, $d_{ii} > 0$ $(i = 1, 2, 3)$ are the rate of competition within the same species.

As we know, predator–prey interaction is a frequently observed phenomenon. Almost all species should exhibit some delays. Considering the inevitability, more and more researchers have taken delay into an ecological model and obtained some nice results [16–19]. Recently, infinite delay has been widely introduced into the ecological model since the works of Volterra to translate the cumulative effect of the past history of a system [20–24]. Chen [22] *et al.* proposed the following model with distributed delays:

$$\begin{cases} dN_1(t)/dt = b_1 N_1(t)(1 - \frac{N_1(t)}{K}) - a_{12}N_1(t)N_2(t), \\ dN_2(t)/dt = -b_2 N_2(t) + a_{21}\int_{-\infty}^{t} K(t-s)N_1(s)N_2(s)\,ds, \end{cases}$$

where the kernel $K : [0,\infty) \to [0,\infty)$ is a normalized L^1 function such that $\int_0^\infty K(s)\,ds = 1$. For distributed delay, MacDonald [25] initially proposed that it was reasonable to use gamma distribution as a kernel function, that is, $f(t) = \frac{t^n \sigma^{n+1} e^{-\sigma t}}{n!}$, where $\sigma > 0$, n is a non-negative integer. If $n = 0$, then the kernel $f(t) = \sigma e^{-\sigma t}$ is called a weak kernel, otherwise it is called a strong kernel.

Motivated by the above discussion, in this paper, we consider a stochastic two-prey one-predator system with distributed delays. For convenience, we mainly consider the weak kernel case, *i.e.*, $f(t) = \sigma e^{-\sigma t}$. Our model is as follows:

$$\begin{cases} dN_1(t) = N_1(t)(r_1 - a_{11}N_1(t) - a_{12}\int_{-\infty}^{t} \sigma_2 e^{-\sigma_2(t-s)}N_2(s)\,ds \\ \qquad - a_{13}\int_{-\infty}^{t} \sigma_3 e^{-\sigma_3(t-s)}N_3(s)\,ds)\,dt + \xi_1 N_1(t)\,d\omega_1(t), \\ dN_2(t) = N_2(t^-)(r_2 - a_{21}\int_{-\infty}^{t} \sigma_1 e^{-\sigma_1(t-s)}N_1(s)\,ds - a_{22}N_2(t) \\ \qquad - a_{23}\int_{-\infty}^{t} \sigma_3 e^{-\sigma_3(t-s)}N_3(s)\,ds)\,dt + \xi_2 N_2(t)\,d\omega_2(t), \\ dN_3(t) = N_3(t)(-r_3 + a_{31}\int_{-\infty}^{t} \sigma_1 e^{-\sigma_1(t-s)}N_1(s)\,ds - a_{33}N_3(t) \\ \qquad + a_{32}\int_{-\infty}^{t} \sigma_2 e^{-\sigma_2(t-s)}N_2(s)\,ds)\,dt + \xi_3 N_3(t)\,d\omega_3(t), \end{cases} \tag{1.1}$$

with the initial data

$$N_i(\theta) = \varphi_i(\theta) \in C((-\infty, 0], R_+), \quad i = 1, 2, 3,$$

where $C((-\infty,0], R_+)$ is the set of all continuous functions from $(-\infty, 0)$ to $R_+ = (0,\infty)$, $\omega_i(t)$ $(i = 1, 2, 3)$ is a standard and independent Brownian motion defined as above. All parameters are positive constants and their biological meanings refer to [6].

Define

$$y_i(t) = \int_{-\infty}^{t} \sigma_i e^{-\sigma_i(t-s)} N_i(s)\,ds, \quad i = 1,2,3.$$

Computing the derivative of $y_i(t)$, then $dy_i(t) = \sigma_i(N_i(t) - y_i(t))\,dt$, $i = 1,2,3$. Using the linear chain technique to (1.1) yields

$$\begin{cases} dN_1(t) = N_1(t)(r_1 - a_{11}N_1(t) - a_{12}y_2(t) - a_{13}y_3(t))\,dt + \xi_1 N_1(t)\,d\omega_1(t), \\ dN_2(t) = N_2(t)(r_2 - a_{21}y_1(t) - a_{22}N_2(t) - a_{23}y_3(t))\,dt + \xi_2 N_2(t)\,d\omega_2(t), \\ dN_3(t) = N_3(t)(-r_3 + a_{31}y_1(t) + a_{32}y_2(t) - a_{33}N_3(t))\,dt + \xi_3 N_3(t)\,d\omega_3(t), \\ dy_1(t) = \sigma_1(N_1(t) - y_1(t))\,dt, \\ dy_2(t) = \sigma_2(N_2(t) - y_2(t))\,dt, \\ dy_3(t) = \sigma_3(N_3(t) - y_3(t))\,dt. \end{cases} \tag{1.2}$$

According to the equivalent property of (1.1) and (1.2), in what follows, we mainly consider (1.2) to reveal the dynamical properties of (1.1). Our main aims are as follows.

Firstly, we study the stability in mean and extinction of all species of (1.2), which have long been and will still be two important topics for the study of stochastic population systems.

Secondly, for a stochastic population system, instead of the positive equilibrium state of the determinate system, it is interesting and important to study the existence and uniqueness of the distribution of (1.2).

The rest work of this paper is organized as follows. Section 2 begins with some notations, definitions, and important lemmas. Section 3 focuses on the stability in mean and extinction of species of (1.2). Section 4 is devoted to the existence and uniqueness of distribution. Some numerical simulations are given in Sect. 5. Finally, we conclude the paper with a brief conclusion and discussion in Sect. 6.

2 Preliminaries

For simplicity, we give the following notations.

$$\begin{aligned}
&\alpha_1 = (a_{11}, a_{21}, -a_{31})^T, \qquad \alpha_2 = (a_{12}, a_{22}, -a_{32})^T, \qquad \alpha_3 = (a_{13}, a_{23}, a_{33})^T, \\
&r = (r_1, r_2, -r_3)^T, \qquad \xi = (\xi_1^2/2, \xi_2^2/2, \xi_3^2/2), \qquad A = \mathtt{det}(\alpha_1, \alpha_2, \alpha_3), \\
&b_1 = r_1 - \xi_1^2/2, \qquad b_2 = r_2 - \xi_2^2/2, \qquad b_3 = -r_3 - \xi_3^2/2, \\
&A_1 = \mathtt{det}(r, \alpha_2, \alpha_3), \qquad A_2 = \mathtt{det}(\alpha_1, r, \alpha_3), \qquad A_3 = \mathtt{det}(\alpha_1, \alpha_2, r), \\
&\tilde{A}_1 = \mathtt{det}(\xi, \alpha_2, \alpha_3), \qquad \tilde{A}_2 = \mathtt{det}(\alpha_1, \xi, \alpha_3), \qquad \tilde{A}_3 = \mathtt{det}(\alpha_1, \alpha_2, \xi), \\
&\Delta_1 = r_2 a_{32} + r_3 a_{22}, \qquad \Delta_2 = r_1 a_{31} + r_3 a_{11}, \qquad \Delta_3 = -r_1 a_{21} + r_2 a_{11}, \\
&\tilde{\Delta}_1 = \frac{\xi_2^2}{2} a_{32} + \frac{\xi_3^2}{2} a_{22}, \qquad \tilde{\Delta}_2 = \frac{\xi_1^2}{2} a_{31} + \frac{\xi_3^2}{2} a_{11}, \qquad \tilde{\Delta}_3 = -\frac{\xi_1^2}{2} a_{21} + \frac{\xi_2^2}{2} a_{11}, \\
&\Delta_1^* = r_2 a_{33} - r_3 a_{23}, \qquad \Delta_2^* = r_1 a_{33} - r_3 a_{13}, \qquad \Delta_3^* = r_1 a_{22} - r_2 a_{12}, \\
&\tilde{\Delta}_1^* = \frac{\xi_2^2}{2} a_{33} - \frac{\xi_3^2}{2} a_{23}, \qquad \tilde{\Delta}_2^* = \frac{\xi_1^2}{2} a_{33} - \frac{\xi_3^2}{2} a_{13}, \qquad \tilde{\Delta}_3^* = \frac{\xi_1^2}{2} a_{22} - \frac{\xi_2^2}{2} a_{12}.
\end{aligned}$$

Throughout this paper, we denote the complement minor of a_{ij} in determinant A by A_{ij} $(i,j = 1,2,3)$, and assume that $A > 0$, $A_i > 0$, *i.e.*, when there is no stochastic perturbation, a positive equilibrium state exists for model (1.1). Further, for convenience, we always assume that K stands for a generic positive constant whose value may be different at different places. And for any function $x(t)$, $t > 0$, we denote

$$\langle x(t)\rangle = t^{-1}\int_0^t x(s)\,ds, \qquad x^* = \limsup_{t\to\infty} x(t), \qquad x_* = \liminf_{t\to\infty} x(t).$$

Now we give assumptions, definitions, and some important lemmas, which are used in our main proof.

Assumption 2.1 $A_{13} > 0, A_{23} < 0, A_{33} > 0, A_{31} < 0, A_{32} > 0$.

Assumption 2.2 $a_{ii} > \sum_{j=1,j\neq i}^{3} a_{ji}$, $i,j = 1,2,3$.

Remark 2.1 Assumption 2.2 means that the intra-specific competitive rates are stronger than the interaction competitive rates or predation rates among different species.

Definition 2.1 Let $P(t) = (N_1(t),N_2(t),N_3(t),y_1(t),y_2(t),y_3(t))^T \in C((-\infty,0],R_+^6)$ be a solution of system (1.2), then

(I) The population $P(t)$ is said to be extinct if $\lim_{t\to\infty} P(t) = 0$;

(II) The population $P(t)$ is said to be stable in mean if $\lim_{t\to\infty}\langle P(t)\rangle = K$, *a.s.*, where K is a constant.

Definition 2.2 Let $P(t) = (N_1(t),N_2(t),N_3(t),y_1(t),y_2(t),y_3(t))^T \in C((-\infty,0],R_+^6)$ and $\bar{P}(t) = (\bar{N}_1(t),\bar{N}_2(t),\bar{N}_3(t),\bar{y}_1(t),\bar{y}_2(t),\bar{y}_3(t))^T \in C((-\infty,0],R_+^6)$ be any two positive solutions of (1.2) with the initial value $P(0) > 0$, $\bar{P}(0) > 0$, then system (1.2) is said to be globally attractive if

$$\lim_{t\to\infty}\left|N_i(t) - \bar{N}_i(t)\right| = 0, \qquad \lim_{t\to\infty}\left|y_i(t) - \bar{y}_i(t)\right| = 0, \quad i = 1,2,3.$$

Lemma 2.1 ([26]) *Suppose that* $Z(t) \in C[\Omega \times [0,+\infty),R_+]$ *and* $\lim_{t\to\infty} F(t)/t = 0$, *a.s.*

(a) *If there exist two positive constants* $T > 0$, $\lambda_0 > 0$ *such that, for all* $t > T$,

$$\ln Z(t) \le \lambda t - \lambda_0\int_0^t z(s)\,ds + F(t), \quad a.s.,$$

$$\text{then} \quad \begin{cases} \langle Z\rangle^* \le \lambda/\lambda_0, & a.s., \text{ if } \lambda \ge 0, \\ \lim_{t\to+\infty} Z(t) = 0, & a.s., \text{ if } \lambda < 0. \end{cases}$$

(b) *If there exist some constants* $T > 0$, $\lambda_0 > 0$, λ *such that, for all* $t > T$,

$$\ln Z(t) \ge \lambda t - \lambda_0\int_0^t z(s)\,ds + F(t), \quad a.s.,$$

$$\text{then} \quad \langle Z\rangle_* \ge \lambda/\lambda_0, \quad a.s.$$

Lemma 2.2 *System* (1.2) *has a unique solution* $P(t) = (N_1(t),N_2(t),N_3(t),y_1(t),y_2(t),y_3(t))^T \in C((-\infty,0],R_+^6)$ *for any given initial data* $P(t_0) \in C((-\infty,0],R_+^6)$, *almost surely.*

Proof The proof is standard. For the readers' convenience, we give the proof in Appendix A.1.

