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Dynamics of a new stage-structured population model with transient and nontransient impulsive effects in a polluted environment

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Abstract

In this paper, we consider a new stage-structured population model with transient and nontransient impulsive effects in a polluted environment. By using the theories of impulsive differential equations, we obtain the globally asymptotically stable condition of a population-extinction solution; we also present the permanent condition for the investigated system. The results indicate that the nontransient and transient impulsive harvesting rate play important roles in system permanence. Finally, numerical analyses are carried out to illustrate the results. Our results provide effective methods for biological resource management in a polluted environment.

Keywords: Stage-structured population model; Transient and nontransient impulsive effects; Polluted environment; Permanence

1 Introduction

In recent years, the aggravation of environmental pollution not only affects human life style, but also poses a serious threat to the long-term survival of the species. The European Environmental Protection Agency released a new "Health and Environmental Assessment Report" on September 8, 2020, which said that 13 percent of the deaths in 28 European countries are related to environmental pollution. Therefore, the study of the population models in polluted environments is becoming more important. To date, some work has been carried out to study the population models in polluted environments [1–7]. Many workers have adopted a mathematical modeling approach to study the influence of environmental pollution on the survival of biological populations [8–10]. Most of the previous studies assumed that the input of the toxicant was continuous. The toxicants, however, are often emitted to the environment in regular pulses [11]. For example, the spraying of agricultural chemicals can be regarded as time-pulse discharge, though the discharge of the toxin is transient, the influence of the toxin will be long lasting.

Currently, the population system with a stage structure has become another focus of many studies [12–15]. Cai [16] presented a stage-structured single-species model with

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pulse input in a polluted environment and revealed that a long mature period of the population in a polluted environment can cause it to go extinct. Kang [17] proposed and studied an age-structured population with nonlocal diffusion. Jiao [18] investigated a stage-structured single-population model with nontransient and transient impulsive effects.

2 The model

In real life, when facing pollutants from the environment, a mature population and an immature population have different reactions. Considering the population with different stage structures has more practical significance. Inspired by the above discussions, we consider a new stage-structured population model with transient and nontransient impulsive effects in a polluted environment:

$$
\begin{cases}
\left.\begin{array}{l}
\frac{dx(t)}{dt} = -(c_1 + d_1)x(t) - \beta_1 c_o(t)x(t), \\
\frac{dy(t)}{dt} = c_1 x(t) - d_2 y(t) - \beta_2 c_o(t)x(t), \\
\frac{dc_o(t)}{dt} = f c_{e1}(t) - (g + m)c_o(t), \\
\frac{dc_{e1}(t)}{dt} = -h_1 c_{e1}(t), \\
\frac{dc_{e2}(t)}{dt} = -h_2 c_{e2}(t),
\end{array}\right\} & t \in \left(n\tau, (n+l)\tau\right], \\
\left.\begin{array}{l}
\Delta x(t) = -u_1 x(t), \\
\Delta y(t) = -u_2 y(t), \\
\Delta c_o(t) = 0, \\
\Delta c_{e1}(t) = 0, \\
\Delta c_{e2}(t) = 0,
\end{array}\right\} & t = (n+l)\tau, \\
\left.\begin{array}{l}
\frac{dx(t)}{dt} = -(c_2 + d_3 + E_1)x(t) - \beta_1 c_o(t)x(t), \\
\frac{dy(t)}{dt} = c_2 x(t) - (d_4 + E_2)y(t) - \beta_2 c_o(t)x(t), \\
\frac{dc_o(t)}{dt} = f c_{e1}(t) - (g + m)c_o(t), \\
\frac{dc_{e1}(t)}{dt} = -h_1 c_{e1}(t), \\
\frac{dc_{e2}(t)}{dt} = -h_2 c_{e2}(t),
\end{array}\right\} & t \in \left((n+l)\tau, (n+1)\tau\right], \\
\left.\begin{array}{l}
\Delta x(t) = y(t)(a - by(t)), \\
\Delta y(t) = 0, \\
\Delta c_o(t) = 0, \\
\Delta c_{e1}(t) = d(c_{e2}(t) - c_{e1}(t)) + v_1, \\
\Delta c_{e2}(t) = d(c_{e1}(t) - c_{e2}(t)) + v_2
\end{array}\right\} & t = (n+1)\tau,
\end{cases}
\tag{2.1}
$$

where it is assumed that system (2.1) consists of two lakes that are connected by underground rivers. Environmental toxins will be dispersed between the two lakes due to weather conditions, such as rainy season or flood outbreaks. $x(t)$, $y(t)$ represent the densities of the immature and mature populations, which depend on drinking the water from lake 1, at time t, respectively. $c_o(t)$ represents the average concentration of toxins in the organism of the immature and mature populations at time t. $c_{e1}(t)$ represents the concentration of environmental toxins in lake 1 at time t. $c_{e2}(t)$ represents the concentration of environmental toxins in lake 2 at time t. $c_1 > 0$ represents the rate of immature population x turning into mature population y on $(n\tau, (n+l)\tau]$. $d_1 > 0$ represents the natural death

rate of population x on $(n\tau, (n + l)\tau]$. $d_2 > 0$ represents the natural death rate of population y on $(n\tau, (n + l)\tau]$. $\beta_1 > 0$ and $\beta_2 > 0$ represent the mortality coefficient of the immature population and the mature population due to the influence of toxins, respectively. $f > 0$ represents the uptake rate of toxin from lake 1 per unit biomass. $g > 0$ represents the toxin-consumption coefficient of the population by means of excretion and so on. $m > 0$ represents the toxin-consumption coefficient in the population by means of biochemical reactions in the body. $h_1 > 0$ represents the consumption coefficient of the environmental toxins with lake 1 as the water source is affected by processes such as chemical hydrolysis, volatilization, microbial degradation and photosynthesis on $((n + l)\tau, (n + 1)\tau]$. $h_2 > 0$ represents the consumption coefficient of environmental toxins with lake 2 as the water source affected by processes such as chemical hydrolysis, volatilization, microbial degradation and photosynthesis on $((n + l)\tau, (n + 1)\tau]$. $0 < u_1 < 1$ represents the transient impulsive harvesting rate of population x at time $t = (n + l)\tau$. $0 < u_2 < 1$ represents the transient impulsive harvesting rate of population y at time $t = (n + l)\tau$. $c_2 > 0$ represents the rate of immature population x turning into mature population y on $((n + l)\tau, (n + 1)\tau]$. $d_3 > 0$ represents the natural death rate of population x on $((n + l)\tau, (n + 1)\tau]$. $d_4 > 0$ represents the natural death rate of population y on $((n + l)\tau, (n + 1)\tau]$. The population is birth pulse as intrinsic rate of nature increase and density dependence rate of the population, which are denoted by $a > 0$, $b > 0$, respectively. $E_1 > 0$ represents the nontransient impulsive harvesting rate of the immature population x on $((n + l)\tau, (n + 1)\tau]$. $E_2 > 0$ represents the nontransient impulsive harvesting rate of the mature population y on $((n + l)\tau, (n + 1)\tau]$. $0 < d < 1$ is the dispersal rate between the two lakes. $v_1 > 0$ represents the concentration of toxins that input into lake 1 due to environmental changes at time $t = (n + 1)\tau$. $v_2 > 0$ represents the concentration of toxins that input into lake 2 due to environmental changes at time $t = (n + 1)\tau$. τ is the period of the population-impulsive harvesting or pulse-input period of toxins.