As to the expectation boundedness and asymptotical properties of the solution of (1.2), we have the following lemma. The proof is similar to that of references [20, 27, 28] and is presented in Appendix A.2. □

Lemma 2.3 *Let* $P(t) = (N_1(t), N_2(t), N_3(t), y_1(t), y_2(t), y_3(t))^T$ *be the solution of* (1.2), *then for any initial data* $P(t_0) \in C((-\infty, 0], R^6_+)$, *there exists a positive constant* $K(p)$ *such that*

$$\limsup_{t\to+\infty} \mathbb{E}\big(N_i(t)^p\big) \le K(p), \qquad \limsup_{t\to+\infty} \mathbb{E}\big(y_i(t)^p\big) \le K(p),$$

further,

$$\lim_{t\to+\infty} \frac{y_i(t)}{t} = 0, \qquad \limsup_{t\to+\infty} \frac{\ln N_i(t)}{t} \le 0, \quad a.s., i = 1, 2, 3.$$

For the following integral equation

$$x(t) = x(t_0) + \int_{t_0}^{t} a\big(s, x(s)\big)\, ds + \sum_{r=1}^{k} \int_{t_0}^{t} b_r\big(s, x(s)\big)\, d\sigma_r(s), \tag{2.1}$$

there is a result as follows.

Lemma 2.4 ([29]) *Suppose that the coefficients of* (2.1) *are independent of t and satisfy*:

$$\big|a(s,x) - a(s,y)\big| + \sum_{r=1}^{k} \big|b_r(s,x) - b_r(s,y)\big| \le K|x-y|,$$

$$\big|a(s,x)\big| + \sum_{r=1}^{k} \big|b_r(s,x)\big| \le K\big(1 + |x|\big),$$

in U_R *for any* $R > 0$, *and there exists a nonnegative* C^2 *function* $V(x)$ *in* R^l *such that*

$$LV(x) \le -1 \tag{2.2}$$

outside some compact set, then system (2.1) *has a solution, which is a stationary Markov process.*

Lemma 2.5 ([30]) *Let* $f(t)$ *be a nonnegative function defined on* $[0, +\infty)$ *such that* $f(t)$ *is integrable on* $[0, +\infty)$ *and is uniformly continuous on* $[0, +\infty)$, *then* $\lim_{t\to\infty} f(t) = 0$.

Lemma 2.6 *Let* $P(t) = (N_1(t), N_2(t), N_3(t), y_1(t), y_2(t), y_3(t))^T$ *be a solution of* (1.2) *with the initial value* $P(0) > 0$, *then almost every sample path of* $P(t)$ *is uniformly continuous on* $t \ge 0$.

Proof For the first equation of (1.2), it is equivalent to the following stochastic integral equation:

$$N_1(s) = N_1(0) + \int_0^t N_1(s)\big(r_1 - a_{11}N_1(s) + a_{12}y_2(s) + a_{13}y_3(s)\big)\, ds + \xi_1 \int_0^t N_1(s)\, d\omega_1(s).$$

By computation, we have

$$\begin{aligned}
&\mathbb{E}\big|N_1(s)\big(r_1 - a_{11}N_1(s) + a_{12}y_2(s) + a_{13}y_3(s)\big)\big|^p \\
&\quad \le \mathbb{E}|N_1|^{2p}/2 + \mathbb{E}\big|r_1 - a_{11}N_1(s) + a_{12}y_2(s) + a_{13}y_3(s)\big|^{2p}/2 \\
&\quad \le \mathbb{E}|N_1|^{2p}/2 + 3^{2p-1}\big(r_1^{2p} + a_{11}\mathbb{E}N_1^{2p} + a_{12}\mathbb{E}y_2^{2p} + a_{13}\mathbb{E}y_3^{2p}\big)/2 \\
&\quad \le K.
\end{aligned}$$

Using the moment inequality for stochastic integrals, for any $0 \le t_1 \le t_2$, $p > 2$, we have

$$\begin{aligned}
\mathbb{E}\left|\int_{t_1}^{t_2} \xi_1 N_1(s)\,d\omega_1(s)\right|^p &\le \sigma_1^p \left(\frac{p(p-1)}{2}\right)^{\frac{p}{2}} (t_2 - t_1)^{\frac{p-2}{2}} \int_{t_1}^{t_2} E\big|N_1(s)\big|^p\,ds \\
&\le \sigma_1^p \left(\frac{p(p-1)}{2}\right)^{\frac{p}{2}} (t_2 - t_1)^{\frac{p-2}{2}} K.
\end{aligned}$$

In the same manner, we can discuss the following five equations of (1.2) and obtain similar inequalities as above. Therefore, by Lemma 2.4 of Refs. [31], we conclude that almost every sample path of $P(t)$ is uniformly continuous. The proof is completed. □

3 Stability in mean and extinction of species

Firstly, we give the following result on stability in mean and extinction of species of model (1.2).

Theorem 3.1 *If Assumptions 2.1 and 2.2 hold, then for system (1.2), we have:*

(i) *If* $b_1 < 0$, $b_2 < 0$, *then* $\lim_{t\to\infty} N_i(t) = 0$, $i = 1, 2, 3$;

(ii) *If* $b_1 < 0$, $b_2 > 0$, $\Delta_1 < \tilde{\Delta}_1$, *then*

$$\lim_{t\to\infty} N_1(t) = 0, \qquad \lim_{t\to\infty}\big\langle N_2(t)\big\rangle = \frac{b_2}{a_{22}}, \qquad \lim_{t\to\infty} N_3(t) = 0;$$

If $b_1 < 0$, $b_2 > 0$, $\Delta_1 > \tilde{\Delta}_1$, *then*

$$\lim_{t\to\infty} N_1(t) = 0, \qquad \lim_{t\to\infty}\big\langle N_2(t)\big\rangle = \frac{\Delta_1^* - \tilde{\Delta}_1^*}{A_{11}}, \qquad \lim_{t\to\infty}\big\langle N_3(t)\big\rangle = \frac{\Delta_1 - \tilde{\Delta}_1}{A_{11}};$$

(iii) *If* $b_1 > 0$, $b_2 < 0$, $\Delta_2 < \tilde{\Delta}_2$, *then*

$$\lim_{t\to\infty}\big\langle N_1(t)\big\rangle = \frac{b_1}{a_{11}}, \qquad \lim_{t\to\infty} N_2(t) = 0, \qquad \lim_{t\to\infty}\big\langle N_3(t)\big\rangle = 0;$$

If $b_1 > 0$, $b_2 < 0$, $\Delta_2 > \tilde{\Delta}_2$, *then*

$$\lim_{t\to\infty} N_2(t) = 0, \qquad \lim_{t\to\infty}\big\langle N_1(t)\big\rangle = \frac{\Delta_2^* - \tilde{\Delta}_2^*}{A_{22}}, \qquad \lim_{t\to\infty}\big\langle N_3(t)\big\rangle = \frac{\Delta_2 - \tilde{\Delta}_2}{A_{22}};$$

(iv) *If* $b_1 > 0$, $b_2 > 0$, $A_i > \tilde{A}_i$ $(i = 1, 2, 3)$, *then*

$$\lim_{t\to\infty}\big\langle N_1(t)\big\rangle = \frac{A_1 - \tilde{A}_1}{A}, \qquad \lim_{t\to\infty}\big\langle N_2(t)\big\rangle = \frac{A_2 - \tilde{A}_2}{A}, \qquad \lim_{t\to\infty}\big\langle N_3(t)\big\rangle = \frac{A_3 - \tilde{A}_3}{A};$$

If $b_1 > 0$, $b_2 > 0$, $A_3 < \tilde{A}_3$, $\Delta_3 > \tilde{\Delta}_3$, *then*

$$\lim_{t\to\infty}\langle N_3(t)\rangle = 0, \qquad \lim_{t\to\infty}\langle N_1(t)\rangle = \frac{\Delta_3^* - \tilde{\Delta}_3^*}{A_{33}}, \qquad \lim_{t\to\infty}\langle N_2(t)\rangle = \frac{\Delta_3 - \tilde{\Delta}_3}{A_{33}};$$

Proof For (1.2), integrating the forth to the sixth equations from 0 to t leads to

$$\frac{y_i(t) - y_i(0)}{t} = \sigma_i\bigl(\langle N_i(t)\rangle - \langle y_i(t)\rangle\bigr), \quad i = 1,2,3.$$

Taking the limit as $t \to \infty$, combining with Lemma 2.3, we have

$$\lim_{t\to\infty}\langle N_i(t)\rangle = \lim_{t\to\infty}\langle y_i(t)\rangle.$$

By utilizing Itô's formula to $\ln N_i(t)$ $(i = 1,2,3)$ and integrating both sides of the first three equations of (1.2) from 0 to t, we obtain

$$\begin{cases} \ln N_1(t) - \ln N_1(0) = b_1 t - a_{11}\int_0^t N_1(s)\,ds - a_{12}\int_0^t N_2(s)\,ds \\ \qquad\qquad - a_{13}\int_0^t N_3(s)\,ds + \xi_1\omega_1(t), \\ \ln N_2(t) - \ln N_2(0) = b_2 t - a_{21}\int_0^t N_1(s)\,ds - a_{22}\int_0^t N_2(s)\,ds \\ \qquad\qquad - a_{23}\int_0^t N_3(s)\,ds + \xi_2\omega_2(t), \\ \ln N_3(t) - \ln N_3(0) = b_3 t + a_{31}\int_0^t N_1(s)\,ds + a_{32}\int_0^t N_2(s)\,ds \\ \qquad\qquad - a_{33}\int_0^t N_3(s)\,ds + \xi_3\omega_3(t). \end{cases} \tag{3.1}$$

Denote $\xi_i(t)\omega_i(t) = \vartheta_i(t)$, then

$$\frac{\ln N_1(t) - \ln N_1(0)}{t} = b_1 - a_{11}\langle N_1(t)\rangle - a_{12}\langle N_2(t)\rangle - a_{13}\langle N_3(t)\rangle + \frac{\vartheta_1(t)}{t}, \tag{3.2}$$

$$\frac{\ln N_2(t) - \ln N_2(0)}{t} = b_2 - a_{21}\langle N_1(t)\rangle - a_{22}\langle N_2(t)\rangle - a_{23}\langle N_3(t)\rangle + \frac{\vartheta_2(t)}{t}, \tag{3.3}$$

and

$$\frac{\ln N_3(t) - \ln N_3(0)}{t} = b_3 + a_{31}\langle N_1(t)\rangle + a_{32}\langle N_2(t)\rangle - a_{33}\langle N_3(t)\rangle \frac{\vartheta_3(t)}{t}. \tag{3.4}$$

We begin with the proof of (i).

It follows from (3.2) and (3.3) that

$$t^{-1}\bigl[\ln N_1(t) - \ln N_1(0)\bigr] \le b_1 - a_{11}\langle N_1(t)\rangle + t^{-1}\vartheta_1(t) \tag{3.5}$$

and

$$t^{-1}\bigl[\ln N_2(t) - \ln N_2(0)\bigr] \le b_2 - a_{22}\langle N_2(t)\rangle + t^{-1}\vartheta_2(t). \tag{3.6}$$

Using Lemma 2.1 to (3.5) and (3.6), then

$$\lim_{t\to\infty} N_i(t) = 0, \quad i = 1,2.$$

Since $b_3 < 0$, (3.4) implies $\lim_{t\to\infty} N_3(t) = 0$, and hence

$$\lim_{t\to\infty} N_i(t) = 0 \quad \text{for } i = 1, 2, 3.$$

Now we prove (ii).

By the proof of (i), if $b_1 < 0$, then $\lim_{t\to\infty} N_1(t) = 0$, and hence (3.3) and (3.4) imply that

$$t^{-1}\big[\ln N_2(t) - \ln N_2(0)\big] = b_2 - a_{22}\langle N_2(t)\rangle - a_{23}\langle N_3(t)\rangle + t^{-1}\vartheta_2(t) \tag{3.7}$$

and

$$t^{-1}\big[\ln N_3(t) - \ln N_3(0)\big] = b_3 + a_{32}\langle N_2(t)\rangle - a_{33}\langle N_3(t)\rangle + t^{-1}\vartheta_3(t). \tag{3.8}$$

By the elimination method, adding (3.7) applied by a_{33} and (3.8) applied by $-a_{23}$ gives

$$\begin{aligned} &t^{-1}\big(a_{33}\big[\ln N_2(t) - \ln N_2(0)\big] - a_{23}\big[\ln N_3(t) - \ln N_3(0)\big]\big) \\ &\quad = (b_2 a_{33} - b_3 a_{23}) - (a_{22}a_{33} + a_{32}a_{23})\langle N_2(t)\rangle + t^{-1}\big(a_{33}\vartheta_2(t) - a_{23}\vartheta_3(t)\big). \end{aligned} \tag{3.9}$$

Applying Lemma 2.1 and Lemma 2.3 to (3.9), we get

$$\langle N_2(t)\rangle^* \le \frac{b_2 a_{33} - b_3 a_{23}}{a_{22}a_{33} + a_{32}a_{23}} = \frac{\Delta_1^* - \tilde{\Delta}_1^*}{A_{11}}.$$

Substituting $\langle N_2(t)\rangle^*$ into (3.8), we obtain

$$t^{-1}\big[\ln N_3(t) - \ln N_3(0)\big] \le b_3 + a_{32}\frac{b_2 a_{33} - b_3 a_{23}}{a_{22}a_{33} + a_{32}a_{23}} - a_{33}\langle N_3(t)\rangle + t^{-1}\vartheta_3(t) \tag{3.10}$$

and

$$\langle N_3(t)\rangle^* \le \frac{b_2 a_{32} + b_3 a_{22}}{a_{22}a_{33} + a_{32}a_{23}} = \frac{\Delta_1 - \tilde{\Delta}_1}{A_{11}}.$$

If $b_1 < 0$, $b_2 > 0$, $\Delta_1 < \tilde{\Delta}_1$, then (3.10) implies $\lim_{t\to\infty} N_3(t) = 0$. By use of (3.7) again, we have

$$t^{-1}\big[\ln N_2(t) - \ln N_2(0)\big] \le b_2 + \varepsilon - a_{22}\langle N_2(t)\rangle + t^{-1}\vartheta_2(t). \tag{3.11}$$

Applying Lemma 2.1 to (3.11) gives

$$\langle N_2(t)\rangle^* \le \frac{b_2}{a_{22}}.$$

Similarly, we have

$$t^{-1}\big[\ln N_2(t) - \ln N_2(0)\big] \ge b_2 - \varepsilon - a_{22}\langle N_2(t)\rangle + t^{-1}\vartheta_2(t)$$

and $\langle N_2(t)\rangle_* \geq \frac{b_2}{a_{22}}$, and hence

$$\lim_{t\to\infty}\langle N_2(t)\rangle = \frac{b_2}{a_{22}}.$$

If $b_1 < 0$, $b_2 > 0$, $\Delta_1 > \tilde{\Delta}_1$, then we can derive from (3.7) and (3.8) that

$$t^{-1}\bigl[\ln N_2(t) - \ln N_2(0)\bigr] \geq b_2 - a_{22}\langle N_2(t)\rangle - a_{23}\langle N_3(t)\rangle^* + t^{-1}\vartheta_2(t) \tag{3.12}$$

and

$$t^{-1}\bigl[\ln N_3(t) - \ln N_3(0)\bigr] \geq b_3 + a_{32}\langle N_2(t)\rangle - a_{33}\langle N_3(t)\rangle^* + t^{-1}\vartheta_3(t). \tag{3.13}$$

Using Lemma 2.1 to (3.12) yields

$$\langle N_2(t)\rangle_* \geq \frac{b_2 - a_{23}\langle N_3(t)\rangle^*}{a_{22}} = \frac{b_2a_{33} - b_3a_{23}}{a_{22}a_{33} + a_{32}a_{23}} = \frac{\Delta_1^* - \tilde{\Delta}_1^*}{A_{11}}. \tag{3.14}$$

From (3.13) and (3.14), then

$$t^{-1}\bigl[\ln N_3(t) - \ln N_3(0)\bigr] \geq b_3 + a_{32}\langle N_2(t)\rangle_* - a_{33}\langle N_3(t)\rangle + t^{-1}\vartheta_3(t). \tag{3.15}$$

Applying Lemma 2.1 to (3.15) yields

$$\langle N_2(t)\rangle_* \geq \frac{b_3 + a_{32}\langle N_2(t)\rangle_*}{a_{33}} = \frac{b_2a_{32} + b_3a_{22}}{a_{22}a_{33} + a_{32}a_{23}} = \frac{\Delta_1 - \tilde{\Delta}_1}{A_{11}}.$$

Then we have

$$\lim_{t\to\infty} N_1(t) = 0, \qquad \lim_{t\to\infty}\langle N_2(t)\rangle = \frac{\Delta_1^* - \tilde{\Delta}_1^*}{A_{11}}, \qquad \lim_{t\to\infty}\langle N_3(t)\rangle = \frac{\Delta_1 - \tilde{\Delta}_1}{A_{11}}.$$

Therefore, case (ii) is proved. The proof of case (iii) is similar to case (ii) and we omit it here.

Next we enter the proof of case (iv). We begin to eliminate $\langle N_1(t)\rangle$, $\langle N_2(t)\rangle$ from (3.2)–(3.4) by the elimination method. By analysis, there exist positive constants $p = A_{13}/A_{33} > 0$, $q = -A_{23}/A_{33} > 0$, multiplying both sides of (3.2)–(3.4) by p, q, and 1, respectively, adding the three inequalities yields

$$\begin{aligned}
&\frac{\ln N_3(t) - \ln N_3(0) + p(\ln N_1(t) - \ln N_1(0)) + q(\ln N_2(t) - \ln N_2(0))}{t} \\
&\quad = b_1p + b_2q + b_3 - (a_{13}p + a_{23}q - a_{33})\langle N_3(t)\rangle + t^{-1}\bigl(\vartheta_1(t)p + \vartheta_2(t)q + \vartheta_3(t)\bigr) \\
&\quad = \frac{A_3 - \tilde{A}_3}{A_{33}} - \frac{A}{A_{33}}\langle N_3(t)\rangle + t^{-1}\bigl(\vartheta_1(t)p + \vartheta_2(t)q + \vartheta_3(t)\bigr).
\end{aligned} \tag{3.16}$$

Similarly, by the elimination method, there exist constants

$$\tilde{p} = \frac{-A_{21}}{A_{11}} < 0, \qquad \tilde{q} = \frac{A_{31}}{A_{11}} < 0, \qquad \bar{p} = \frac{-A_{12}}{A_{22}} < 0, \qquad \bar{q} = \frac{-A_{32}}{A_{22}} < 0$$

such that

$$\begin{aligned}&\frac{\ln N_1(t)-\ln N_1(0)+\tilde{p}(\ln N_2(t)-\ln N_2(0))+\tilde{q}(\ln N_3(t)-\ln x_3(0))}{t}\\&\quad=\frac{A_1-\tilde{A}_1}{A_{11}}-\frac{A}{A_{11}}\langle N_1(t)\rangle+t^{-1}\bigl(\vartheta_1(t)\tilde{p}+\vartheta_2(t)\tilde{q}+\vartheta_3(t)\bigr)\end{aligned}\tag{3.17}$$

and

$$\begin{aligned}&\frac{\ln N_2(t)-\ln N_2(0)+\bar{p}(\ln N_1(t)-\ln N_1(0))+\bar{q}(\ln N_3(t)-\ln N_3(0))}{t}\\&\quad=\frac{A_2-\tilde{A}_2}{A_{22}}-\frac{A}{A_{22}}\langle N_2(t)\rangle+t^{-1}\bigl(\vartheta_1(t)\bar{p}+\vartheta_2(t)\bar{q}+\vartheta_3(t)\bigr).\end{aligned}\tag{3.18}$$

Using Lemma 2.3 in equality (3.16), for arbitrarily $\varepsilon>0$, there exists $T>0$, for all $t>T$, we have

$$\begin{aligned}&t^{-1}\bigl(p\bigl[\ln N_1(t)-\ln N_1(0)\bigr]+q\bigl[\ln N_2(t)-\ln N_2(0)\bigr]\bigr)\\&\quad\le t^{-1}\bigl(p\ln N_1(t)+q\ln N_2(t)\bigr)+\varepsilon\le\varepsilon.\end{aligned}\tag{3.19}$$

Substituting (3.19) into (3.16) leads to

$$\begin{aligned}&t^{-1}\bigl[\ln N_3(t)-\ln N_3(0)\bigr]\\&\quad\ge\frac{A_3-\tilde{A}_3}{A_{33}}-\varepsilon-\frac{A}{A_{33}}\langle N_3(t)\rangle+t^{-1}\bigl(\vartheta_1(t)p+\vartheta_2(t)q+\vartheta_3(t)\bigr).\end{aligned}\tag{3.20}$$

Since $A_3>\tilde{A}_3$, letting $\varepsilon>0$ be small enough such that $A_3-\tilde{A}_3-\varepsilon>0$, then by Lemma 2.1, we have

$$\langle N_3(t)\rangle_*\ge\frac{A_3-\tilde{A}_3}{A}.$$

Similarly, we derive from (3.2) and (3.3) that

$$\begin{aligned}&t^{-1}\bigl[\ln N_1(t)-\ln N_1(0)\bigr]\\&\quad\le\frac{A_1-\tilde{A}_1}{A_{11}}+\varepsilon-\frac{A}{A_{11}}\langle N_1(t)\rangle+t^{-1}\bigl(\vartheta_1(t)\tilde{p}+\vartheta_2(t)\tilde{q}+\vartheta_3(t)\bigr)\end{aligned}\tag{3.21}$$

and

$$\begin{aligned}&t^{-1}\bigl[\ln N_2(t)-\ln N_2(0)\bigr]\\&\quad\le\frac{A_2-\tilde{A}_2}{A_{22}}+\varepsilon-\frac{A}{A_{22}}\langle N_2(t)\rangle+t^{-1}\bigl(\vartheta_1(t)\bar{p}+\vartheta_2(t)\bar{q}+\vartheta_3(t)\bigr).\end{aligned}\tag{3.22}$$

Applying Lemma 2.1 to (3.21) and (3.22) again, for sufficiently large t, we obtain

$$\langle N_1(t)\rangle^*\le\frac{A_1-\tilde{A}_1}{A},\qquad\langle N_2(t)\rangle^*\le\frac{A_2-\tilde{A}_2}{A}.$$

By the definition of sup limit, we deduce from (3.4) that

$$t^{-1}\big[\ln N_3(t) - \ln N_3(0)\big] \le b_3 + a_{31}\langle N_1(t)\rangle^* + a_{32}\langle N_2(t)\rangle^* - a_{33}\langle N_3(t)\rangle + t^{-1}\vartheta_3(t).$$

Therefore, Lemma 2.1 implies

$$\langle N_3(t)\rangle^* \le \frac{A_3 - \tilde{A}_3}{A}.$$

By the same way, from (3.2) and (3.3), we obtain

$$\begin{aligned} &t^{-1}\big[\ln N_1(t) - \ln N_1(0)\big] \\ &\quad\ge b_1 - a_{11}\langle N_1(t)\rangle - a_{12}\langle N_2(t)\rangle^* - a_{13}\langle N_3(t)\rangle^* + t^{-1}\vartheta_1(t) \end{aligned} \tag{3.23}$$

and

$$\begin{aligned} &t^{-1}\big[\ln N_2(t) - \ln N_2(0)\big] \\ &\quad\ge b_2 - a_{21}\langle N_1(t)\rangle^* - a_{22}\langle N_2(t)\rangle - a_{23}\langle N_3(t)\rangle^* + t^{-1}\vartheta_2(t). \end{aligned} \tag{3.24}$$

Substituting $\langle N_1(t)\rangle^* \le \frac{A_1-\tilde{A}_1}{A}$, $\langle N_2(t)\rangle^* \le \frac{A_2-\tilde{A}_2}{A}$, $\langle N_3(t)\rangle^* \le \frac{A_3-\tilde{A}_3}{A}$ into (3.23) and (3.24) and using Lemma 2.1, we have

$$\langle N_1(t)\rangle_* \ge \frac{A_1 - \tilde{A}_1}{A}, \qquad \langle N_2(t)\rangle_* \ge \frac{A_2 - \tilde{A}_2}{A}.$$

Therefore,

$$\lim_{t\to\infty}\langle N_1(t)\rangle = \frac{A_1 - \tilde{A}_1}{A}, \qquad \lim_{t\to\infty}\langle N_2(t)\rangle = \frac{A_2 - \tilde{A}_2}{A}, \qquad \lim_{t\to\infty}\langle N_3(t)\rangle = \frac{A_3 - \tilde{A}_3}{A},$$

which is the required assertion.