3 The dynamics

Denoting $U(t) = (x(t), y(t), c_o(t), c_{e1}(t), c_{e2}(t))^T$, the solution of system (2.1), is a piecewise continuous $U : R_+ \to R_+^5$, where $R_+ = [0, \infty)$, $R_+^5 = \{Z \in R^5 : U > 0\}$. $U(t)$ is continuous on $(n\tau, (n + l)\tau]$ and $((n + l)\tau, (n + 1)\tau]$. According to Ref. [19], the global existence and uniqueness of the solution of system (2.1) is guaranteed by the smoothness properties of f_1, which denotes the mapping defined by the right-side of system (2.1).

The subsystem of system (2.1) is

$$
\begin{cases}
\left.\begin{array}{l}
\frac{dx(t)}{dt} = -(c_1 + d_1)x(t), \\
\frac{dy(t)}{dt} = c_1 x(t) - d_2 y(t),
\end{array}\right\} \quad t \in \left(n\tau, (n + l)\tau\right], \\
\left.\begin{array}{l}
\Delta x(t) = -u_1 x(t), \\
\Delta y(t) = -u_2 y(t),
\end{array}\right\} \quad t = (n + l)\tau, \\
\left.\begin{array}{l}
\frac{dx(t)}{dt} = -(c_2 + d_3 + E_1)x(t), \\
\frac{dy(t)}{dt} = c_2 x(t) - (d_4 + E_2)y(t),
\end{array}\right\} \quad t \in \left((n + l)\tau, (n + 1)\tau\right], \\
\left.\begin{array}{l}
\Delta x(t) = y(t)(a - by(t)), \\
\Delta y(t) = 0,
\end{array}\right\} \quad t = (n + 1)\tau.
\end{cases}
\tag{3.1}
$$

Considering the first and second equations and the fifth and sixth equations of system (3.1), we can obtain the analytic solution of system (3.1) between pluses as

$$
\begin{cases}
x(t) = \begin{cases}
x(n\tau^+)e^{-(c_1+d_1)(t-n\tau)}, & t \in (n\tau, (n+l)\tau], \\
x((n+l)\tau^+)e^{-(c_2+d_3+E_1)(t-(n+l)\tau)}, & t \in ((n+l)\tau, (n+1)\tau],
\end{cases} \\
y(t) = \begin{cases}
e^{-d_2(t-n\tau)}\big[\frac{c_1(1-e^{-(c_1+d_1-d_2)(t-n\tau)})}{c_1+d_1-d_2}x(n\tau^+)+y(n\tau^+)\big], \\
\quad t \in (n\tau, (n+l)\tau], \\
e^{-(d_4+E_2)(t-(n+l)\tau)}\big[\frac{c_2(1-e^{-(c_2+d_3+E_1-d_4-E_2)(t-(n+l)\tau)})}{c_2+d_3+E_1-d_4-E_2}x \\
\quad \times ((n+l)\tau^+)+y((n+l)\tau^+)\big], \\
\quad t \in ((n+l)\tau, (n+1)\tau].
\end{cases}
\end{cases}
\tag{3.2}
$$

Considering the third and fourth equations and the seventh and eighth equations of system (3.1), we obtain the stroboscopic map of system (3.1)

$$
\begin{cases}
x((n+1)\tau^+) = (aA+C)x(n\tau^+)+aBy(n\tau^+)-b(Ax(n\tau^+)+By(n\tau^+))^2, \\
y((n+1)\tau^+) = Ax(n\tau^+)+By(n\tau^+),
\end{cases}
\tag{3.3}
$$

where

$$
\begin{aligned}
A &= e^{-(d_4+E_2)(1-l)\tau}\Big[\frac{c_2(1-u_1)e^{-(c_1+d_1)l\tau}(1-e^{-(c_2+d_3+E_1-d_4-E_2)(1-l)\tau})}{c_2+d_3+E_1-d_4-E_2} \\
&\quad + \frac{c_1(1-u_2)e^{-d_2l\tau}(1-e^{-(c_1+d_1-d_2)l\tau})}{c_1+d_1-d_2}\Big], \\
B &= (1-u_2)e^{-[d_2l\tau+(d_4+E_2)(1-l)\tau]} < 1, \\
C &= (1-u_1)e^{-[(c_1+d_1)l\tau+(c_2+d_3+E_1)(1-l)\tau]} < 1.
\end{aligned}
$$

The system (3.3) has two fixed points as $F_1(0,0)$ and $F_2(x^*, y^*)$, where

$$
\begin{cases}
x^* = \frac{(1-B)[aA-(1-B)(1-C)]}{bA^2}, & aA-(1-B)(1-C) > 0, \\
y^* = \frac{aA-(1-B)(1-C)}{bA}, & aA-(1-B)(1-C) > 0.
\end{cases}
\tag{3.4}
$$

Theorem 1 (*i*) *If* $aA-(1-B)(1-C) < 0$, *the fixed point* $F_1(0,0)$ *is globally asymptotically stable*;

(*ii*) *If* $aA-(1-B)(1-C) > 0$, *the fixed point* $F_2(x^*, y^*)$ *is globally asymptotically stable.*

Proof For convenience, we make a notation as $(x^n, y^n) = (x(n\tau^+), y(n\tau^+))$. The linear form of (3.3) can be written as

$$
\begin{pmatrix} x^{n+1} \\ y^{n+1} \end{pmatrix} = J_1 \begin{pmatrix} x^n \\ y^n \end{pmatrix}.
\tag{3.5}
$$

Obviously, the near dynamics of $F_1(0,0)$ and $F_2(x^*, y^*)$ of (3.3) are determined by the linear system (3.5). The stabilities of the two fixed points of (3.3) are determined by the

eigenvalues of J_1 less than 1. We can determine the eigenvalue of J_1 less than 1, if J_1 satisfies the Jury criteria [20]

$$1 - \operatorname{tr} J_1 + \det J_1 > 0. \tag{3.6}$$

(i) If $aA - (1 - B)(1 - C) < 0$, $F_1(0,0)$ is the unique fixed point of (3.3), we have

$$J_1 = \begin{pmatrix} aA + C & aB \\ A & B \end{pmatrix}. \tag{3.7}$$

Calculating

$$\begin{aligned}
1 - \operatorname{tr} J_1 + \det J_1 &= 1 - \big[(aA + C) + B\big] + \big[B(aA + C) - aAB\big] \\
&= 1 - aA - C - B + BC \\
&= -aA + (1 - B)(1 - C) \\
&= -\big[aA - (1 - B)(1 - C)\big] > 0.
\end{aligned}$$