If $b_1 > 0$, $b_2 > 0$, $A_3 < \tilde{A}_3$, then the proof is similar to case (iii), and we omit it here. The proof is completed. □

Remark 3.1 By the process of our proof, if considering the effect of Lévy jumps, one can also establish sufficient conditions preserving the stability in mean and extinction of all species. Here we move some restricting conditions like $R > 0$ and $b_1 > b_2$, which appeared in [6].

4 Stability in distribution

Theorem 4.1 *The solution of model* (1.2) *is a stationary Markov process, that is, there exists a stationary distribution for system* (1.2) *if Assumption* 2.2 *holds.*

Proof Define

$$\hat{V} = R_1\frac{N_1^p}{p} + R_2\frac{N_2^p}{p} + R_3\left(\frac{N_3^p}{p} + \frac{a_{31}y_1^{p+1}}{\sigma_1(p+1)} + \frac{a_{32}y_2^{p+1}}{\sigma_2(p+1)}\right),$$

where R_1, R_2, R_3 are positive constants defined later. By Itô's formula, we have

$$\begin{aligned}
L\hat{V}(t) &= R_1N_1(t)^p\left(r_1 - a_{11}N_1(t) - a_{12}y_2(t) - a_{13}y_3(t) + \frac{p-1}{2}\xi_1^2\right) \\
&\quad + R_2N_2(t)^p\left(r_2 - a_{21}y_1(t) - a_{22}N_2(t) - a_{23}y_3(t) + \frac{p-1}{2}\xi_2^2\right) \\
&\quad + R_3N_3(t)^p\left(-r_3 + a_{31}y_1(t) + a_{32}y_2(t) - a_{33}N_3(t) + \frac{p-1}{2}\xi_1^2\right) \\
&\quad + R_3\frac{a_{31}}{p+1}\left(N_1^{p+1}(t) - y_1^{p+1}(t)\right) + R_3\frac{a_{32}}{p+1}\left(N_2^{p+1}(t) - y_2^{p+1}(t)\right) \\
&\le R_1N_1(t)^p\left(r_1 + \frac{p-1}{2}\xi_1^2 - a_{11}N_1(t)\right) + R_2N_2(t)^p\left(r_2 + \frac{p-1}{2}\xi_2^2 - a_{22}N_2(t)\right) \\
&\quad + R_3N_3(t)^p\left(-r_3 + \frac{p-1}{2}\xi_1^2 - a_{33}N_3(t)\right) + R_3a_{31}\frac{pN_3^{p+1}(t) + y_1^{p+1}(t)}{p+1} \\
&\quad + R_3a_{32}\frac{pN_3^{p+1}(t) + y_2^{p+1}(t)}{p+1} + R_3\frac{a_{31}}{p+1}\left(N_1^{p+1}(t) - y_1^{p+1}(t)\right) \\
&\quad + R_3\frac{a_{32}}{p+1}\left(N_2^{p+1}(t) - y_2^{p+1}(t)\right) \\
&= R_1N_1(t)^p\left(r_1 + \frac{p-1}{2}\xi_1^2 - a_{11}N_1(t)\right) + R_2N_2(t)^p\left(r_2 + \frac{p-1}{2}\xi_2^2 - a_{22}N_2(t)\right) \\
&\quad + R_3N_3(t)^p\left(-r_3 + \frac{p-1}{2}\xi_1^2 - a_{33}N_3(t)\right) + R_3\frac{pN_3^{p+1}(t)}{p+1}(a_{31} + a_{32}) \\
&\quad + R_3\frac{a_{31}}{p+1}N_1^{p+1}(t) + R_3\frac{a_{32}}{p+1}N_2^{p+1}(t) \\
&= R_1N_1(t)^p\left(r_1 + \frac{p-1}{2}\xi_1^2\right) + R_2N_2(t)^p\left(r_2 + \frac{p-1}{2}\xi_2^2\right) \\
&\quad + R_3N_3(t)^p\left(-r_3 + \frac{p-1}{2}\xi_1^2\right) + \left(-R_1a_{11} + R_3\frac{a_{31}}{p+1}\right)N_1^{p+1}(t) \\
&\quad + \left(-R_2a_{22} + R_3\frac{a_{32}}{p+1}\right)N_2^{p+1}(t) + \left(-R_3a_{33} + R_3\frac{p(a_{31} + a_{32})}{p+1}\right)N_3^{p+1}(t).
\end{aligned}$$

By Assumption 2.2, there exist positive constants

$$\begin{aligned}
&R_1 = \frac{p+1+R_3a_{31}}{(p+1)a_{11}} > 0, \qquad R_2 = \frac{p+1+R_3a_{32}}{(p+1)a_{22}} > 0, \\
&R_3 = \frac{p+1}{(p+1)a_{33} - p(a_{31} + a_{32})} > 0
\end{aligned}$$

such that

$$\begin{aligned}
L\hat{V} &\le -\left(N_1^{p+1}(t) + N_2^{p+1}(t) + N_3^{p+1}(t)\right) + R_1N_1(t)^p\left(r_1 + \frac{p-1}{2}\xi_1^2\right) \\
&\quad + R_2N_2(t)^p\left(r_2 + \frac{p-1}{2}\xi_2^2\right) + R_3N_3(t)^p\left(-r_3 + \frac{p-1}{2}\xi_1^2\right).
\end{aligned}$$

Define $\check{V} = \hat{V} + \sum_{i=1}^{3} \frac{y_i^{p+1}}{2\sigma_i}$, then

$$\begin{aligned} L\check{V} \le & -\frac{1}{2}\big(N_1^{p+1}(t) + N_2^{p+1}(t) + N_3^{p+1}(t) + y_1^{p+1}(t) + y_2^{p+1}(t) + y_3^{p+1}(t)\big) \\ & + R_1 N_1(t)^p \left(r_1 + \frac{p-1}{2}\xi_1^2\right) + R_2 N_2(t)^p \left(r_2 + \frac{p-1}{2}\xi_2^2\right) \\ & + R_3 N_3(t)^p \left(-r_3 + \frac{p-1}{2}\xi_1^2\right). \end{aligned}$$

Let

$$\tilde{V}(t) = \sum_{i=1}^{3}\left(\frac{1}{N_i^{\iota}(t)} - \ln y_i(t)\right).$$

Applying Itô's formula to $\tilde{V}(t)$ yields

$$\begin{aligned} L\tilde{V} = & -\frac{\sigma_1(N_1(t)-y_1(t))}{y_1(t)} - \frac{\sigma_2(N_2(t)-y_2(t))}{y_2(t)} - \frac{\sigma_3(N_3(t)-y_3(t))}{y_3(t)} \\ & - \iota\big(N_1(t)\big)^{-\iota}\left(r_1 - a_{11}N_1(t) - a_{12}y_2(t) - a_{13}y_3(t) - \frac{\iota+1}{2}\xi_1^2\right) \\ & - \iota\big(N_2(t)\big)^{-\iota}\left(r_2 - a_{21}y_1(t) - a_{22}N_2(t) - a_{23}y_3(t) - \frac{\iota+1}{2}\xi_2^2\right) \\ & - \iota\big(N_3(t)\big)^{-\iota}\left(-r_3 + a_{31}y_1(t) + a_{32}y_2(t) - a_{33}N_3(t) - \frac{\iota+1}{2}\xi_3^2\right) \\ \le & -\iota\frac{r_1 - \frac{\iota+1}{2}\xi_1^2}{N_1^{\iota}} + a_{11}\iota N_1^{1-\iota}(t) + a_{12}\iota\frac{y_2^2 + N_1^{-2\iota}}{2} + a_{13}\iota\frac{y_3^2 + N_1^{-2\iota}}{2} \\ & - \frac{\sigma_1(N_1(t)-y_1(t))}{y_1(t)} + a_{22}\iota N_2^{1-\iota}(t) + a_{21}\iota\frac{y_1^2 + N_2^{-2\iota}}{2} + a_{23}\iota\frac{y_3^2 + N_2^{-2\iota}}{2} \\ & - \frac{\sigma_2(N_2(t)-y_2(t))}{y_2(t)} - \iota\frac{-r_3 - \frac{\iota+1}{2}\xi_3^2}{N_3^{\iota}} + a_{33}\iota N_3^{1-\iota}(t) - \frac{\sigma_3(N_3(t)-y_3(t))}{y_3(t)} \\ = & \ \sigma_1 + \sigma_2 + \sigma_3 - \iota\frac{r_1 - \frac{\iota+1}{2}\xi_1^2}{N_1^{\iota}} - \iota\frac{r_2 - \frac{\iota+1}{2}\xi_2^2}{N_2^{\iota}} - \iota\frac{-r_3 - \frac{\iota+1}{2}\xi_3^2}{N_3^{\iota}} \\ & + a_{21}\iota y_1^2/2 + a_{12}\iota y_2^2/2 + (a_{13}+a_{23})\iota y_3^2/2 + (a_{12}+a_{13})\iota N_1^{-2\iota}/2 \\ & + (a_{21}+a_{23})\iota N_2^{-2\iota}/2 + a_{11}\iota N_1^{1-\iota}(t) \\ & + a_{22}\iota N_2^{1-\iota}(t) + a_{33}\iota N_3^{1-\iota}(t) - \frac{\sigma_1 N_1(t)}{y_1(t)} - \frac{\sigma_2 N_2(t)}{y_2(t)} - \frac{\sigma_3 N_3(t)}{y_3(t)}. \end{aligned}$$

Define $V(t) = \check{V}(t) + \tilde{V}(t)$, then

$$\begin{aligned} LV = & \ L\check{V} + L\tilde{V} \\ \le & \ \sigma + M - \frac{1}{4}\big(N_1^{p+1}(t) + N_2^{p+1}(t) + N_3^{p+1}(t) + y_1^{p+1}(t) + y_2^{p+1}(t) + y_3^{p+1}(t)\big) \\ & - \iota\frac{r_1 - \frac{\iota+1}{2}\xi_1^2}{N_1^{\iota}} - \iota\frac{r_2 - \frac{\iota+1}{2}\xi_2^2}{N_2^{\iota}} - \iota\frac{-r_3 - \frac{\iota+1}{2}\xi_3^2}{N_3^{\iota}} \end{aligned}$$

$$-\frac{\sigma_1 N_1(t)}{y_1(t)}-\frac{\sigma_2 N_2(t)}{y_2(t)}-\frac{\sigma_3 N_3(t)}{y_3(t)},$$

where $\sigma=\sigma_1+\sigma_2+\sigma_3$, and

$$\begin{aligned}M=\max_{N_i,y_i\in R_+^6}\Bigg\{&-\frac{1}{4}\sum_{i=1}^{3}\big(N_i^{p+1}(t)+y_i^{p+1}(t)\big)+R_1N_1(t)^p\left(r_1+\frac{p-1}{2}\xi_1^2\right)\\&+R_2N_2(t)^p\left(r_2+\frac{p-1}{2}\xi_2^2\right)+R_3N_3(t)^p\left(-r_3+\frac{p-1}{2}\xi_1^2\right)\\&+\frac{a_{21}\iota y_1^2}{2}+\frac{a_{12}\iota y_2^2}{2}+\frac{(a_{13}+a_{23})\iota y_3^2}{2}+\frac{(a_{12}+a_{13})\iota N_1^{-2\iota}}{2}\\&+\frac{(a_{21}+a_{23})\iota N_2^{-2\iota}}{2}+a_{11}\iota N_1^{1-\iota}(t)+a_{22}\iota N_2^{1-\iota}(t)+a_{33}\iota N_3^{1-\iota}(t)\Bigg\}.\end{aligned}$$

Choose $\varepsilon>0$ small enough such that

$$0<\varepsilon<\min\left\{\left(\frac{\iota(r_i-\frac{\xi_i^2}{2}(\iota+1))}{\sigma+M+1}\right)^{\frac{1}{\iota}},\left(\frac{1}{4(\sigma+M+1)}\right)^{\frac{1}{p+1}},\frac{\sigma_i}{\sigma+M+1},i=1,2,3\right\}.$$

Define the following bounded closed set:

$$D_\varepsilon=\left\{(N_1,N_2,N_3,y_1,y_2,y_3)\in R_+^6\,\middle|\,\varepsilon<N_i<\frac{1}{\varepsilon},\varepsilon^2<y_i<\frac{1}{\varepsilon},i=1,2,3\right\},$$

and for $i=1,2,3$, denote

$$\begin{aligned}D_\varepsilon^i&=\left\{(N_1,N_2,N_3,y_1,y_2,y_3)\in R_+^6\,\middle|\,N_i>\frac{1}{\varepsilon}\right\},\\D_\varepsilon^{i+3}&=\left\{(N_1,N_2,N_3,y_1,y_2,y_3)\in R_+^6\,\middle|\,y_i>\frac{1}{\varepsilon}\right\},\\D_\varepsilon^{i+6}&=\left\{(N_1,N_2,N_3,y_1,y_2,y_3)\in R_+^6\,\middle|\,0<N_i<\varepsilon\right\},\\D_\varepsilon^{i+9}&=\left\{(N_1,N_2,N_3,y_1,y_2,y_3)\in R_+^6\,\middle|\,\varepsilon<N_i<\frac{1}{\varepsilon},0<y_i<\varepsilon^2\right\}.\end{aligned}$$

Denote the complement of D_ε by D_ε^C, then it is easy to get $D_\varepsilon^C=\bigcup_{j=1}^{12}D_\varepsilon^j$. For all $(N_1,N_2,N_3,y_1,y_2,y_3)\in D_\varepsilon^C$, we discuss as follows.

(i) If $(N_1,N_2,N_3,y_1,y_2,y_3)\in D_\varepsilon^i$, $i=1,2,3$, then

$$LV\le\sigma+M-\frac{1}{4}N_i^p\le\sigma+M-\frac{1}{4\varepsilon^{p+1}}<-1;$$

(ii) If $(N_1,N_2,N_3,y_1,y_2,y_3)\in D_\varepsilon^{i+3}$, $i=1,2,3$, then

$$LV\le\sigma+M-\frac{1}{4}y_i^p\le\sigma+M-\frac{1}{4\varepsilon^{p+1}}<-1;$$

(iii) If $(N_1,N_2,N_3,y_1,y_2,y_3)\in D_\varepsilon^{i+6}$, $i=1,2,3$, then

$$LV\le \sigma+M-\iota\frac{r_i-\frac{\xi_i^2}{2}(\iota+1)}{N_i^\iota}\le \sigma+M-\iota\frac{r_i-\frac{\xi_i^2}{2}(\iota+1)}{\varepsilon^\iota}<-1;$$

(iv) If $(N_1,N_2,N_3,y_1,y_2,y_3)\in D_\varepsilon^{i+9}$, $i=1,2,3$, then

$$LV\le \sigma+M-\sigma_i N_i/y_i\le \sigma+M-\sigma_i\varepsilon/\varepsilon^2<-1.$$

Consequently, for any $(N_1,N_2,N_3,y_1,y_2,y_3)\in D_\varepsilon^C$, we have

$$\sup_{(N_1,N_2,N_3,y_1,y_2,y_3)\in R_+^6} LV(N_1,N_2,N_3,y_1,y_2,y_3)\le -1.$$

Therefore, it follows from Lemma 2.4 that there exists a stationary distribution for system (1.2). The proof is completed. □

Theorem 4.2 *Under Assumption 2.2, solutions of model (1.2) are globally attractive.*

Proof Firstly, let $N(t)=N(t,N(\phi))$ and $\bar N(t)=\bar N(t,\bar N(\phi))$ be any two solutions of model (1.1) with the initial data $N(\phi),\bar N(\phi)\in C([-\tau,0],R_+^3)$. We only need to prove $\lim_{t\to\infty}\mathbb{E}|N_i(t)-\bar N_i(t)|=0$ for $i=1,2,3$.

Define

$$V(t)=\sum_{i=1}^3 D_i\bigl|\ln N_i(t)-\ln\bar N_i(t)\bigr|,$$

where D_i $(i=1,2,3)$ is defined later in the proof. By computing the upper right derivative of $V(t)$, then

$$\begin{aligned} D^+V(t)\le{}& D_1\operatorname{sign}\bigl(N_1(t)-\bar N_1(t)\bigr)\bigl[-a_{11}\bigl(N_1(t)-\bar N_1(t)\bigr)-a_{12}\bigl(y_2(t)-\bar y_2(t)\bigr)\\ &-a_{13}\bigl(y_3(t)-\bar y_3(t)\bigr)\bigr]\,dt+D_2\operatorname{sign}\bigl(N_2(t)-\bar N_2(t)\bigr)\\ &\times\bigl[-a_{21}\bigl(y_1(t)-\bar y_1(t)\bigr)-a_{22}\bigl(N_2(t)-\bar N_2(t)\bigr)-a_{23}\bigl(y_3(t)-\bar y_3(t)\bigr)\bigr]\,dt\\ &+D_3\operatorname{sign}\bigl(N_3(t)-\bar N_3(t)\bigr)\bigl[a_{31}\bigl(y_1(t)-\bar y_1(t)\bigr)+a_{32}\bigl(y_2(t)-\bar y_2(t)\bigr)\\ &-a_{33}\bigl(Ny_3(t)-\bar N_3(t)\bigr)\bigr]\,dt. \end{aligned}\tag{4.1}$$

On the other hand, by (1.2), we have

$$\frac{d(y_i-\bar y_i)}{dt}=\sigma_i(N_i-\bar N_i)-\sigma_i(y_i-\bar y_i),\quad i=1,2,3,$$

that is,

$$y_i(t)-\bar y_i(t)=e^{-\sigma_1 t}\bigl(y_i(0)-\bar y_i(0)\bigr)+\sigma_1e^{-\sigma_1 t}\int_0^t e^{\sigma_1 s}\bigl(N_i(s)-\bar N_i(s)\bigr)\,ds,\quad i=1,2,3.$$

Therefore,

$$\bigl|y_i(t)-\bar y_i(t)\bigr|\le e^{-\sigma_1 t}\bigl|y_i(0)-\bar y_i(0)\bigr|+\sigma_1e^{-\sigma_1 t}\int_0^t e^{\sigma_1 s}\bigl|N_i(s)-\bar N_i(s)\bigr|\,ds,\quad i=1,2,3.$$

Integrating two sides of the above inequality from 0 to t, we have

$$\begin{aligned}\int_0^t \left|y_i(t)-\bar{y}_i(t)\right| &\le \frac{1-e^{-\sigma_1 t}}{\sigma_i}\left|y_i(0)-\bar{y}_i(0)\right| + \sigma_i\int_0^t dv\int_0^v e^{\sigma_1(s-v)}\left|N_i(s)-\bar{N}_i(s)\right|ds \\ &= \frac{1-e^{-\sigma_1 t}}{\sigma_i}\left|y_i(0)-\bar{y}_i(0)\right| + \int_0^t \left(1-e^{\sigma_1(s-t)}\right)\left|N_i(s)-\bar{N}_i(s)\right|ds \\ &\le \frac{1}{\sigma_i}\left|y_i(0)-\bar{y}_i(0)\right| + \int_0^t \left|N_i(s)-\bar{N}_i(s)\right|ds, \quad i=1,2,3.\end{aligned}$$

Integrating both sides of (4.1) from 0 to t and taking expectations give

$$\begin{aligned}V(t) \le{}& V(0) + D_1\left(-a_{11}\int_0^t \left|N_1(s)-\bar{N}_1(s)\right|ds + a_{12}\int_0^t\left|y_2(s)-\bar{y}_2(s)\right|ds \right.\\ &\left.+ a_{13}\int_0^t\left|y_3(s)-\bar{y}_3(s)\right|ds\right) + D_2\left(-a_{22}\int_0^t\left|N_2(s)-\bar{N}_2(s)\right|ds\right.\\ &\left.+ a_{21}\int_0^t\left|y_1(s)-\bar{y}_1(s)\right|ds + a_{23}\int_0^t\left|y_3(s)-\bar{y}_3(s)\right|ds\right)\\ &+ D_3\left(-a_{33}\int_0^t\left|N_3(s)-\bar{N}_3(s)\right|ds + a_{31}\int_0^t\left|y_1(s)-\bar{y}_1(s)\right|ds\right.\\ &\left.+ a_{32}\int_0^t\left|y_2(s)-\bar{y}_2(s)\right|ds\right)\\ \le{}& V(0) + (D_2a_{31}+D_3a_{21}-D_1a_{11})\int_0^t\left|N_1(s)-\bar{N}_1(s)\right|ds\\ &+ (D_1a_{12}+D_3a_{32}-D_2a_{22})\int_0^t\left|N_2(s)-\bar{N}_2(s)\right|ds\\ &+ (D_1a_{13}+D_2a_{23}-D_3a_{33})\int_0^t\left|N_3(s)-\bar{N}_3(s)\right|ds\\ &+ \frac{D_2a_{21}+D_3a_{31}}{\sigma_1}\left|y_1(0)-\bar{y}_1(0)\right| + \frac{D_1a_{12}+D_3a_{32}}{\sigma_2}\left|y_2(0)-\bar{y}_2(0)\right|\\ &+ \frac{D_1a_{13}+D_2a_{23}}{\sigma_3}\left|y_3(0)-\bar{y}_3(0)\right|.\end{aligned}$$

For the following equations,

$$\begin{cases} D_1a_{11}-D_2a_{21}-D_3a_{31}=1,\\ -D_1a_{12}+D_2a_{22}-D_3a_{32}=1,\\ -D_1a_{13}-D_2a_{23}+D_3a_{33}=1,\end{cases}$$

under Assumption 2.2, the coefficient matrix of D_1, D_2, and D_3 is a nonsingular M-matrix, then by M-matrix theory, there exists $D_i>0$ ($i=1,2,3$) satisfying the equation. Therefore,

$$\begin{aligned}&V(t)+\int_0^t\left|N_1(s)-\bar{N}_1(s)\right|ds+\int_0^t\left|N_2(s)-\bar{N}_2(s)\right|ds+\int_0^t\left|N_3(s)-\bar{N}_3(s)\right|ds\\ &\quad\le V(0)+\frac{D_2a_{21}+D_3a_{31}}{\sigma_1}\left|y_1(0)-\bar{y}_1(0)\right|+\frac{D_1a_{12}+D_3a_{32}}{\sigma_2}\left|y_2(0)-\bar{y}_2(0)\right|\end{aligned}$$

$$+\frac{D_1a_{13}+D_2a_{23}}{\sigma_3}\left|y_3(0)-\bar{y}_3(0)\right|$$
$$<+\infty,$$

which means $|N_i(t)-\bar{N}_i(t)| \in \mathbb{L}_1[0,+\infty)$. Consequently, we can derive from Lemma 2.5 and Lemma 2.6 that

$$\lim_{t\to\infty}\left|N_i(t)-\bar{N}_i(t)\right| = 0, \quad i = 1,2,3.$$

The proof is completed. □

Remark 4.1 Combining the existence of distribution and the global attractivity of solutions of (1.2), we conclude that system (1.2) has a unique distribution, which is stable.