From the Jury criteria, $F_1(0,0)$ is locally stable. Then, it is globally asymptotically stable. (ii) If $aA - (1 - B)(1 - C) > 0$, $F_1(0,0)$ is obviously unstable, $F_2(x^*, y^*)$ exists, and

$$J_1 = \begin{pmatrix} aA + C - 2bA(Ax^* + By^*) & aB - 2bB(Ax^* + By^*) \\ A & B \end{pmatrix}. \tag{3.8}$$

Calculating

$$\begin{aligned}
1 - \operatorname{tr} J_1 + \det J_1 &= 1 - \big[aA + C - 2bA(Ax^* + By^*) + B\big] \\
&\quad + \big\{B\big[aA + C - 2bA(Ax^* + By^*)\big] - A\big[aB - 2bB(Ax^* + By^*)\big]\big\} \\
&= 1 - aA - C - B + BC + 2bA(Ax^* + By^*) \\
&= -\big[aA + (1 - B)(1 - C)\big] + 2\big[aA - (1 - B)(1 - C)\big] \\
&= aA - (1 - B)(1 - C) > 0.
\end{aligned}$$

From the Jury criteria, $F_2(x^*, y^*)$ is locally stable. Then, it is globally asymptotically stable. This completes the proof. □

According to Theorem 1, and similar to reference [18], the following lemma can be easily proved.

Theorem 2 (*i*) *If* $aA - (1 - B)(1 - C) < 0$, *the triviality periodic solution* $(0,0)$ *of system* (3.1) *is globally asymptotically stable*;

(ii) If $aA - (1 - B)(1 - C) > 0$, the periodic solution $(\widetilde{x(t)}, \widetilde{y(t)})$ of system (3.1) is globally asymptotically stable, where

$$
\begin{cases}
\widetilde{x(t)} = \begin{cases} x^* e^{-(c_1+d_1)(t-n\tau)}, & t \in (n\tau, (n+l)\tau], \\ x^{**} e^{-(c_2+d_3+E_1)(t-(n+l)\tau)}, & t \in ((n+l)\tau, (n+1)\tau], \end{cases} \\
\widetilde{y(t)} = \begin{cases} e^{-d_2(t-n\tau)}\big[\frac{c_1(1-e^{-(c_1+d_1-d_2)(t-n\tau)})}{c_1+d_1-d_2} x^* + y^*\big], & t \in (n\tau, (n+l)\tau], \\ e^{-(d_4+E_2)(t-(n+l)\tau)}\big[\frac{c_2(1-e^{-(c_2+d_3+E_1-d_4-E_2)(t-(n+l)\tau)})}{c_2+d_3+E_1-d_4-E_2} x^{**} + y^{**}\big], \\ \qquad t \in ((n+l)\tau, (n+1)\tau], \end{cases}
\end{cases}
\tag{3.9}
$$

and

$$
\begin{cases}
x^{**} = (1 - u_1)e^{-(c_1+d_1)l\tau} x^*, \\
y^{**} = (1 - u_2)e^{-d_2 l\tau}\big[\frac{c_1(1-e^{-(c_1+d_1-d_2)l\tau})}{c_1+d_1-d_2} x^* + y^*\big].
\end{cases}
$$

Remark 3 From Theorem 2, For any $\varepsilon > 0$, there exists a positive number t_0, when $t > t_0$, we have

$$
\widetilde{x(t)} - \varepsilon \leq x(t) \leq \widetilde{x(t)} + \varepsilon,
$$
$$
\widetilde{y(t)} - \varepsilon \leq y(t) \leq \widetilde{y(t)} + \varepsilon,
$$

then

$$
m_1 \leq x(t) \leq M_1,
$$
$$
m_2 \leq y(t) \leq M_2,
$$

where

$$
m_1 = \big[x^* + x^{**}\big] - \varepsilon,
$$
$$
M_1 = \big[x^* e^{-(c_1+d_1)l\tau} + x^{**} e^{-(c_2+d_3+E_1)(1-l)\tau}\big] + \varepsilon,
$$
$$
m_2 = \big[y^* + y^{**}\big] - \varepsilon,
$$
$$
M_2 = \Bigg\{ e^{-d_2 l\tau}\Bigg[\frac{c_1(1 - e^{-(c_1+d_1-d_2)l\tau})}{c_1 + d_1 - d_2} x^* + y^*\Bigg]
$$
$$
+ e^{-(d_4+E_2)(1-l)\tau}\Bigg[\frac{c_2(1 - e^{-(c_2+d_3+E_1-d_4-E_2)(1-l)\tau})}{c_2 + d_3 + E_1 - d_4 - E_2} x^{**} + y^{**}\Bigg]\Bigg\} + \varepsilon.
$$

Another subsystem of system (2.1) is also obtained as follows:

$$
\begin{cases}
\frac{dc_o(t)}{dt} = f c_{e1}(t) - (g + m)c_o(t), \\
\frac{dc_{e1}(t)}{dt} = -h_1 c_{e1}(t), \\
\frac{dc_{e2}(t)}{dt} = -h_2 c_{e2}(t),
\end{cases} \quad t \in \big(n\tau, (n+1)\tau\big], \\
\begin{cases}
\Delta c_o(t) = 0, \\
\Delta c_{e1}(t) = d(c_{e2}(t) - c_{e1}(t)) + v_1, \\
\Delta c_{e2}(t) = d(c_{e1}(t) - c_{e2}(t)) + v_2
\end{cases} \quad t = n\tau.
\tag{3.10}
$$

The first, second and third equations of system (3.10) integrate over the interval $(n\tau, (n+1)\tau]$, we have

$$
\begin{cases}
c_o(t) = e^{-(g+m)(t-n\tau)}\left[\frac{f(1-e^{-(h_1-g-m)(t-n\tau)})}{h_1-g-m}c_{e1}(n\tau^+) + c_o(n\tau^+)\right], \\
\quad t \in (n\tau, (n+1)\tau], \\
c_{e1}(t) = c_{e1}(n\tau^+)e^{-h_1(t-n\tau)}, \quad t \in (n\tau, (n+1)\tau], \\
c_{e2}(t) = c_{e2}(n\tau^+)e^{-h_2(t-n\tau)}, \quad t \in (n\tau, (n+1)\tau].
\end{cases}
\tag{3.11}
$$

Considering the fourth, fifth and sixth equations of system (3.10), the stroboscopic map of system (3.11) is obtained as

$$
\begin{cases}
c_o((n+1)\tau^+) = e^{-(g+m)\tau}\left[\frac{f(1-e^{-(h_1-g-m)\tau})}{h_1-g-m}c_{e1}(n\tau^+) + c_o(n\tau^+)\right], \\
c_{e1}((n+1)\tau^+) = (1-d)c_{e1}(n\tau^+)e^{-h_1 n\tau} + dc_{e2}(n\tau^+)e^{-h_2 n\tau} + v_1, \\
c_{e2}((n+1)\tau^+) = (1-d)c_{e2}(n\tau^+)e^{-h_2 n\tau} + dc_{e1}(n\tau^+)e^{-h_1 n\tau} + v_2.
\end{cases}
\tag{3.12}
$$