5 Numerical simulations

In this section, we give some numerical simulations to validate our theoretical results. By the Milstein higher order method proposed by Higham [32], we numerically simulate the solutions of system (1.2). Using discretization Brownian path over $[0,T]$ and writing efficient Matlab codes, we can obtain the corresponding simulation figures one by one.

Let

$$A = \begin{vmatrix} 0.4 & 0.1 & 0.15 \\ 0.2 & 0.4 & 0.1 \\ -0.15 & -0.1 & 0.4 \end{vmatrix} = 0.0645, \qquad \sigma_1 = \sigma_2 = \sigma_3 = 1,$$
$$r_1 = 0.31, \qquad r_2 = 0.31, \qquad r_3 = 0.01.$$

In the following, without special mention, we only change the parameter of white noise and keep the rest of parameters unchanged so as to clearly see the dynamical effect of white noise.

Case (i) $b_1 < 0$, $b_2 < 0$.

Let $\xi_1 = 0.8$, $\xi_2 = 0.8$, $\xi_3 = 0.5292$, then an easy computation yields $b1 = -0.01$, $b_2 = -0.01$, $b_3 = -0.15$. It follows from Theorem 3.1 that all species are extinct, illustrated in Fig. 1.

Case (ii) $b_1 < 0$, $b_2 > 0$.

Let $\xi_1 = 0.8$, $\xi_2 = 0.4$, $\xi_3 = 0.01$, then $b1 = -0.01$, $b_2 = 0.23$, $b_3 = -0.01$, and $\Delta_1^* = 0.019$, $\tilde{\Delta}_1^* = 0.093$, $A_{11} = 0.17$, $\frac{\Delta_1-\tilde{\Delta}_1}{A_{11}} = 0.019/0.17 = 0.1118$, $\frac{\Delta_1^*-\tilde{\Delta}_1^*}{A_{11}} = 0.093/0.17 = 0.5471$. By Theorem 3.1, then

$$\lim_{t\to\infty} N_1(t) = 0, \qquad \lim_{t\to\infty}\langle N_2(t)\rangle = 0.5471,$$
$$\lim_{t\to\infty}\langle N_3(t)\rangle = 0.1118,$$

which is illustrated in Fig. 2(a).

If $r_3 = 0.1$, $\xi_1 = 0.8$, $\xi_2 = 0.469$, $\xi_3 = 0.01$, then $b1 = -0.01$, $b_2 = 0.2$, $b_3 = -0.1$, $b_2a_{32} + b_3a_{22} = -0.02 < 0$. Hence, Theorem 3.1 implies $N_1(t)$, $N_3(t)$ are both extinct, and species $N_2(t)$ is stable in mean, and $\lim_{t\to\infty}\langle N_2(t)\rangle = 0.5$, see Fig. 2(b).

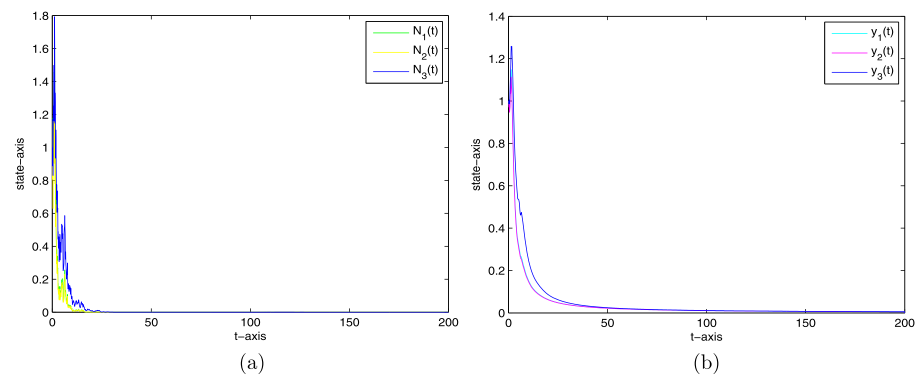

Figure 1 Extinction of all species of system (1.2) with $\xi_1 = 0.8$, $\xi_2 = 0.8$, $\xi_3 = 0.5292$. (**a**) Extinction of N_1, N_2, N_3. (**b**) Extinction of y_1, y_2, y_3

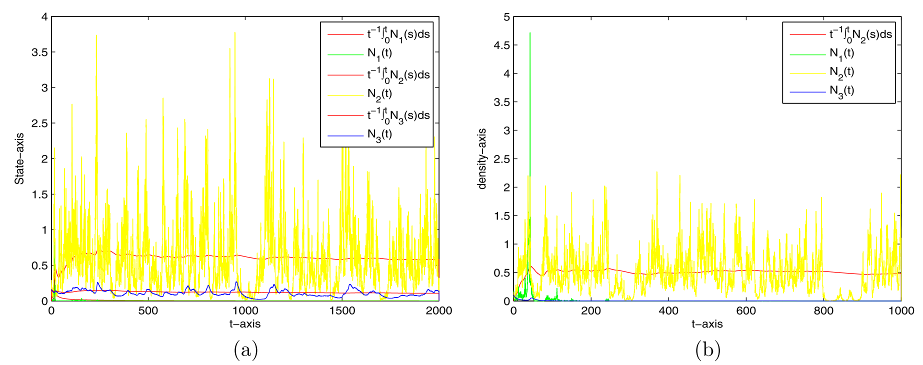

Figure 2 Dynamical behavior of (1.2). (**a**) is for (1.2) with $\xi_1 = 0.8$, $\xi_2 = 0.4$, $\xi_3 = 0.01$, then $\lim_{t\to\infty} N_1(t) = 0$, $\lim_{t\to\infty}\langle N_2(t)\rangle = 0.5471$, $\lim_{t\to\infty}\langle N_3(t)\rangle = 0.1118$, (**b**) is for (1.2) with $r_3 = 0.1$, $\xi_1 = 0.8$, $\xi_2 = 0.469$, $\xi_3 = 0.01$, then $N_1(t)$, $N_3(t)$ are both extinct, $\lim_{t\to\infty}\langle N_2(t)\rangle = 0.5$

Case (iii) $b_1 > 0$, $b_2 < 0$.

Let $\xi_1 = 0.4$, $\xi_2 = 0.8$, $\xi_3 = 0.01$, then $b1 = 0.23$, $b_2 = -0.01$, $b_3 = -0.01$. By computation, $\Delta_2 - \tilde{\Delta}_2 = 0.0305$, $\Delta_2^* - \tilde{\Delta}_2^* = 0.0935$, $A_{22} = 0.1825$. Hence, it follows from Theorem 3.1 that $N_2(t)$ is extinct, and

$$\lim_{t\to\infty}\langle N_1(t)\rangle = \frac{\Delta_2^* - \tilde{\Delta}_2^*}{A_{22}} = 0.1671, \qquad \lim_{t\to\infty}\langle N_3(t)\rangle = \frac{\Delta_2 - \tilde{\Delta}_2}{A_{22}} = 0.5123.$$

Figure 3(a) verifies it correctly.

If $r_3 = 0.1$, $\xi_1 = 0.469$, $\xi_2 = 0.8$, $\xi_3 = 0.01$, then $b1 = 0.2$, $b_2 = -0.01$, $b_3 = -0.1$, and $b_1 a_{31} + b_3 a_{11} - 0.01$. Theorem 3.1 indicates that $N_2(t)$, $N_3(t)$ are both extinct and $\lim_{t\to\infty}\langle N_1(t)\rangle = 0.2/0.4 = 0.5$, see Fig. 3(b).

Case (iv) $b_1 > 0$, $b_2 > 0$.

We choose $\xi_1 = 0.1414$, $\xi_2 = 0.1414$, $\xi_3 = 0.2$ such that $b1 = 0.3$, $b_2 = 0.3$, $b_3 = -0.03$. By computation, then $A_1 - \tilde{A}_1 = 0.036$, $A_2 - \tilde{A}_2 = 0.0266$, $A_3 - \tilde{A}_3 = 0.0153$. Therefore, by Theorem 3.1, we have

$$\lim_{t\to\infty}\langle N_1(t)\rangle = \frac{A_1 - \tilde{A}_1}{A} = 0.5581, \qquad \lim_{t\to\infty}\langle N_2(t)\rangle = \frac{A_2 - \tilde{A}_2}{A} = 0.4124,$$

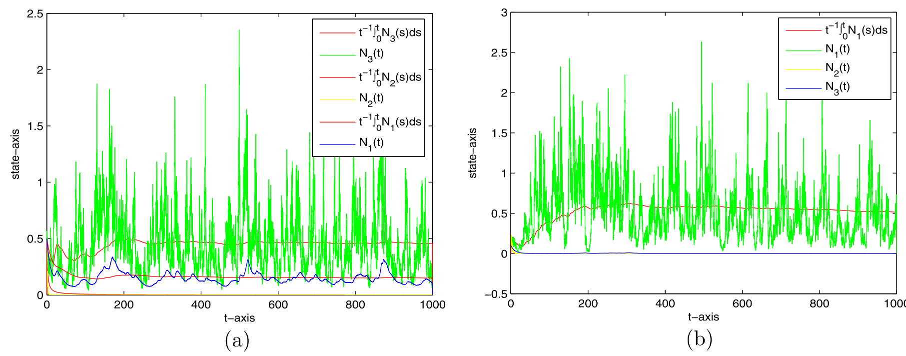

Figure 3 Dynamical behavior of (1.2). (**a**) is for system (1.2) with $\xi_1 = 0.4$, $\xi_2 = 0.8$, $\xi_3 = 0.01$, then $N_2(t)$ is extinct, and $\lim_{t\to\infty}\langle N_1(t)\rangle = 0.1671$, $\lim_{t\to\infty}\langle N_3(t)\rangle = 0.5123$. (**b**) is for system (1.2) with $r_3 = 0.1$, $\xi_1 = 0.469$, $\xi_2 = 0.8$, $\xi_3 = 0.01$, then $N_2(t)$, $N_3(t)$ are both extinct and $\lim_{t\to\infty}\langle N_1(t)\rangle = 0.5$

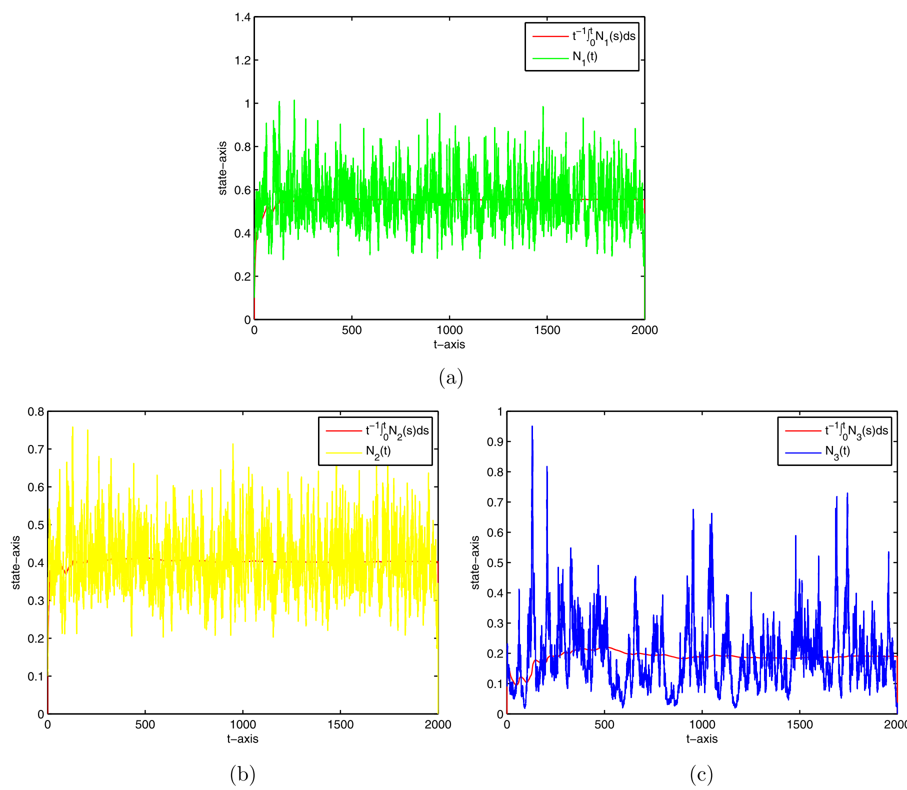

Figure 4 Stable case of (1.2) with $\xi_1 = 0.1414$, $\xi_2 = 0.1414$, $\xi_3 = 0.2$. Theorem 3.1 shows that $\lim_{t\to\infty}\langle N_1(t)\rangle = \frac{A_1-\tilde{A}_1}{A} = 0.5581$, $\lim_{t\to\infty}\langle N_2(t)\rangle = \frac{A_2-\tilde{A}_2}{A} = 0.4124$, $\lim_{t\to\infty}\langle N_3(t)\rangle = \frac{A_3-\tilde{A}_3}{A} = 0.2372$

$$\lim_{t\to\infty}\langle N_3(t)\rangle = \frac{A_3 - \tilde{A}_3}{A} = 0.2372,$$

which is illustrated in Fig. 4.