The unique fixed point of (3.12) is obtained as $F(c_o^*, c_{e1}^*, c_{e2}^*)$, where

$$
\begin{cases}
c_o^* = \frac{e^{-(g+m)\tau}f(1-e^{-(h_1-g-m)\tau})v_2 de^{-h_2\tau}+v_1[1-(1-d)e^{-h_2\tau}]}{(h_1-g-m)(1-e^{-(g+m)\tau})[1-(1-d)e^{-h_1\tau}](1-e^{-h_2\tau})+de^{-h_2\tau}(1-e^{-h_1\tau})}, \\
c_{e1}^* = \frac{v_2 de^{-h_2\tau}+v_1[1-(1-d)e^{-h_2\tau}]}{[1-(1-d)e^{-h_1\tau}](1-e^{-h_2\tau})+de^{-h_2\tau}(1-e^{-h_1\tau})}, \\
c_{e2}^* = \frac{v_1 de^{-h_1\tau}+v_2[1-(1-d)e^{-h_1\tau}]}{[1-(1-d)e^{-h_1\tau}](1-e^{-h_2\tau})+de^{-h_2\tau}(1-e^{-h_1\tau})}.
\end{cases}
\tag{3.13}
$$

Theorem 4 *If $d > 1/2$, the unique fixed point $F(c_o^*, c_{e1}^*, c_{e2}^*)$ is globally asymptotically stable.*

Proof Making a notation as $(c_o^n, c_{e1}^n, c_{e2}^n) = (c_o(n\tau^+), c_{e1}(n\tau^+), c_{e2}(n\tau^+))$, we rewrite the linear form of (3.12) as

$$
\begin{pmatrix} c_o^{n+1} \\ c_{e1}^{n+1} \\ c_{e2}^{n+1} \end{pmatrix} = J_2 \begin{pmatrix} c_o^n \\ c_{e1}^n \\ c_{e2}^n \end{pmatrix}.
\tag{3.14}
$$

Obviously, the near dynamics of $F(c_o^*, c_{e1}^*, c_{e2}^*)$ is determined by the linear system (3.14). The stabilities of the fixed point of (3.12) are determined by the eigenvalues of J_2 less than 1. According to the condition of this theorem and $0 < e^{-h_1\tau} < 1$, $0 < e^{-h_2\tau} < 1$, it is easy to obtain the eigenvalues of

$$
J_2 = \begin{pmatrix} e^{-(g+m)\tau} & e^{-(g+m)\tau}\frac{f(1-e^{-(h_1-g-m)\tau})}{h_1-g-m} & 0 \\ 0 & (1-d)e^{-h_1\tau} & de^{-h_2\tau} \\ 0 & de^{-h_1\tau} & (1-d)e^{-h_2\tau} \end{pmatrix},
\tag{3.15}
$$

which are

$$
\lambda_1 = e^{-(g+m)\tau} < 1,
$$

$$
\lambda_2 = \frac{(1-d)(e^{-h_1\tau} + e^{-h_2\tau}) + \sqrt{[d(e^{-h_1\tau} + e^{-h_2\tau})]^2 - 4(1-2d)e^{-(h_1+h_2)\tau}}}{2}
$$

$$< \frac{(1-d)(e^{-h_1\tau} + e^{-h_2\tau}) + d(e^{-h_1\tau} + e^{-h_2\tau})}{2}$$

$$= \frac{e^{-h_1\tau} + e^{-h_2\tau}}{2} < 1,$$

$$\lambda_3 = \frac{(1-d)(e^{-h_1\tau} + e^{-h_2\tau}) - \sqrt{[d(e^{-h_1\tau} + e^{-h_2\tau})]^2 - 4(1-2d)e^{-(h_1+h_2)\tau}}}{2}$$

$$< \frac{(1-d)(e^{-h_1\tau} + e^{-h_2\tau})}{2} < 1.$$

Hence, $F(c_o^*, c_{e1}^*, c_{e2}^*)$ is locally stable. Then, it is globally asymptotically stable. □

Theorem 5 *If $d > 1/2$, the periodic solution $(\widetilde{c_o(t)}, \widetilde{c_{e1}(t)}, \widetilde{c_{e2}(t)})$ of system* (3.10) *is globally asymptotically stable, where*

$$\begin{cases} \widetilde{c_o(t)} = e^{-(g+m)(t-n\tau)}\left[\frac{f(1-e^{-(h_1-g-m)(t-n\tau)})}{h_1-g-m}c_{e1}^* + c_o^*\right], & t \in (n\tau, (n+1)\tau], \\ \widetilde{c_{e1}(t)} = c_{e1}^* e^{-h_1(t-n\tau)}, & t \in (n\tau, (n+1)\tau], \\ \widetilde{c_{e2}(t)} = c_{e2}^* e^{-h_2(t-n\tau)}, & t \in (n\tau, (n+1)\tau], \end{cases} \quad (3.16)$$

Here, $c_o^*, c_{e1}^*, c_{e2}^*$ are defined as (3.13).

Remark 6 From Theorem 5, for any $\varepsilon > 0$, there exists a positive number t_0, when $t > t_0$, we have

$$\widetilde{c_o(t)} - \varepsilon \le c_o(t) \le \widetilde{c_o(t)} + \varepsilon,$$

$$\widetilde{c_{e1}(t)} - \varepsilon \le c_{e1}(t) \le \widetilde{c_{e1}(t)} + \varepsilon,$$

$$\widetilde{c_{e2}(t)} - \varepsilon \le c_{e2}(t) \le \widetilde{c_{e2}(t)} + \varepsilon,$$

then

$$m_o \le c_o(t) \le M_o,$$

$$m_{e1} \le c_{e1}(t) \le M_{e1},$$

$$m_{e2} \le c_{e2}(t) \le M_{e2},$$

where

$$m_0 = \frac{f}{h_1 - g - m}c_{e1}^* + c_o^* - \varepsilon,$$

$$M_0 = e^{-(g+m)\tau}\left[\frac{f(1 - e^{-(h_1-g-m)\tau})}{h_1 - g - m}c_{e1}^* + c_o^*\right] + \varepsilon,$$

$$m_{e1} = c_{e1}^* - \varepsilon,$$

$$M_{e1} = c_{e1}^* e^{-h_1\tau} + \varepsilon,$$

$$m_{e2} = c_{e2}^* - \varepsilon,$$

$$M_{e2} = c_{e2}^* e^{-h_2\tau} + \varepsilon.$$

Considering the first and second equations and the eleventh and twelfth equations of system (2.1), we obtain

$$
\begin{cases}
\left.\begin{aligned}
\frac{dx(t)}{dt} &\le -(c_1 + d_1)x(t), \\
\frac{dy(t)}{dt} &\le c_1 x(t) - d_2 y(t),
\end{aligned}\right\} \quad t \in \big(n\tau, (n+l)\tau\big], \\[2mm]
\left.\begin{aligned}
\Delta x(t) &= -u_1 x(t), \\
\Delta y(t) &= -u_2 y(t),
\end{aligned}\right\} \quad t = (n+l)\tau, \\[2mm]
\left.\begin{aligned}
\frac{dx(t)}{dt} &\le -(c_2 + d_3 + E_1)x(t), \\
\frac{dy(t)}{dt} &\le c_2 x(t) - (d_4 + E_2)y(t),
\end{aligned}\right\} \quad t \in \big((n+l)\tau, (n+1)\tau\big], \\[2mm]
\left.\begin{aligned}
\Delta x(t) &= y(t)(a - by(t)), \\
\Delta y(t) &= 0,
\end{aligned}\right\} \quad t = (n+1)\tau,
\end{cases}
\tag{3.17}
$$

and

$$
\begin{cases}
\left.\begin{aligned}
\frac{dx(t)}{dt} &\ge -(c_1 + d_1 + \beta_1(\widetilde{c_o(t)} + \varepsilon))x(t), \\
\frac{dy(t)}{dt} &\ge c_1 x(t) - (d_2 + \beta_2(\widetilde{c_o(t)} + \varepsilon))y(t),
\end{aligned}\right\} \quad t \in \big(n\tau, (n+l)\tau\big], \\[2mm]
\left.\begin{aligned}
\Delta x(t) &= -u_1 x(t), \\
\Delta y(t) &= -u_2 y(t),
\end{aligned}\right\} \quad t = (n+l)\tau, \\[2mm]
\left.\begin{aligned}
\frac{dx(t)}{dt} &\ge -(c_2 + d_3 + E_1 + \beta_1(\widetilde{c_o(t)} + \varepsilon))x(t), \\
\frac{dy(t)}{dt} &\ge c_2 x(t) - (d_4 + E_2 + \beta_2(\widetilde{c_o(t)} + \varepsilon))y(t),
\end{aligned}\right\} \quad t \in \big((n+l)\tau, (n+1)\tau\big], \\[2mm]
\left.\begin{aligned}
\Delta x(t) &= y(t)(a - by(t)), \\
\Delta y(t) &= 0,
\end{aligned}\right\} \quad t = (n+1)\tau.
\end{cases}
\tag{3.18}
$$

Then, we can obtain the comparative differential equation of system (3.17)

$$
\begin{cases}
\left.\begin{aligned}
\frac{dx_1(t)}{dt} &= -(c_1 + d_1)x_1(t), \\
\frac{dy_1(t)}{dt} &= c_1 x_1(t) - d_2 y_1(t),
\end{aligned}\right\} \quad t \in \big(n\tau, (n+l)\tau\big], \\[2mm]
\left.\begin{aligned}
\Delta x_1(t) &= -u_1 x_1(t), \\
\Delta y_1(t) &= -u_2 y_1(t),
\end{aligned}\right\} \quad t = (n+l)\tau, \\[2mm]
\left.\begin{aligned}
\frac{dx_1(t)}{dt} &= -(c_2 + d_3 + E_1)x_1(t), \\
\frac{dy_1(t)}{dt} &= c_2 x_1(t) - (d_4 + E_2)y_1(t),
\end{aligned}\right\} \quad t \in \big((n+l)\tau, (n+1)\tau\big], \\[2mm]
\left.\begin{aligned}
\Delta x_1(t) &= y_1(t)(a - by_1(t)), \\
\Delta y_1(t) &= 0,
\end{aligned}\right\} \quad t = (n+1)\tau,
\end{cases}
\tag{3.19}
$$

and the comparative differential equation of system (3.18)

$$
\begin{cases}
\begin{rcases}
\frac{dx_2(t)}{dt} = -(c_1 + d_1 + \beta_1(\widetilde{c_o(t)} + \varepsilon))x_2(t), \\
\frac{dy_2(t)}{dt} = c_1 x(t) - (d_2 + \beta_2(\widetilde{c_o(t)} + \varepsilon))y_2(t),
\end{rcases} & t \in (n\tau, (n+l)\tau], \\[2mm]
\begin{rcases}
\Delta x_2(t) = -u_1 x_2(t), \\
\Delta y_2(t) = -u_2 y_2(t),
\end{rcases} & t = (n+l)\tau, \\[2mm]
\begin{rcases}
\frac{dx_2(t)}{dt} = -(c_2 + d_3 + E_1 + \beta_1(\widetilde{c_o(t)} + \varepsilon))x_2(t), \\
\frac{dy_2(t)}{dt} = c_2 x_2(t) - (d_4 + E_2 + \beta_2(\widetilde{c_o(t)} + \varepsilon))y_2(t),
\end{rcases} & t \in ((n+l)\tau, (n+1)\tau], \\[2mm]
\begin{rcases}
\Delta x_2(t) = y_2(t)(a - by_2(t)), \\
\Delta y_2(t) = 0,
\end{rcases} & t = (n+1)\tau.
\end{cases}
\tag{3.20}
$$

Similarly with Theorem 1 and Theorem 2, we have:

Theorem 7 (*i*) *If $aA_1 - (1 - B_1)(1 - C_1) < 0$, the triviality periodic solution of system* (3.20) *is globally asymptotically stable*;

(*ii*) *If $aA_1 - (1 - B_1)(1 - C_1) > 0$, the triviality periodic solution of system* (3.20) *is unstable, and the periodic solution $(\overline{x_2(t)}, \overline{y_2(t)})$ of system* (3.20) *is globally asymptotically stable, where*

$$
\begin{cases}
\overline{x_2(t)} =
\begin{cases}
x_2^* e^{-(c_1 + d_1 + \beta_1 M_o)(t - n\tau)}, & t \in (n\tau, (n+l)\tau], \\
x_2^{**} e^{-(c_2 + d_3 + E_1 + \beta_1 M_o)(t - (n+l)\tau)}, & t \in ((n+l)\tau, (n+1)\tau],
\end{cases} \\[4mm]
\overline{y_2(t)} =
\begin{cases}
e^{-(d_2 + \beta_2 M_o)(t - n\tau)} \left[\frac{c_1(1 - e^{-(c_1 + d_1 + \beta_1 M_o - d_2 - \beta_2 M_o)(t - n\tau)})}{c_1 + d_1 + \beta_1 M_o - d_2 - \beta_2 M_o} x_2^* + y_2^* \right], \\
\qquad t \in (n\tau, (n+l)\tau], \\
e^{-(d_4 + E_2 + \beta_2 M_o)(t - (n+l)\tau)} \\
\qquad \times \left[\frac{c_2(1 - e^{-(c_2 + d_3 + E_1 + \beta_1 M_o - d_4 - E_2 - \beta_2 M_o)(t - (n+l)\tau)})}{c_2 + d_3 + E_1 + \beta_1 M_o - d_4 - E_2 - \beta_2 M_o} x_2^{**} + y_2^{**} \right], \\
\qquad t \in ((n+l)\tau, (n+1)\tau],
\end{cases}
\end{cases}
\tag{3.21}
$$