By use of Theorems 4.1 and 4.2, we know that system (1.2) has a unique distribution, which is revealed in Figs. 5 and 6. Figure 5 is the probability density function of preys $N_1(t)$, $N_2(t)$ and predator $N_3(t)$, respectively. Figure 6 shows the attractivity of the solutions. They

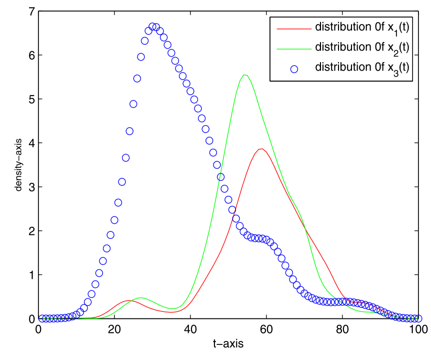

Figure 5 Probability density function of preys $N_1(t)$, $N_2(t)$ and predator $N_3(t)$ of (1.2) with $\xi_1 = 0.1414$, $\xi_2 = 0.1414$, $\xi_3 = 0.2$

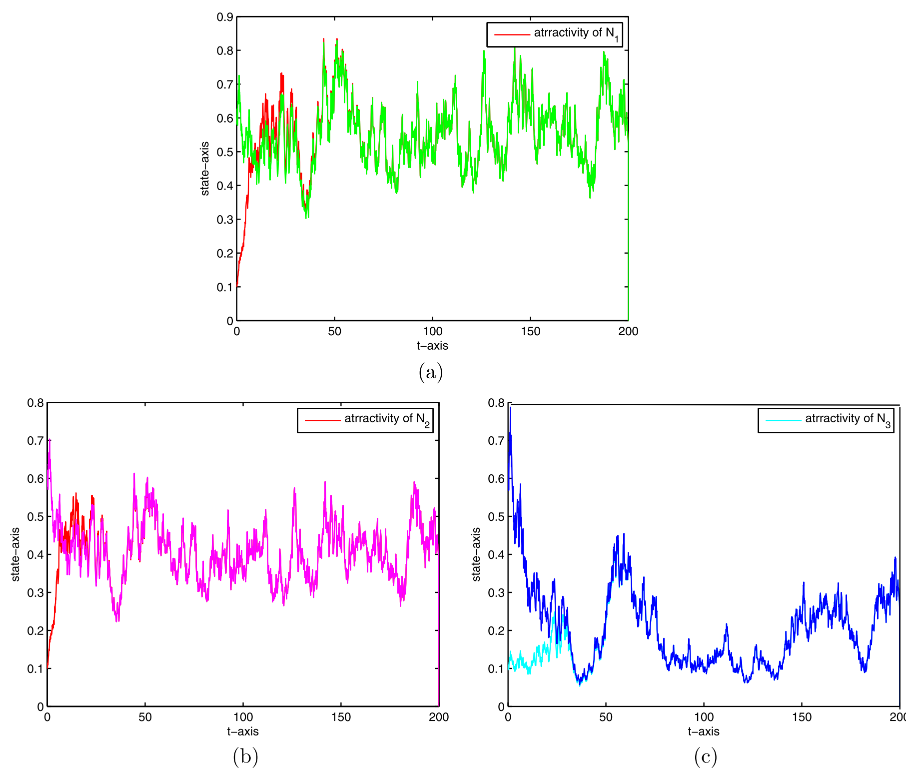

Figure 6 Attractivity of solutions of system (1.2)

both indicate the existence and stability of stationary distribution function. The simulation results verify that when the condition is satisfied, that is, the white noise is relatively small, system (1.2) is stable.

If $\xi_1 = 0.1414$, $\xi_2 = 0.1414$, $\xi_3 = 0.5292$, then $b1 = 0.3$, $b_2 = 0.3$, $b_3 = -0.15$. By computation, we have $A_3 - \tilde{A}_3 = -0.0015 < 0$, $\Delta_3 - \tilde{\Delta}_3 = 0.06$, $\Delta_3^* - \tilde{\Delta}_3^* = 0.09$, $A_{33} = 0.14$, which guarantees that the condition of case (iv) holds, and hence Theorem 3.1 implies the two-prey are stable in mean and the predator N_3 is extinct, further,

$$\lim_{t\to\infty}\langle N_1(t)\rangle = \frac{\Delta_3 - \tilde{\Delta}_3}{A_{33}} = 0.4286, \qquad \lim_{t\to\infty}\langle N_2(t)\rangle = \frac{\Delta_3^* - \tilde{\Delta}_3^*}{A_{33}} = 0.6429$$

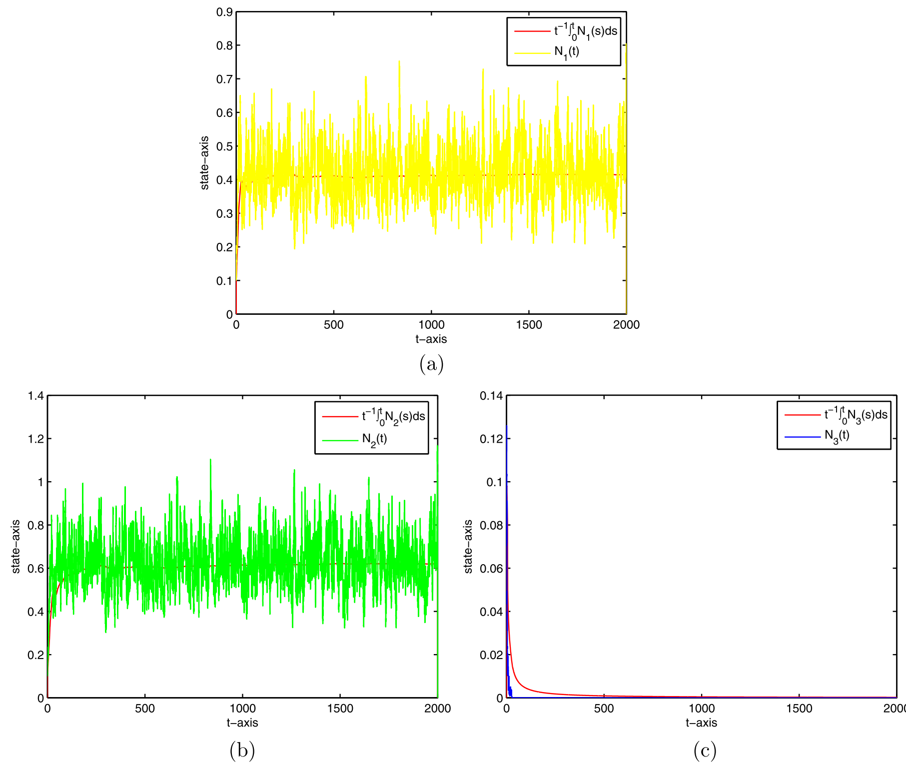

Figure 7 Dynamical behavior of (1.2) with $\xi_1 = 0.1414$, $\xi_2 = 0.1414$, $\xi_3 = 0.5292$. Theorem 3.1 implies the two prey are stable in mean and the predator N_3 is extinct, and $\lim_{t\to\infty}\langle N_1(t)\rangle = \frac{\Delta_3-\tilde{\Delta}_3}{A_{33}} = 0.4286$, $\lim_{t\to\infty}\langle N_2(t)\rangle = \frac{\Delta_3^*-\tilde{\Delta}_3^*}{A_{33}} = 0.6429$, $\lim_{t\to\infty}\langle N_3(t)\rangle = 0$

and

$$\lim_{t\to\infty}\langle N_3(t)\rangle = 0.$$

Figure 7 indicates the result is true.

6 Conclusion and discussion

In this paper, we consider a three-species stochastic predator–prey system with distributed delays. Theorem 3.1 gives sufficient conditions of the stability in mean and extinction of each species. Theorems 4.1 and 4.2 give the existence and uniqueness of distribution of each species. Finally, by numerical simulations, we illustrate the validity of our theoretical results.

Theorem 3.1 implies that stochastic parameter ξ_i ($i = 1, 2, 3$) has some important influences to the extinction, stability in mean of all species of (1.2), which is illustrated by our simulations clearly. Simulations reveal that small intensity of white noise strengthens the stability of (1.2), while large intensity of white noise will bring serious influence to the dynamical behavior.

Recently, regime switching appears in a biological system frequently, and many nice results have been obtained by many researchers. How about the white noise affecting the dynamical behavior of a predator–prey system with regime switching? We believe it is very interesting and leave it for our future work.

Appendix

A.1 Proof of Lemma 2.2

Let $N_i(t) = e^{x_i(t)}$, $y_i(t) = e^{M_i(t)}$, then (1.2) is transformed to the following equivalent system:

$$\begin{cases} dx_1(t) = (r_1 - a_{11}e^{x_1(t)} - a_{12}e^{M_2(t)} - a_{13}e^{M_3(t)})\,dt + \xi_1\,d\omega_1(t), \\ dx_2(t) = (r_2 - a_{21}e^{M_1(t)} - a_{22}e^{x_2(t)} - a_{23}e^{M_3(t)})\,dt + \xi_2\,d\omega_2(t), \\ dx_3(t) = (-r_3 + a_{31}e^{M_1(t)} + a_{32}e^{M_2(t)} - a_{33}e^{x_3(t)})\,dt + \xi_3\,d\omega_3(t), \\ dM_1(t) = \sigma_1(e^{x_1(t)-M_1(t)} - 1)\,dt, \\ dM_2(t) = \sigma_2(e^{x_2(t)-M_2(t)} - 1)\,dt, \\ dM_3(t) = \sigma_3(e^{x_3(t)-M_3(t)} - 1)\,dt. \end{cases} \tag{A.1}$$

Clearly the coefficients of (A.1) obey the local Lipschitz condition, then it has a unique local solution on $[0, \tau_e)$, where τ_e is the explosion time. According to Itô's formula, we can see that $N_i(t) = e^{x_i(t)}$, $y_i(t) = e^{M_i(t)}$ $(i = 1, 2, 3)$ is the unique positive local solution of (1.2). So we only need to prove $\tau_e = \infty$. To this end, we employ the method of Theorem 3.1 Mao et al. [33] and Zuo et al. [20]. The key step is to construct a nonnegative C^2-function $V : R^6_+ \to R_+$ such that

$$\liminf_{(N_1,N_2,N_3,y_1,y_2,y_3)\in R^6_+\setminus U_k, k\to\infty} V(N_1,N_2,N_3,y_1,y_2,y_3) = \infty$$

and

$$LV(N_1,N_2,N_3,y_1,y_2,y_3) \le M,$$

where $U_k = (\frac{1}{k}, k) \times (\frac{1}{k}, k) \times (\frac{1}{k}, k) \times (\frac{1}{k}, k) \times (\frac{1}{k}, k) \times (\frac{1}{k}, k)$ and M is a positive constant.