Here, $x_2^*, y_2^*, x_2^{**}, y_2^{**}$ are defined as follows:

$$
\begin{cases}
x_2^* = \frac{(1 - B_1)[aA_1 - (1 - B_1)(1 - C_1)]}{bA_1^2}, & aA_1 - (1 - B_1)(1 - C_1) > 0, \\[2mm]
y_2^* = \frac{aA_1 - (1 - B_1)(1 - C_1)}{bA_1}, & aA_1 - (1 - B_1)(1 - C_1) > 0,
\end{cases}
\tag{3.22}
$$

and

$$
\begin{cases}
x_2^{**} = (1 - u_1) e^{-(c_1 + d_1 + \beta_1 M_o)l\tau} x_2^*, \\[2mm]
y_2^{**} = (1 - u_2) e^{-(d_2 + \beta_2 M_o)l\tau} \left[\frac{c_1(1 - e^{-(c_1 + d_1 + \beta_1 M_o - d_2 - \beta_2 M_o)l\tau})}{c_1 + d_1 + \beta_1 M_o - d_2 - \beta_2 M_o} x_2^* + y_2^* \right].
\end{cases}
\tag{3.23}
$$

where

$$
A_1 = e^{-(d_4 + E_2 + \beta_2 M_o)(1 - l)\tau} \left[\frac{c_2(1 - u_1) e^{-(c_1 + d_1 + \beta_1 M_o)l\tau} (1 - e^{-(c_2 + d_3 + E_1 + \beta_1 M_o - d_4 - E_2 - \beta_2 M_o)(1 - l)\tau})}{c_2 + d_3 + E_1 + \beta_1 M_o - d_4 - E_2 - \beta_2 M_o} \right.
$$
$$
\left. + \frac{c_1(1 - u_2) e^{-(d_2 + \beta_2 M_o)l\tau} (1 - e^{-(c_1 + d_1 + \beta_1 M_o - d_2 - \beta_2 M_o)l\tau})}{c_1 + d_1 + \beta_1 M_o - d_2 - \beta_2 M_o} \right],
$$

$$B_1 = (1 - u_2)\mathrm{e}^{-[(d_2+\beta_2 M_o)l\tau+(c_1+d_1+\beta_1 M_o-d_2-\beta_2 M_o)(1-l)\tau]} < 1,$$

$$C_1 = (1 - u_1)\mathrm{e}^{-[(c_1+d_1+\beta_1 M_o)l\tau+(c_2+d_3+E_1+\beta_1 M_o)(1-l)\tau]} < 1.$$

Remark 8 From Theorem 7, For any $\varepsilon > 0$, there exists a positive number t_0, such that for $t > t_0$,

$$\overline{x_2(t)} - \varepsilon \le x_2(t) \le \overline{x_2(t)} + \varepsilon,$$

$$\overline{y_2(t)} - \varepsilon \le y_2(t) \le \overline{y_2(t)} + \varepsilon,$$

then

$$m_{21} \le x_2(t) \le M_{21},$$

$$m_{22} \le y_2(t) \le M_{22},$$

where

$$m_{21} = \left[x_2^* + x_2^{**}\right] - \varepsilon,$$

$$M_{21} = \left[x_2^* \mathrm{e}^{-(c_1+d_1+\beta_1 M_o)l\tau} + x_2^{**}\mathrm{e}^{-(c_2+d_3+E_1+\beta_1 M_o)(1-l)\tau}\right] + \varepsilon,$$

$$m_{22} = \left[y_2^* + y_2^{**}\right] - \varepsilon,$$

$$M_{22} = \mathrm{e}^{-(d_2+\beta_2 M_o)l\tau}\left[\frac{c_1(1 - \mathrm{e}^{-(c_1+d_1+\beta_1 M_o-d_2-\beta_2 M_o)l\tau})}{c_1 + d_1 + \beta_1 M_o - d_2 - \beta_2 M_o}x_2^* + y_2^*\right]$$

$$+ \mathrm{e}^{-(d_4+E_2+\beta_2 M_o)(1-l)\tau}\left[\frac{c_2(1 - \mathrm{e}^{-(c_2+d_3+E_1+\beta_1 M_o-d_4-E_2-\beta_2 M_o)(1-l)\tau})}{c_2 + d_3 + E_1 + \beta_1 M_o - d_4 - E_2 - \beta_2 M_o}x_2^{**} + y_2^{**}\right] + \varepsilon.$$

From the above theorems and remarks, we present an important theorem in this paper.

Theorem 9 (*i*) $aA - (1 - B)(1 - C) < 0$, *the population $x(t)$ and $y(t)$ go extinct*;
(*ii*) *If* $aA_1 - (1 - B_1)(1 - C_1) > 0$, *the system is permanent.*

Proof (i) In the condition of $aA - (1 - B)(1 - C) < 0$, the trivial periodic solution is globally asymptotically stable, that is when $t \to \infty$, we have $x(t) \to 0$ and $y(t) \to 0$. According to (3.17), (3.19) and comparison with the theorem of the impulsive equation [21], we know that $0 \le x(t) \le x_1(t) = x(t)$ and $0 \le y(t) \le y_1(t) = y(t)$. These show that the populations $x(t)$ and $y(t)$ go extinct.

(ii) By the condition $aA_1 - (1 - B_1)(1 - C_1) > 0$, it is easy to show that $aA - (1 - B)(1 - C) > 0$. According to (18)–(20) and with the comparison theorem of the impulsive equation [21], we can obtain that $\overline{x_2(t)} - \varepsilon \le x_2(t) \le x(t), x(t) \le x_1(t) \le \widetilde{x(t)} + \varepsilon$. From Remark 3 and Remark 8, we have $m_{21} \le x_2(t)$, $x_1(t) \le M_1$, then $m_{21} \le x(t) \le M_1$. Similarly, $m_{22} \le y(t) \le M_2$, From Remark 6, we have $m_o \le c_o(t) \le M_o, m_{e1} \le c_{e1}(t) \le M_{e1}, m_{e2} \le c_{e2}(t) \le M_{e2}$. This completes the proof. □

4 Numerical simulations
Using numerical simulations, we analyze the influences of E_1 and u_1 on system (2.1). If it is assumed that $x(t) = 1$, $y(t) = 0.5$, $c_o(t) = 0.5$, $c_{e1}(t) = 0.4$, $c_1 = 0.1$, $d_1 = 0.1$, $\beta_1 = 0.01$,
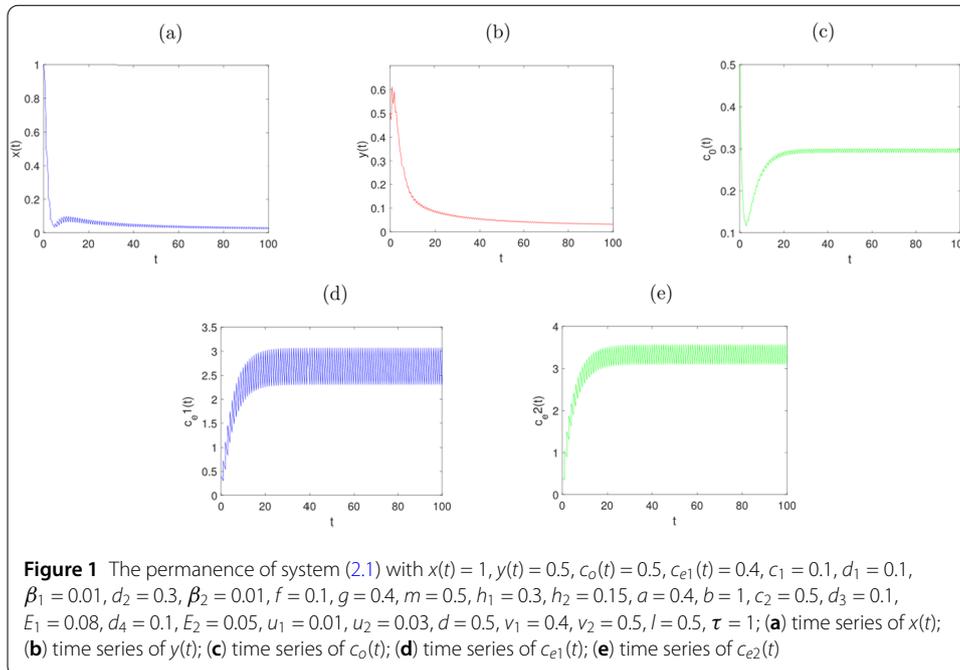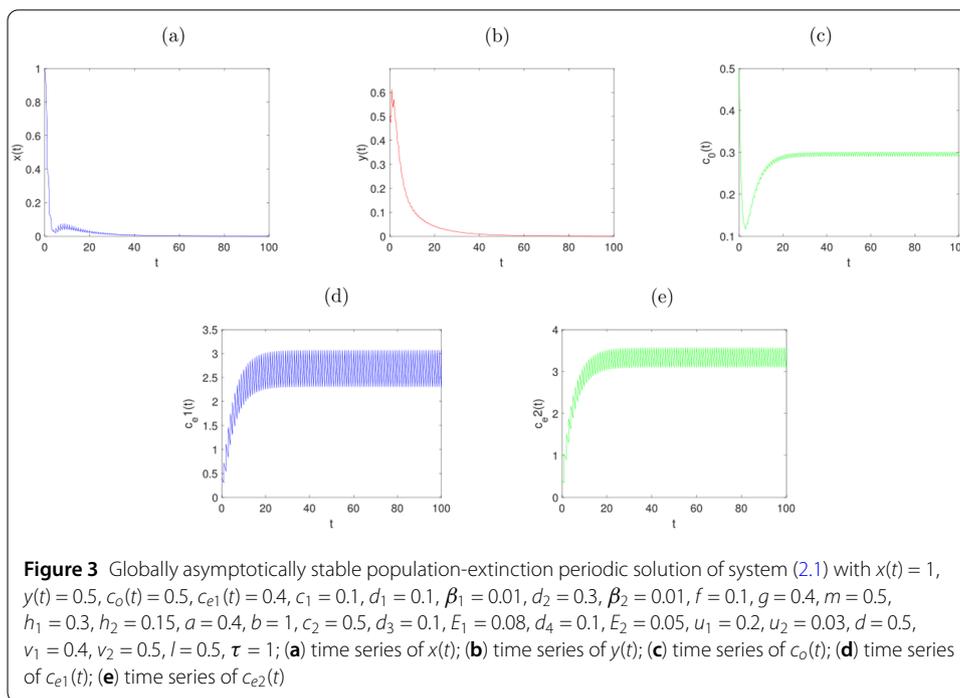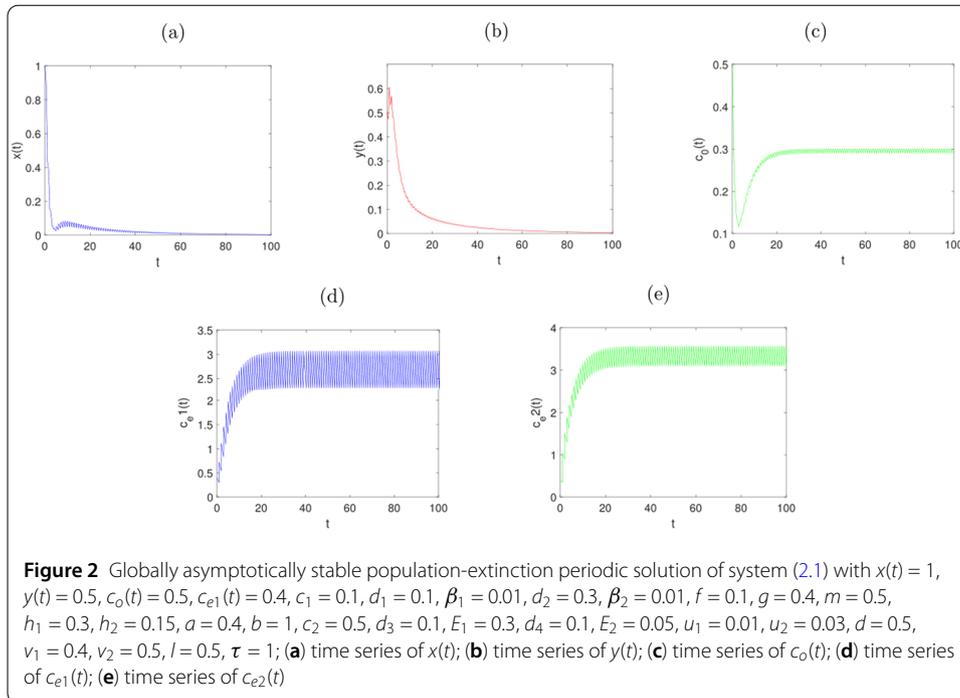
Figure 1 The permanence of system (2.1) with $x(t) = 1$, $y(t) = 0.5$, $c_o(t) = 0.5$, $c_{e1}(t) = 0.4$, $c_1 = 0.1$, $d_1 = 0.1$, $\beta_1 = 0.01$, $d_2 = 0.3$, $\beta_2 = 0.01$, $f = 0.1$, $g = 0.4$, $m = 0.5$, $h_1 = 0.3$, $h_2 = 0.15$, $a = 0.4$, $b = 1$, $c_2 = 0.5$, $d_3 = 0.1$, $E_1 = 0.08$, $d_4 = 0.1$, $E_2 = 0.05$, $u_1 = 0.01$, $u_2 = 0.03$, $d = 0.5$, $v_1 = 0.4$, $v_2 = 0.5$, $l = 0.5$, $\tau = 1$; (**a**) time series of $x(t)$; (**b**) time series of $y(t)$; (**c**) time series of $c_o(t)$; (**d**) time series of $c_{e1}(t)$; (**e**) time series of $c_{e2}(t)$

$d_2 = 0.3$, $\beta_2 = 0.01$, $f = 0.1$, $g = 0.4$, $m = 0.5$, $h_1 = 0.3$, $h_2 = 0.15$, $a = 0.4$, $b = 1$, $c_2 = 0.5$, $d_3 = 0.1$, $E_1 = 0.08$, $d_4 = 0.1$, $E_2 = 0.05$, $u_1 = 0.01$, $u_2 = 0.03$, $d = 0.5$, $v_1 = 0.4$, $v_2 = 0.5$, $l = 0.5$, $\tau = 1$, the system is permanent, as shown in Fig. 1.