Define

$$\begin{aligned} V &= R_1\frac{N_1^p}{p} + R_2\frac{N_2^p}{p} + R_3\left(\frac{N_3^p}{p} + \frac{a_{31}y_1^{p+1}}{\sigma_1(p+1)} + \frac{a_{32}y_2^{p+1}}{\sigma_2(p+1)}\right) + \sum_{i=1}^{3}\frac{y_i^{p+1}}{2\sigma_i} \\ &\quad - \sum_{i=1}^{3}\ln N_i - \sum_{i=1}^{3}\ln y_i \\ &= \check{V} - \sum_{i=1}^{3}\ln N_i - \sum_{i=1}^{3}\ln y_i, \end{aligned}$$

where $p > 1$, R_1, R_2, R_3 are positive constants defined in Theorem 4.1.

Obviously,

$$\liminf_{(N_1,N_2,N_3,y_1,y_2,y_3)\in R^6_+\setminus U_k, k\to\infty} V(N_1,N_2,N_3,y_1,y_2,y_3) \to \infty \quad \text{as } k \to \infty$$

since

$$-\ln N \to \infty \quad \text{as } N \to 0^{+} \quad \text{and} \quad kN^{p} - \ln N \to \infty \quad \text{as } N \to \infty, \text{where} k > 0, p > 1.$$

On the other hand, by the proof of Theorem 4.1, we have

$$\begin{aligned} L\check{V}(t) \le{}& -\frac{1}{2}\bigl(N_1^{p+1}(t) + N_2^{p+1}(t) + N_3^{p+1}(t) + y_1^{p+1}(t) + y_2^{p+1}(t) + y_3^{p+1}(t)\bigr) \\ &+ R_1 N_1(t)^p\left(r_1 + \frac{p-1}{2}\xi_1^2\right) + R_2 N_2(t)^p\left(r_2 + \frac{p-1}{2}\xi_2^2\right) \\ &+ R_3 N_3(t)^p\left(-r_3 + \frac{p-1}{2}\xi_1^2\right). \end{aligned}$$

By Itô's formula, we have

$$\begin{aligned} &L\left(-\sum_{i=1}^{3}\ln N_i - \sum_{i=1}^{3}\ln y_i\right) \\ &\quad= -\sum_{i=1}^{2} r_i + r_3 + \sum_{i=1}^{3}\sigma_i + \sum_{i=1}^{3}\frac{\xi_i^2}{2} + \sum_{i=1}^{3} a_{ii}N_i + a_{12}y_2 + a_{13}y_3 \\ &\qquad + a_{21}y_1 + a_{23}y_3 - a_{31}y_1 - a_{32}y_2 - \sum_{i=1}^{3}\frac{N_i}{y_i} \\ &\quad\le r_3 + \sum_{i=1}^{3}\sigma_i + \sum_{i=1}^{3}\frac{\xi_i^2}{2} + \sum_{i=1}^{3} a_{ii}N_i + a_{12}y_2 + a_{13}y_3 + a_{21}y_1 + a_{23}y_3. \end{aligned}$$

Therefore,

$$LV \le r_3 + \sum_{i=1}^{3}\sigma_i + \sum_{i=1}^{3}\frac{\xi_i^2}{2} + \tilde{M},$$

where $\tilde{M} = \max_{N_i, y_i \in (0,\infty)}\{-\frac{1}{2}(N_1^{p+1}(t) + N_2^{p+1}(t) + N_3^{p+1}(t) + y_1^{p+1}(t) + y_2^{p+1}(t) + y_3^{p+1}(t)) + R_1N_1(t)^p(r_1 + \frac{p-1}{2}\xi_1^2) + R_2N_2(t)^p(r_2 + \frac{p-1}{2}\xi_2^2) + R_3N_3(t)^p(-r_3 + \frac{p-1}{2}\xi_1^2) + \sum_{i=1}^{3} a_{ii}N_i + a_{12}y_2 + a_{13}y_3 + a_{21}y_1 + a_{23}y_3\}$. Let $M = r_3 + \sum_{i=1}^{3}\sigma_i + \sum_{i=1}^{3}\frac{\xi_i^2}{2} + \tilde{M}$, then $LV(N_1, N_2, N_3, y_1, y_2, y_3) \le M$. The proof is completed.

A.2 Proof of Lemma 2.3

The methods applied here are motivated by [20]. Let

$$\check{V}(t) = R_1\frac{N_1^p}{p} + R_2\frac{N_2^p}{p} + R_3\left(\frac{N_3^p}{p} + \frac{a_{31}y_1^{p+1}}{\sigma_1(p+1)} + \frac{a_{32}y_2^{p+1}}{\sigma_2(p+1)}\right) + \sum_{i=1}^{3}\frac{y_i^{p+1}}{2\sigma_i},$$

as defined in Theorem 4.1, where

$$\begin{aligned} R_1 &= \frac{p+1+R_3a_{31}}{(p+1)a_{11}} > 0, \qquad R_2 = \frac{p+1+R_3a_{32}}{(p+1)a_{22}} > 0, \\ R_3 &= \frac{p+1}{(p+1)a_{33} - p(a_{31}+a_{32})} > 0. \end{aligned}$$

Then, by the proof of Theorem 4.1, we have

$$\begin{aligned}L\check{V}(t) &\le -\frac{1}{2}\big(N_1^{p+1}(t)+N_2^{p+1}(t)+N_3^{p+1}(t)+y_1^{p+1}(t)+y_2^{p+1}(t)+y_3^{p+1}(t)\big)\\&\quad+R_1N_1(t)^p\left(r_1+\frac{p-1}{2}\xi_1^2\right)+R_2N_2(t)^p\left(r_2+\frac{p-1}{2}\xi_2^2\right)\\&\quad+R_3N_3(t)^p\left(-r_3+\frac{p-1}{2}\xi_1^2\right).\end{aligned}$$

For any $k>0$, we compute

$$\begin{aligned}L\big(e^{kt}\check{V}(t)\big) &= ke^{kt}\check{V}(t)+e^{kt}L\check{V}(t)\\&\le e^{kt}\left(-\frac{1}{2}\big(N_1^{p+1}(t)+N_2^{p+1}(t)+N_3^{p+1}(t)\big)\right.\\&\quad+\left(-\frac{1}{2}+k\left(\frac{R_3a_{31}}{\sigma_1(p+1)}+\frac{1}{2\sigma_1}\right)\right)y_1^{p+1}\\&\quad+\left(-\frac{1}{2}+k\left(\frac{R_3a_{32}}{\sigma_1(p+1)}+\frac{1}{2\sigma_2}\right)\right)y_2^{p+1}+\left(-\frac{1}{2}+\frac{k}{2\sigma_3}\right)y_3^{p+1}(t)\\&\quad+R_1N_1(t)^p\left(r_1+\frac{p-1}{2}\xi_1^2+\frac{k}{p}\right)+R_2N_2(t)^p\left(r_2+\frac{p-1}{2}\xi_2^2+\frac{k}{p}\right)\\&\quad\left.+R_3N_3(t)^p\left(-r_3+\frac{p-1}{2}\xi_1^2+\frac{k}{p}\right)\right).\end{aligned}$$

Choosing k sufficiently small such that

$$\frac{1}{2}-k\left(\frac{R_3a_{31}}{\sigma_1(p+1)}-\frac{1}{2\sigma_1}\right)>0,\qquad \frac{1}{2}-k\left(\frac{R_3a_{32}}{\sigma_1(p+1)}-\frac{1}{2\sigma_2}\right),\qquad \frac{1}{2}-\frac{k}{2\sigma_3}>0,$$

we have

$$L\big(e^{kt}\check{V}(t)\big)\le Me^{kt},$$

where

$$\begin{aligned}M=\max_{N_i(t)>0,i=1,2,3}&\left\{-\frac{1}{2}N_1^{p+1}(t)+R_1\left(r_1+\frac{p-1}{2}\xi_1^2+\frac{k}{p}\right)N_1(t)^p-\frac{1}{2}N_2^{p+1}(t)\right.\\&\left.+R_2\left(r_2+\frac{p-1}{2}\xi_2^2+\frac{k}{p}\right)N_2(t)^p-\frac{1}{2}N_3^{p+1}(t)+R_3\left(-r_3+\frac{p-1}{2}\xi_1^2+\frac{k}{p}\right)N_3(t)^p\right\}.\end{aligned}$$

Applying the same method of Lemma 5.1 in [17] and integrating both sides of $L(e^{kt}\check{V}(t))$ and taking expectation lead to

$$E\big(\check{V}(t)\big)\le H,\quad t\ge 0,\text{a.s.},$$

where H is a constant. By the monotonicity of the expectation, we can derive that

$$E\big(N_i^p\big)\le\frac{pH}{R_i}\quad\text{and}\quad E\big(y_i^{p+1}\big)\le 2\sigma_iH,\quad i=1,2,3.$$

By the Cauchy–Schwarz inequality, there exists $\varrho(p)$ such that

$$E(y_i^p) \le \varrho(p)\big[E(y_i^{p+1}\big]^{\frac{p}{p+1}} \le \varrho(p)(2\sigma_i H)^{\frac{p}{p+1}}.$$

Denote $K(p) = \max\{\frac{pH}{R_i}, \varrho(p)(2\sigma_i H)^{\frac{p}{p+1}}, i = 1, 2, 3\}$, then

$$E(N_i^p) \le K(p) \quad \text{and} \quad E(y_i^p) \le K(p).$$

Next computing the derivative of $\check{V}(t)$ reads

$$d\check{V}(t) = L\check{V}(t)\,dt + \sum_{i=1}^{3} R_i\xi_i N_i^p(t)\,dw_i(t), \quad i = 1, 2, 3.$$

For small $\tau > 0$ enough and $n = 1, 2, \dots$, we integrate both sides of $d\check{V}(t)$ from $n\tau$ to t and take expectation, then

$$E\Big(\sup_{n\tau \le t \le (n+1)\tau} \check{V}(t)\Big) \le E\big(\check{V}(n\tau)\big) + E\bigg(\sup_{n\tau \le t \le (n+1)\tau}\bigg|\int_{n\tau}^{t} L\check{V}(s)\,ds\bigg|\bigg) + \sum_{i=1}^{3} R_i\xi_i E\Big(\sup_{n\tau \le t \le (n+1)\tau} \int_{n\tau}^{t} N_i^p\,dw_i(s)\Big|.$$

Again using a similar proof of Lemma 5.1 in [17], for any positive constant ϵ and any finitely many n, one can derive that

$$\sup_{n\tau \le t \le (n+1)\tau} \check{V}(t)) \le (n\tau)^{1+\epsilon}, \quad \text{a.s.}$$

Letting $\epsilon \to 0$ leads to

$$\limsup_{t\to\infty} \frac{\ln \check{V}(t)}{\ln t} \le 1, \quad \text{a.s.},$$

which implies

$$\limsup_{t\to\infty} \frac{\ln y_i(t)}{\ln t} \le \frac{1}{p+1}, \quad i = 1, 2, 3, \text{a.s.}$$

Fixing $\epsilon_0 = \frac{p}{2(p+1)}$, then there exists $T > 0$ such that $\ln y_i(t) \le (\frac{1}{p+1} + \epsilon_0)\ln t$ for all $t > T$. Therefore,

$$\limsup_{t\to\infty} \frac{y_i(t)}{t} \le \limsup_{t\to\infty} t^{\epsilon_0 - \frac{p}{p+1}} = 0, \quad \text{a.s.}$$

Together with the positivity of $y_i(t)$, we have

$$\lim_{t\to\infty} \frac{y_i(t)}{t} = 0.$$

Similarly, we can derive that

$$\limsup_{t\to\infty}\frac{\ln\frac{R_i}{p}N_i^p(t)}{\ln t}\le 1,\quad \text{a.s.},$$

that is, $\limsup_{t\to\infty}\frac{\ln N_i(t)}{\ln t}\le\frac{1}{p}$, a.s. By the same deduction, we have

$$\limsup_{t\to\infty}\frac{\ln N_i(t)}{t}\le\left(\frac{1}{p}+\epsilon_0\right)\limsup_{t\to\infty}\frac{\ln t}{t}=0,\quad i=1,2,3,\text{a.s.}$$

The proof is confirmed.

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Authors' contributions
JZ carried out all studies and drafted the manuscript. YS conceived of the study and participated in its design and coordination and helped to draft the manuscript, YS performed the simulation analysis. All authors read and approved the final manuscript. All authors contributed equally to the writing of this paper.

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