4.1 The simulation of system (2.1) affected by parameter E_1

Assuming that $x(t) = 1$, $y(t) = 0.5$, $c_o(t) = 0.5$, $c_{e1}(t) = 0.4$, $c_1 = 0.1$, $d_1 = 0.1$, $\beta_1 = 0.01$, $d_2 = 0.3$, $\beta_2 = 0.01$, $f = 0.1$, $g = 0.4$, $m = 0.5$, $h_1 = 0.3$, $h_2 = 0.15$, $a = 0.4$, $b = 1$, $c_2 = 0.5$, $d_3 = 0.1$, $E_1 = 0.3$, $d_4 = 0.1$, $E_2 = 0.05$, $u_1 = 0.01$, $u_2 = 0.03$, $d = 0.5$, $v_1 = 0.4$, $v_2 = 0.5$, $l = 0.5$, $\tau = 1$, the population-extinction periodic solution $(0, 0)$ of system (2.1) is globally asymptotically stable, as shown in Fig. 2. From Figs. 1 and 2, if all parameters of system (2.1) are fixed, when $E_1 = 0.08$, we can obtain $aA - (1 - B)(1 - C) = 0.0120 > 0$, then the condition of Theorem 9(ii) is satisfied, and the system is permanent. When $E_1 = 0.3$, we can obtain $aA - (1 - B)(1 - C) = -0.0108 < 0$, then the condition of Theorem 9(i)is satisfied, and the populations $x(t)$ and $y(t)$ go extinct.

4.2 The simulation of system (2.1) affected by parameter u_1

Assuming that $x(t) = 1$, $y(t) = 0.5$, $c_o(t) = 0.5$, $c_{e1}(t) = 0.4$, $c_1 = 0.1$, $d_1 = 0.1$, $\beta_1 = 0.01$, $d_2 = 0.3$, $\beta_2 = 0.01$, $f = 0.1$, $g = 0.4$, $m = 0.5$, $h_1 = 0.3$, $h_2 = 0.15$, $a = 0.4$, $b = 1$, $c_2 = 0.5$, $d_3 = 0.1$, $E_1 = 0.08$, $d_4 = 0.1$, $E_2 = 0.05$, $u_1 = 0.2$, $u_2 = 0.03$, $d = 0.5$, $v_1 = 0.4$, $v_2 = 0.5$, $l = 0.5$, $\tau = 1$, the population-extinction periodic solution $(0, 0)$ of system (2.1) is globally asymptotically stable, as shown in Fig. 3. From Figs. 1 and 3, if all parameters of system (2.1) are fixed, when $u_1 = 0.01$, we can obtain $aA - (1 - B)(1 - C) = 0.0120 > 0$, then the condition of Theorem 9(ii) is satisfied, and the system is permanent. When $u_1 = 0.2$, we can obtain $aA - (1 - B)(1 - C) = -0.0035 < 0$, then the condition of Theorem 9(i) is satisfied, and the populations $x(t)$ and $y(t)$ go extinct.

Figure 2 Globally asymptotically stable population-extinction periodic solution of system (2.1) with $x(t) = 1$, $y(t) = 0.5$, $c_o(t) = 0.5$, $c_{e1}(t) = 0.4$, $c_1 = 0.1$, $d_1 = 0.1$, $\beta_1 = 0.01$, $d_2 = 0.3$, $\beta_2 = 0.01$, $f = 0.1$, $g = 0.4$, $m = 0.5$, $h_1 = 0.3$, $h_2 = 0.15$, $a = 0.4$, $b = 1$, $c_2 = 0.5$, $d_3 = 0.1$, $E_1 = 0.3$, $d_4 = 0.1$, $E_2 = 0.05$, $u_1 = 0.01$, $u_2 = 0.03$, $d = 0.5$, $v_1 = 0.4$, $v_2 = 0.5$, $l = 0.5$, $\tau = 1$; (**a**) time series of $x(t)$; (**b**) time series of $y(t)$; (**c**) time series of $c_o(t)$; (**d**) time series of $c_{e1}(t)$; (**e**) time series of $c_{e2}(t)$

Figure 3 Globally asymptotically stable population-extinction periodic solution of system (2.1) with $x(t) = 1$, $y(t) = 0.5$, $c_o(t) = 0.5$, $c_{e1}(t) = 0.4$, $c_1 = 0.1$, $d_1 = 0.1$, $\beta_1 = 0.01$, $d_2 = 0.3$, $\beta_2 = 0.01$, $f = 0.1$, $g = 0.4$, $m = 0.5$, $h_1 = 0.3$, $h_2 = 0.15$, $a = 0.4$, $b = 1$, $c_2 = 0.5$, $d_3 = 0.1$, $E_1 = 0.08$, $d_4 = 0.1$, $E_2 = 0.05$, $u_1 = 0.2$, $u_2 = 0.03$, $d = 0.5$, $v_1 = 0.4$, $v_2 = 0.5$, $l = 0.5$, $\tau = 1$; (**a**) time series of $x(t)$; (**b**) time series of $y(t)$; (**c**) time series of $c_o(t)$; (**d**) time series of $c_{e1}(t)$; (**e**) time series of $c_{e2}(t)$

5 Discussion

In this paper, we propose a new stage-structured population model with impulsive effects in a polluted environment. The condition for the globally asymptotic stability of the triviality periodic solution $(0, 0)$ of system (2.1) is obtained, and the permanent condition of system (2.1) is also obtained. It can be seen from the analyses that the nontransient harvesting rate and transient impulsive harvesting rate play important roles in system (2.1).

From the numerical simulation of Figs. 1 and 2, we can deduce that there must exist a nontransient impulsive harvesting population threshold E_1^*, which satisfies $0.08 < E_1^* < 0.3$. If $E_1 < E_1^*$, the system (2.1) is permanent, and if $E_1 > E_1^*$, the populations go extinct. From the numerical simulation of Figs. 1 and 3, we can also deduce that there exists a transient impulsive harvesting population threshold u_1^*, which satisfies $0.01 < u_1^* < 0.2$. If $u_1 < u_1^*$, the system (2.1) is permanent, and if $u_1 > u_1^*$, the populations go extinct. Comparing the figures, it is obvious that under the condition of system persistence, the nontransient impulsive harvesting rate E_1 and the transient impulsive harvesting rate u_1 are both smaller than under the condition of population extinction. Hence, we can protect biological diversity by reducing the amount of transient pulse harvest, or reducing the nontransient impulsive harvesting intensity.

Availability of data and materials
Data sharing is not applicable to this article as no datasets were generated or analyzed during the current study.

Declarations

Competing interests
The authors declare that they have no competing interests.

Authors' contributions
All authors read and approved the final manuscript.

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